# Information Learning in Social Networks, and Coordination Game

Zaruhi Hakobyan

National Research University Higher School of Economics, Moscow, Russia

**Abstract.** My model is the extension of Morris and Shin (2002) model.In my benchmark I try to understand the impact of social networks on information structure, find an optimal/ minimal sufficient network depends on preferences of agents. I show in which case, it will be better if your two friends know each other and show that in networks could be information loses if they know each other.
**JEL classification:** D7, D81, D82, D83, D85
**Keywords:** Economic Networks, games on Network, Coordination game

## 1 introduction

My model is an extension Morris and Shin(2002) Benchmark. Morris and Shin shows, when there is imperfect information, the welfare effects of increased public information are more equivocal:

- When the agents have no private information- so that the only source of information for the agents is the public information- then the greater precision of the public information always increases social welfare.
- However, if the agents have access to some private information, it is not always the case that greater precision of public information is desirable. Over some ranges, increased precision of public information is detrimental to welfare. Specifically, the greater the precision of the agents' private information, the more likely it is that increased provision of public information lowers social welfare.

In my benchmark I try to understand the impact of social networks on information structure, find an optimal/ minimal sufficient network depends on preferences of agents (did agents put more wait on coordination motive, or they put more wait to know the real state of the world. I used the different type of information source: the first I find the general solution for my model with public and private information which are normally distributed, then I used probabilistic information structure(which you can see, in Setup section), I used binomial distribution structure to show how agents results change than people in the same network, can get different private signals.

## 2 Setup

**Citizen payoff** There are $N$ agents. Agent $i$ chooses an action $a_i$. We use quadratic loss function for describing agents utility function.

The loss function for individual $i$ has two components. **The first component** is a standard quadratic loss in the distance between the underlying state $\theta$ and his action $a_i$. **The second component** is the "beauty contest" term. There is an externality in which an individual tries to second-guess the decisions of other individuals in the economy. The payoff function for agent $i$ is given by:

$$u_i(a_i, \theta) = -(1-r)(a_i - \theta)^2 - r[\gamma \sum_{j}^{N_i} (a_j - a_i)^2 + (1-\gamma) \sum_{k}^{N_{-i}} (a_k - a_i)^2] \tag{1}$$

$\theta$ is government type. We start with an example where $\theta$ takes value 0 with probability $\frac{1}{2}$ and 1 with probability $\frac{1}{2}$.

We assume that regime is autocratic so agents would like to overthrow the regime if they can, so if government type is 1(We assume that $\theta = 1$ government not strong enough), agents taking action to overthrow the regime. If $\theta = 0$, government strong enough, and agents will better off to take action 0.

$r$ is a constant, with $0 < r < 1$. The parameter $r$ gives the weight on this second-guessing motive. The larger is $r$; the more severe is the externality.

$\alpha$ is the weight of conformity. Agent $i$ puts more weight on guessing his friends action, then others, so $\alpha > \frac{1}{2}$.

$N_i$ agents who are in agents $i$ networks, we assume that there are no self-loops.We assume that $N_i$ depends on degree centrality.

$N_{-i}$ agents who are not in agents $i$ network.

In perfect information benchmark, we get Morris & Shin result; there is no conflict between individually rational action and the socially optimal actions. And in perfect information benchmark all agent's take action equal to $\theta$. So $a_i^* = \theta$.

## 3 Information structure:Private and Public Information Benchmark

We use Information structure which was introduced in Morris and Shin online appendix We are considering a case where agents face uncertainty about government type $\theta$.And agents get public and private signals about government type. Citizens get a public signal:

$$y = \begin{cases} \theta, & q \in [\frac{1}{2}, 1] \\ 1 - \theta, & 1 - q \end{cases} \tag{2}$$

A binary public signal is equal to the true state with probability $q \in [\frac{1}{2}, 1]$ and is incorrect with probability $1 - q$.

