

Governmental restrictions on Enter to industry: Is it really so bad?

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1 Introduction

The presented paper uses an additive separable utility function of general type. The main focus of this research is on welfare aspects, e.g., how many oligopolies are needed to foster the Social Welfare? The common wisdom tells us that, *ceteris paribus*, the tougher is competition (or, the more oligopolies enter to industry), the higher will be consumers' well-being. In monopolistic competitive models the only condition to cease the enter is Zero-Profit condition, which is quite natural for case of myriads small firms seeking the way to earn any profit, even very small. It would be very expensive for government to control the enter of these myriads. The case of relatively small number of firms (oligopolies) is quite different. Some types of enterprise activity require a governmental license, which may be given or withdrawn by various reasons. Typical examples of such industrial branches are cellular connection, television/radio broadcasting, air traffic, etc. This paper shows that in most cases the limitation of enter may be beneficial for social welfare, in other words, the socially optimal of competing oligopolies is strictly less than number determined by Zero-profit condition under Free entry. As a natural policy implication of this result is an apology of governmental restrictions on firms' entrance at proper level.

2 The model

Firms and consumers

Consider the one-sector economy with horizontally differentiated good and one production factor - labor. There is a continuum $[0, L]$ of identical consumers endowed with one unit of labor. The labor market is perfectly competitive and labor is chosen as the numéraire. There is a finite number $N \geq 2$ of oligopolistic firms, which means that impact of single firm to market statistics is not negligible and should be strategically taken into account by other competitors. Each variety is produced by a single firm and each firm produces a single variety, thus the horizontally differentiated good $\mathbf{x} = (x_1, \dots, x_N) \in \mathbb{R}_+^N$. To operate every firm needs a fixed requirement $f > 0$ and a marginal

requirement $c > 0$ of labor, which may be normalized to 1 without loss of generality. Wage is also normalized to 1, thus the cost of producing q_i units of variety $i \in \{1, \dots, N\}$ is equal to $f + 1 \cdot q_i$.

Consumers share the same additive preferences given by

$$U(\mathbf{x}) = \sum_{i=1}^N u(x_i) \quad (1)$$

where $u(\cdot)$ is thrice continuously differentiable, strictly increasing, strictly concave, and such that $u(0) = 0$. Following [3], we define the relative love for variety (RLV) as follows:

$$r_u(x) = -\frac{xu''(x)}{u'(x)}$$

which is strictly positive for all $x > 0$.

A consumer's income is equal to her wage plus her share in total profits. Since we focus on symmetric equilibria, consumers must have the same income, which means that profits have to be uniformly distributed across consumers. In this case, a consumer's income y is given by

$$y = 1 + \frac{1}{L} \sum_{i=1}^N \Pi_i \geq 1 \quad (2)$$

where the profit made by the oligopoly selling amount q_i of variety $i \in \{1, \dots, N\}$ at price p_i is given by

$$\Pi_i = (p_i - 1)q_i - f \quad (3)$$

Evidently, the income level varies with firms' strategies. A consumer's budget constraint is given by

$$\sum_{i=1}^N p_i x_i = y \quad (4)$$

Market equilibrium

The market equilibrium is defined by the following conditions:

- (i) each consumer maximizes her utility (1) subject to her budget constraint (4),
- (ii) each firm k maximizes its profit (3) with respect to p_k ,
- (iii) product market clearing:

$$Lx_k = q_k \quad \text{for all } k \in \{1, \dots, N\},$$

- (iv) labor market clearing:

$$Nf + \sum_{i=1}^N q_i = L.$$

Market equilibrium is *symmetric* when $q_k = q_j$, $p_k = p_j$ for all $k \neq j$.

Conditions (iii) and (iv) imply that

$$\bar{x} \equiv \frac{1}{N} - \frac{f}{L} \quad (5)$$

are the only candidate symmetric equilibrium demands for “oligopolistic” varieties.

As mentioned above, the income level influences firms’ demands, whence their profits. As a result, firms can manipulate the income level, whence their “true” demands, through their own strategies with the aim of maximizing profits (see e.g. [2]). This feedback effect is known as the *Ford effect* (see [1]).

