Equilibrium Existence in Krugman's Trade Model

New Trade theory emerged, when Krugman (1979, 1980) and other scholars adapted Dixit and Stiglitz's (1977) models of monopolistic competition to international/interregional economy. Numerous further developments by Melitz (2003), Melitz and Ottaviano (2008) and others included heterogeneous firms into the analysis, to highlight the selection effects of trade. However, the basic Krugman's model remains an interesting approach for some research questions, e.g., microfoundations of gravity equations, and also for revealing the trade elasticity. We are interested in the general version of the model having variable elasticity of substitution (VES). This version is helpful for anaysis of pro-competitive effects alike Mrázová and Neary (2014), Bertoletti and Epifani (2014), and gravity analysis alike Arkolakis et al. (2015).

Despite the fact that Krugman's model has been exploited extensively, we are unaware of any proof of the equilibrium existence in this model. This paper provides such a proof in a sufficiently general setting. The key element of our setting is the maximal attainable operating profit function (AOP), dependent on the firm's variable cost and on some market aggregator. In particular, such a function can be generated by the traditional additive preferences with variable elasticity (see Mrázová and Neary 2014), but we suppose it to be applicaple to other approaches (e.g., the dual-additive approach by Bertoletti and Etro. (2014)). In the traditional case, the market aggregator is the consumer's marginal utility of income, and whenever the elementary utility satisfies the choke-price assumption (a finite derivative of utility at zero), then the resulting AOP must satisfy our key assumptions needed for equilibrium existence: it has to be monotonically decreasing from infinity to zero in its argument.

We include N countries, having populations $l_i, ..., l_N$ and a trade costs matrix $\{\tau_{ij}\}_{i \leq N, j \leq N}$. Under our high level of abstraction, related Krugman's model is described by the (partial) AOP $\pi(\tilde{c}_{ij}, \lambda_j)$ which denotes the amount of profit from selling an i-firm's good in country j, where the marginal cost $\tilde{c}_{ij} = \tau_{ij} w_i c_i$ depends on the firms marginal cost c_i, the wage w_j in this country and the usual "iceberg" trade coefficient $\tau_{ij} \geq 1$. Then the equilibrium equations consist of two groups: the zero profit conditions for all countries

$$\pi(x^{j1}, \ldots, x^{jK}) = \sum_{k=1}^{K} \frac{\tilde{c}_j x^{jk}}{1 - r_u(x^{jk})} \cdot l^k - f w_j \quad \forall j$$

(where $f > 0$ is a fixed cost required to start a company, measured in labor) and their trade balances

$$w_i l_i = \sum_{j=1}^{N} \left(\frac{R_{ji}}{\sum_{k=1}^{N} R_{jk}} \right) w_j l_j \quad \forall i$$

Here R_{ji} is the share of income in country j spent on the goods from the country i. Among the two groups of equilibrium variables $\{\lambda_i, w_i\}_{i \leq N}$, wages

1

can be determined only up to normalization (because we have N countries and actually only $N-1$ independent restricting conditions, therefore we need one additional equation to complete the system). That is why we can normalize wages as

$$\sum w_i = 1.$$

Generally, a system of non-linear equation need not have a solution. However, our assumptions include choke-price existance and monotonicity of the profit function, these are strong enough, remaining quite general, applicable to many particular demand systems. So, we achieve the existence proposition. If the preferences display a choke-price, and the profit function (AOP) is monotonic with respect to its argument, then N countries trade always result in at least one equilibrium: set of $\{\lambda_i, w_i\}_{i \leq N} \gg 0$ which ensures posititive wages for all the countries and non-negative source-destination profits for all directions of trade, that satisfy the equilibrium conditions.

The firther direction of study can include the question of uniqueness of the equilibrium, and probably monoponicity of the AOP will turn out sufficient for this.

The fruit of these two findings for theory is that they would justify logical consistency of using such general version of Krugman's model in theory and empirics. For empirics, they suggest, what kind of preferences (or demand structures) are consistent with theory in any attempts to calibrate the model on real data.

Arkolakis C., Costinot A., Donaldson D. and Rodríguez-Clare A. (2015) "The elusive pro-competitive effects of trade," NBER WP 21370.

Bertoletti P. and Etro F., (2014) "A general theory of endogeneous market structures", Journal of Economic Literature, ISSN: 1827-3580 No. 09/WP/2014

Bertoletti P. and Epifani P. (2014) "Monopolistic competition: CES redux?" Journal of International Economics, 93(2): 227-238.

Bykadorov I., Ellero A., Funari S., Kokovin S. and Molchanov P. (2016) "Painful Birth of Trade under Classical Monopolistic Competition," NRU Higher School of Economics, WP BRP 132/EC/2016.

Krugman P.R. (1979) "Increasing returns, monopolistic competition, and international trade," Journal of international Economics, 9(4): 469-479.

Melitz M.J. (2003) "The impact of trade on intra-industry reallocations and aggregate industry productivity," Econometrica, 71(6): 1695-1725.

Melitz M.J. and Ottaviano G.I. (2008) "Market size, trade, and productivity," The review of economic studies, 75(1): 295-316.

Mrázová M. and Neary J.P (2014) "Together at Last: Trade Costs, Demand Structure, and Welfare," American Economic Review, 104(5): 298-303.