$y$ is a public signal in the sense that the actual realization of $y$ is common knowledge to all agents.

In addition to public signal $y$, agents observes the realisation of a private signal. For example agent $i$'s private signal is:

$$x_i = \begin{cases} \theta, & p_i \in [\frac{1}{2}, 1] \\ 1 - \theta, & 1 - p_i \end{cases} \tag{3}$$

Where each agents observe a private signal that is correct with probability $p \in [\frac{1}{2}, 1]$ and is incorrect with probability $1 - p$. We assumed that agents see the private signal of their friends(If there is a direct link between agents, they can see private signal).

Agents made their decision after observing signals. The action which they take we denote: $a_i(I)$, where $I$ is information set $(y, x_i, \sum_j x_j)$.

$$a_i = \frac{1-r}{\phi} E(\theta | I(x_i, y)) + \frac{r}{\phi} \gamma E \sum_j^{N_i} a_j | I(x_i, y) + \frac{r}{\phi}(1-\gamma) E \sum_k^{N_{-i}} a_k | I(x_i, y) \tag{4}$$

$\phi = (1 - r) + r(\gamma N_i + (1 - \gamma) N_{-i})$

Table 1: The player 1's conditional probability of state 1

| Public signal $y$ | Agent private signal $x_1$ | Friends private signal $x$ | Probability $\theta = 1$ |
|---|---|---|---|
| 0 | 0 | 0 | $\frac{(1-q)(1-p)^{N_i+1}}{qp^{N_i+1}+(1-q)(1-p)^{N_i+1}}$ |
| 0 | 1 | 1 | $\frac{(1-q)p^{N_i+1}}{q(1-p)^{N_i+1}+(1-q)p^{N_i+1}}$ |
| 1 | 0 | 0 | $\frac{q(1-p)^{N_i+1}}{q(1-p)^{N_i+1}+(1-q)p^{N_i+1}}$ |
| 1 | 1 | 1 | $\frac{qp^{N_i+1}}{qp^{N_i+1}+(1-q)(1-p)^{N_i+1}}$ |

As we assume that agent $i$ knows his friends, signal his conditional expectation that agents, who are not in his network( $N_{-i}$) observe private signal 1 is:

Agent $i$ consider one of the following strategy, which agent's not from his network can play.

Table 2: The player 1's conditional probability of $N_{-i}$ gets private signal 1

| Public signal $y$ | Agent $i$ and $N_i$ private signal | Probability $x_{N_{-i}} = 1$ |
|---|---|---|
| 0 | 0 | $\frac{qp^{N_i+1}(1-p)^{N_{-i}}+(1-q)(1-p)^{N_i+1}p^{N_{-i}}}{qp^{N_i+1}++(1-q)(1-p)^{N_i+1}}$ |
| 0 | 1 | $\frac{q(1-p)^{N}+(1-q)p^{N}}{q(1-p)^{N_i+1}+(1-q)p^{N_i+1}}$ |
| 1 | 0 | $\frac{q(1-p)^{N_i+1}p^{N_{-i}}+(1-q)p^{N_i+1}(1-p)^{N_{-i}}}{q(1-p)^{N_i+1}+(1-q)p^{N_i+1}}$ |
| 1 | 1 | $\frac{qp^{N}+(1-q)(1-p)^{N}}{qp^{N_i+1}+(1-q)(1-p)^{N_i+1}}$ |

Table 3: Strategy $N_{-i}$ can play

| Public signal $y$ | $N_{-i}$ private signal | Action |
|---|---|---|
| 0 | 0 | $1-\bar{a}$ |
| 0 | 1 | $1-\underline{a}$ |
| 1 | 0 | $\underline{a}$ |
| 1 | 1 | $\bar{a}$ |