The generalized *Bertrand equilibrium* is a vector \mathbf{p}^* such that p_i^* maximizes $\Pi_i(p_i, \mathbf{p}_{-i}^*)$ for all $i \in \{1, \dots, N\}$. Under assumption of Ford effect we obtain

$$m(N) = \frac{p(N) - 1}{p(N)} = r_u \left(\frac{1}{N} - \frac{f}{L} \right) \frac{N}{N - 1}, \quad (6)$$

while case of income-taking firms

$$m_0(N) = \frac{p_0(N) - 1}{p_0(N)} = r_u \left(\frac{1}{N} - \frac{f}{L} \right) \frac{N}{N - 1 + r_u \left(\frac{1}{N} - \frac{f}{L} \right)}. \quad (7)$$

Free Enter and Governmental restrictions

Using (6) and (7) we can calculate the firm’s profit at symmetric equilibrium and result depends on firm’s behavior: in case of Ford effect

$$\bar{\Pi}(N) = L(p(N) - 1)\bar{x}(N) - f = L \frac{r_u \left(\frac{1}{N} - \frac{f}{L} \right) - (N - 1) \frac{f}{L}}{\left(1 - r_u \left(\frac{1}{N} - \frac{f}{L} \right) \right) N - 1}$$

for income-takers

$$\bar{\Pi}_0(N) = L \frac{(1 - f/L) \cdot r_u \left(\frac{1}{N} - \frac{f}{L} \right) - (N - 1) \frac{f}{L}}{\left(1 - r_u \left(\frac{1}{N} - \frac{f}{L} \right) \right) (N - 1)} < \bar{\Pi}.$$

This means that under Ford effect more firms could enter into industry until profit drops down to zero. But is it wise to allow them to enter?

3 Consumer's Welfare under Free Enter and Governmental restrictions

Consider the following Social Welfare (actually, an indirect utility)

$$V(N) = Nu(\bar{x}) = Nu\left(\frac{1}{N} - \frac{f}{L}\right), \quad (8)$$

as a function of firm's number. In what follows we shall use notion $\varphi \equiv f/L$. The first order condition

$$u\left(\frac{1}{N} - \varphi\right) - \frac{1}{N}u'\left(\frac{1}{N} - \varphi\right) = 0$$

determines the Social optimum of firms' number $N^*(\varphi)$. It is obvious that for CES utility with $u(x) = x^\rho$ this Social optimum is

$$N^*(\varphi) = \frac{1 - \rho}{\varphi}.$$

On the other hand, the Free Entry "number" of firms, which determined by zero-profit condition is

$$\widehat{N}(\varphi) = \frac{1 - \rho}{\varphi} + 1$$

under Ford effect, while the maximum "number" of income-taking firms

$$\widehat{N}_0(\varphi) = \frac{1 - \rho}{\varphi} + \rho.$$

This means that Social optimum

$$N^*(\varphi) < \widehat{N}_0(\varphi) < \widehat{N}(\varphi) \quad (9)$$

is less than Free Entry number in both cases. Note that in case of monopolistic competition with CES utility equilibrium (Free Entry) mass of firm coincides with Social optimum. To formulate the main result of this paper consider elasticity of utility function

$$\varepsilon(x) \equiv \frac{x \cdot \partial u}{\partial x}$$

and let determine the limit value (finite or infinite)

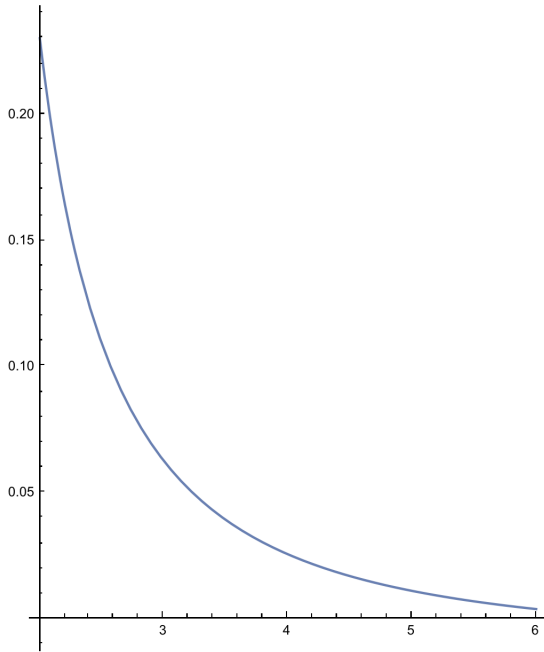
$$\varepsilon'(0) = \lim_{x \rightarrow 0} \varepsilon'(x)$$

of its derivative at zero.