Suppose player 1 observed public signal 1, and private signal 1. His expectation of $\theta$ would be

$$\frac{qp^{N_i+1}}{qp^{N_i+1}+(1-q)(1-p)^{N_i+1}} \tag{5}$$

His expectation his friend's action would be

$$\frac{qp^{N_j+1}}{qp^{N_j+1}+(1-q)(1-p)^{N_j+1}} \tag{6}$$

His expectation about others who are not in his network would be:

$$E(\bar{a}_k) = (\frac{qp^{N}+(1-q)(1-p)^{N}}{qp^{N_i+1}+(1-q)(1-p)^{N_i+1}})\bar{a} + (1 - \frac{qp^{N}+(1-q)(1-p)^{N}}{qp^{N_i+1}+(1-q)(1-p)^{N_i+1}})\underline{a} \tag{7}$$

For equilibrium we must have

$$\bar{a} = \frac{1-r}{\phi}\frac{qp^{N_i+1}}{qp^{N_i+1}+(1-q)(1-p)^{N_i+1}} + \frac{rN_i}{\phi}\frac{qp^{N_j+1}}{qp^{N_j+1}+(1-q)(1-p)^{N_j+1}} + \frac{rN_k}{\phi}E(\bar{a}_k) \tag{8}$$

Similarly, agent $i$ observed public signal 1, and private signal 0. His expectation of $\theta$ would be

$$\frac{q(1-p)^{N_i+1}}{q(1-p)^{N_i+1}+(1-q)p^{N_i+1}} \tag{9}$$

His expectation his friend's action would be

$$E(a_j) = \frac{q(1-p)^{N_j+1}}{q(1-p)^{N_j+1}+(1-q)p^{N_j+1}} \tag{10}$$

His expectation about others who are not in his network would be:

$$E(\underline{a}_k) = \frac{q(1-p)^{N_i+1}p^{N_{-i}}+(1-q)p^{N_i+1}(1-p)^{N_{-i}}}{q(1-p)^{N_i+1}+(1-q)p^{N_i+1}}\bar{a} + (1 - \frac{q(1-p)^{N_i+1}p^{N_{-i}}+(1-q)p^{N_i+1}(1-p)^{N_{-i}}}{q(1-p)^{N_i+1}+(1-q)p^{N_i+1}})\underline{a} \tag{11}$$

For equilibrium, we must have

$$\underline{a} = \frac{1-r}{\phi}\frac{q(1-p)^{N_i+1}}{q(1-p)^{N_i+1}+(1-q)p^{N_i+1}} + \frac{rN_i}{\phi}E(a_j) + \frac{rN_{-i}}{\phi}E(\underline{a}_k) \tag{12}$$

From equation (30) and (34) we can find $\bar{a}$ and $\underline{a}$, and then we can do welfare analyze.

$$W[\alpha, r, q, p, N] = -\frac{1}{2}[qp^N(1-\bar{a})^2 + q(1-p)^N(1-\underline{a})^2 + (1-q)p^N(\underline{a})^2 + (1-q)(1-p)\bar{a}^2 + \\ + qp^N(\bar{a}-1)^2 + q(1-p)^N(\underline{a}-1)^2 + (1-q)p^N(-\underline{a})^2 + (1-q)(1-p)^N(-\bar{a})^2] \quad (13)$$

Let's consider the network with three agents.

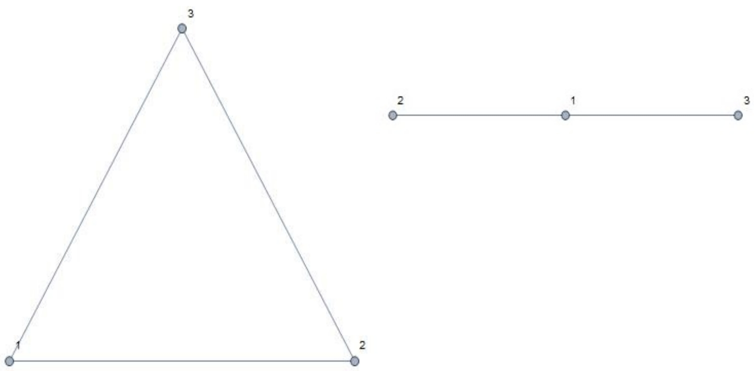



Fig. 1: Network with 3 agents. Agent number 1 have two friends who knows each other, and the second networks 1'st agent friends doesn't know each other.

Let's consider the first case: Agent number 1 have two friends who know each other. So the probability that 1'st agent see the public signal 1, and private signal 1 is the follows $\frac{qp^3}{qp^3+(1-q)(1-p)^3}$.

The optimal action which takes each agent, as follows:

$$\begin{aligned} \bar{a}(I) &= \frac{qp^3}{qp^3 + (1-q)(1-p)^3} \\ \underline{a}(I) &= \frac{q(1-p)^3}{q(1-p)^3 + (1-q)p^3} \end{aligned} \quad (14)$$

The exciting thing we get, in 3 agent problem, our result doesn't depend on r, because if the population consist of 3 agents, and everyone knows each other, it means $\gamma = 1$. And the optimal action doesn't depend on coordination motive.

What is welfare, i.e., the expected value of $-(a_i - \theta)^2$ under this strategy?

$$\begin{aligned} W_1 = &-[qp^3(1-\bar{a})^2 + (1-q)p^3\underline{a} + (1-q)(1-p)^3\bar{a} + q(1-p)^3\underline{a}] = \\ &= -\frac{(1-p)^9 q^3}{(p^3(1-q) + (1-p)^3 q)^2} - \frac{p^3(1-p)^6(1-q)q^2}{(p^3(1-q) + (1-p)^3 q)^2} - \\ &- p^3 q\left(1 - \frac{p^3 q}{p^3 q + (1-p)^3(1-q)}\right)^2 - \frac{p^6(1-p)^3(1-q)q^2}{(p^3 q + (1-p)^3(1-q))^2}] \end{aligned} \quad (15)$$

So solving the welfare, we can find that the maximum value for welfare will be equal to 0. Look Appendix.

Public information is damaging($\frac{dW(p,q)}{dq} < 0$) everywhere, beside the following situation

$$\begin{aligned} \frac{1}{2}(\sqrt{5}-1) < p < 1 \\ 0.45 < q < 0.55 \end{aligned} \quad (16)$$

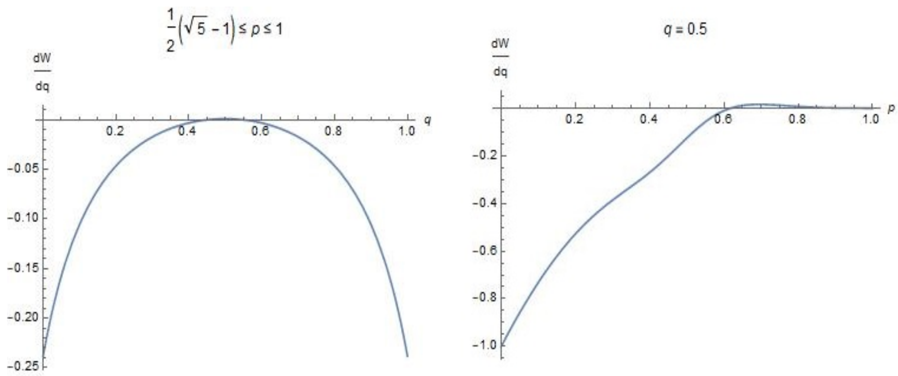



Fig. 2: Comparative statics $\frac{dW}{dq}$ depends on $p, q$

Increased probability of public information is beneficial only when the probability of private information of the agents is not very high. If the agents have access to a very high probability of private information, then any increase in the probability of the public information will be harmful. Thus, as a rule of thumb, when the private sector agents are already very well informed, the official sector would be well advised to keep the probability of public information closer to $\frac{1}{2}$. Than Welfare will be maximum.