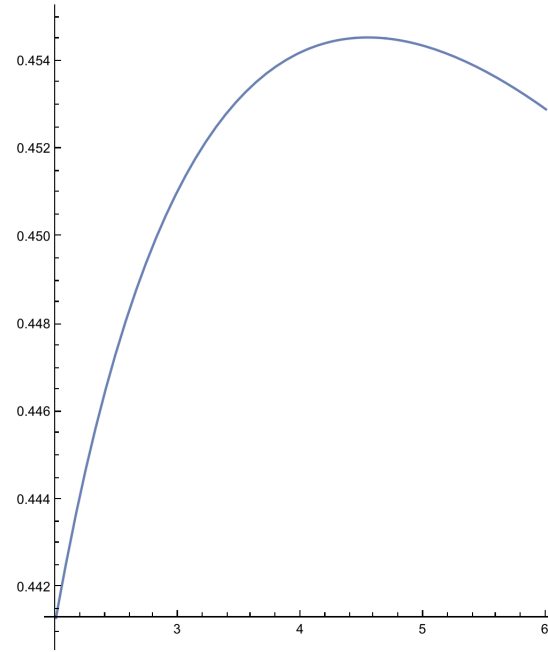
Theorem. *Assume that $\varepsilon'_u(0) < r_u(0)$ holds, then $N^*(\varphi) < \widehat{N}(\varphi)$, moreover if $\varepsilon'_u(0) < r_u(0)(1 - r_u(0))$, then $N^*(\varphi) < \widehat{N}_0(\varphi) < \widehat{N}(\varphi)$ holds for all sufficiently small φ .*

It is easy to see that both conditions, weak and strong, are satisfied for all “pro-competitive” classes of utility functions: CES $u(x) = x^\rho \Rightarrow \varepsilon'_u(0) = 0 < r_u(0) = 1 - \rho < r_u(0)(1 - r_u(0)) = \rho(1 - \rho)$, HARA $u(x) = (x + \alpha)^\rho - \alpha^\rho \Rightarrow \varepsilon'_u(0) = -(1 - \rho)/2\alpha < r_u(0) = 0 < r_u(0)(1 - r_u(0))\rho(1 - \rho) = 0$, CARA $u(x) = 1 - e^{-\alpha x} \Rightarrow \varepsilon'_u(0) = -\alpha/2 < r_u(0) = 0 < r_u(0)(1 - r_u(0))\rho(1 - \rho) = 0$, quadratic functions $u(x) = \alpha x - x^2/2 \Rightarrow \varepsilon'_u(0) = -1/2\alpha < r_u(0) = 0 < r_u(0)(1 - r_u(0))\rho(1 - \rho) = 0$.

To illustrate this result numerically, we consider the model with HARA utility $u(x) = \sqrt{x + 1} - 1$ and $\varphi = f/L = 0.01$. It is easy to see that industry may accommodate with positive profit up to 6 firms, while the optimum number is 4.



Plot of $\Pi(N)$



Plot of $V(N)$

References

- [1] d’Aspremont, C., Dos Santos Ferreira, R. and Gerard-Varet, L.: On monopolistic competition and involuntary unemployment. *The Quarterly Journal of Economics*. **105(4)**, 895–919 (1990)
- [2] Gabszewicz, J. and Vial, J.: Oligopoly à la Cournot in general equilibrium analysis. *Journal of Economic Theory*. **4**, 381–400 (1972)
- [3] Zhelobodko, E., Kokovin, S., Parenti M., and Thisse, J.-F.: Monopolistic competition in general equilibrium: Beyond the constant elasticity of substitution. *Econometrica*. **80**, 2765–2784 (2012)