Let's consider the second network structure. Now network consist of 3 agents.

The first agent will play the same strategy as agents in the first network structure. So the optimal action which takes agent number 1 will look like as equation (14).

But Agent number 2 and number 3 will take a different action than 1st agents. Optimal action for both actions is the same, because they have one neighbor, whose signal they can see, and there are one more agents whose signal they can't see.

$$\begin{aligned}\bar{a} &= (1-r)\frac{qp^2}{qp^2+(1-q)(1-p)^2} + r\frac{qp^3}{qp^3+(1-q)(1-p)^3} + r\bar{a}\frac{qp^3+(1-q)(1-p)^3}{qp^2+(1-q)(1-p)^2} + \\ &\quad + r\underline{a}(1-\frac{qp^3+(1-q)(1-p)^3}{qp^2+(1-q)(1-p)^2}) \\ \underline{a} &= (1-r)\frac{q(1-p)^2}{q(1-p)^2+(1-q)p^2} + r\frac{q(1-p)^3}{q(1-p)^3+(1-q)p^3} + r\bar{a}\frac{q(1-p)^2p+(1-q)p^2(1-p)}{q(1-p)^2+(1-q)p^2} + \\ &\quad + r\underline{a}(1-\frac{q(1-p)^2p+(1-q)p^2(1-p)}{q(1-p)^2+(1-q)p^2})\end{aligned} \tag{17}$$

We can find $\bar{a}$ and $\underline{a}$. And then we can use $\bar{a}$ and $\underline{a}$ and calculate welfare:

$$\begin{aligned}W_2[\alpha, r, q, p, N] &= -\frac{1}{2}[qp^N(1-\bar{a})^2 + q(1-p)^N(1-\underline{a})^2 + (1-q)p^N(\underline{a})^2 + (1-q)(1-p)\bar{a}^2 + \\ &\quad + qp^N(\bar{a}-1)^2 + q(1-p)^N(\underline{a}-1)^2 + (1-q)p^N(-\underline{a})^2 + (1-q)(1-p)^N(-\bar{a})^2]\end{aligned} \tag{18}$$

Depends on $p, q, r$ Welfare looks the different way, but we can see that Welfare is closer to optimum, than $r$ is closer to 0.

Figure 1 shows $W(p)$ than $q$ and $r$ are small enough.

Figure 2 shows that if we raise the coordination motive, Welfare will always be low 0.

In Figure 3 the graph number 3 shows, $W(q)$ then $p = 0.8, q = 0.6$, So we can see, if the probability of public and private signal is high enough, Welfare will be maximum if r is close to 0. We can see, that Welfare can be maximized if the probability of private signal is high enough, in that time probability of public signal is small.

Let's do comparative statics, which shows, how welfare depends on $p, q, r$.

Let's made a comparative statics, do the second network structure is better than the first one. Let's calculate $W_2 - W_1$ and find the intervals than welfare in the second case is higher than the welfare in the first case.

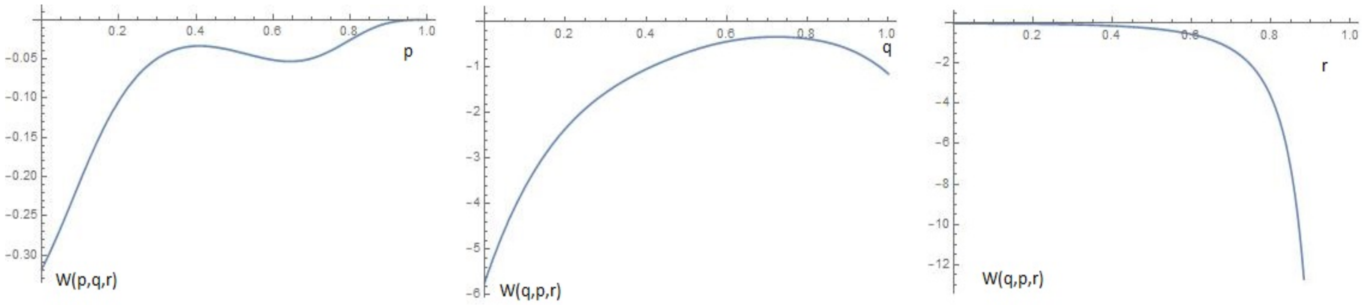



Fig. 3: Welfare depends on $q, p, r$

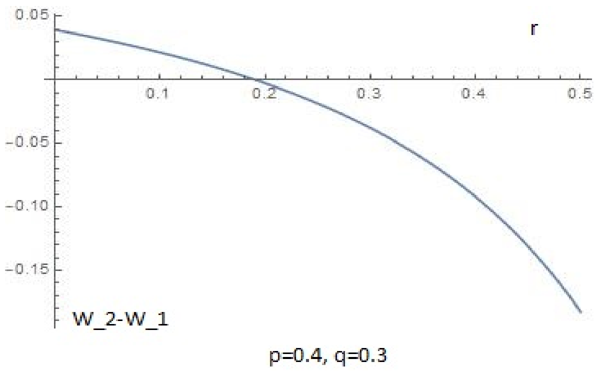



Fig. 4: Welfare on second network structure is higher than the coordination motive is very low.

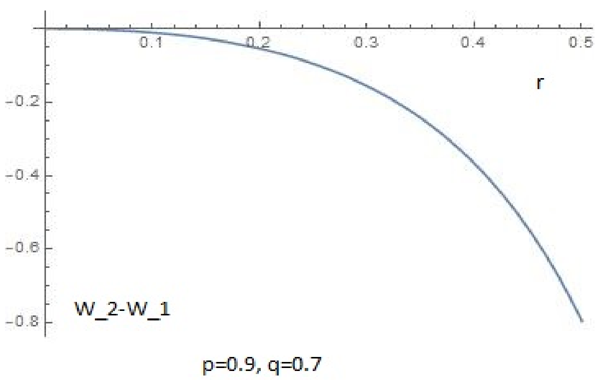
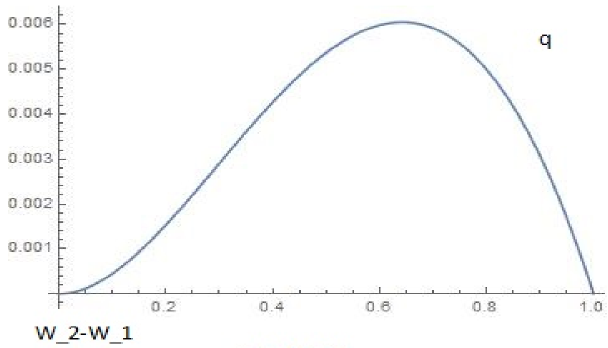


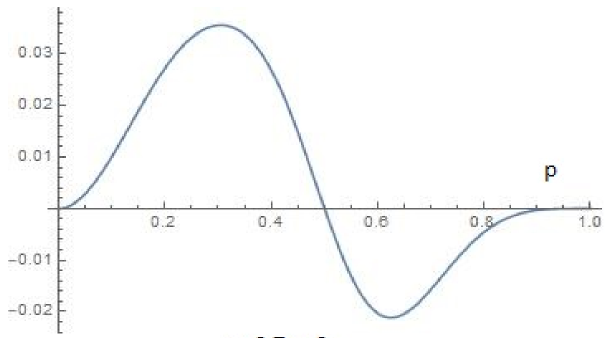

Fig. 5: If $q, p$ is high enough, the second network structure will be chosen only if $r$ is low

## References

# 4 Appendix




p=o.48, r=0




q=0.7, r=0