Confidence Sets for the Break Date in Cointegrating Regressions

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Abstract

This paper proposes constructing a confidence set for the change point in cointegrating regressions. Cointegration has long been an important concept for investigating the long-run relationships among macroeconomic variables. To capture the long-run relationship, data over relatively long time-frames are often used in such an investigation. In this case, we should take into account that the economic structure may change during the sample period. For example, Campos, Ericsson, and Hendry (1996) investigate the effect of structural change on cointegration tests and Gregory and Hansen (1996a, b) propose tests for the null hypothesis of no cointegration that are robust to the existence of structural change, while tests for the null hypothesis of cointegration with a structural break are proposed by Carrion-i-Silvestre and Sansó (2006) and Arai and Kurozumi (2007). On the contrary, tests for structural change in the framework of cointegrating regressions have been proposed by Bai, Lumsdaine, and Stock (1998, BLS hereafter) and Kejriwal and Perron (2010). By using the tests presented in the literature in addition to the careful inspection of original data and economic events, we may find cointegrating relations with structural change. In this case, a statistical inference about the change point can be made by using the method proposed by BLS in the case of a single break, while multiple breaks were investigated by Kejriwal and Perron (2008a).

In the case of regressions using stationary variables, the break point is estimated by minimizing the sum of the squared residuals or by using the quasi-maximum likelihood method, while the confidence interval is constructed by using the limiting distribution of the break point estimator, as suggested by Bai (1997) and Bai and Perron (1998). In this case, the

crucial assumption made for the construction of the confidence interval is that the magnitude of the structural break shrinks to 0 at a rate slower than $1/\sqrt{T}$, as also assumed in BLS and Kejriwal and Perron (2008a). However, as demonstrated by Elliott and Müller (2007) and Chang and Perron (2015), a confidence interval based on the limiting distribution of the break point estimator tends to be too liberal when the magnitude of the break is not so large. Instead of using the limiting distribution of the change point estimator, Elliott and Müller (2007) propose constructing a confidence interval by inverting the test for the break location, which helps control the coverage rate. However, the drawback of their method, as pointed out by Chang and Perron (2015), is that the confidence interval tends to be too wide. Indeed, it covers most of the sample period in some cases, thereby offering no useful information in practice. To overcome this drawback, Yamamoto (2016) pays attention to the estimation of the long-run variance for the construction of the test for the break location and proposes estimating it by taking the estimated break point into account, while Kurozumi and Yamamoto (2015) consider a similar method to Elliott and Müller (2007) but propose inverting the sup-type, average-type, and exponential-type tests for the break location, which can be obtained by maximizing the average power of a test. By Monte Carlo simulations, it is shown that these methods can better control the coverage rate and that the length of the confidence set becomes close to or smaller than that based on Bai (1997). On the contrary, Eo and Morley (2015) investigate a confidence set based on the likelihood ratio, while Harvey and Leybourne (2015) propose constructing a confidence set for the date of a break in level and trend that is valid for both I(0) and I(1) processes. Further, Kurozumi (2016) extends the method of Kurozumi and Yamamoto (2015) to linear regression models with non-homogeneous regressors, particularly with a linear trend.

As in the above case of stationary regressions, controlling the coverage rate of the confidence interval of the break date in the case of cointegrating regressions may be difficult based on the methods of BLS and Kejriwal and Perron (2008a). Indeed, the simulation results reported in these papers are not necessarily satisfactory. Therefore, in this paper, we propose constructing a confidence set by inverting the test for the break location in cointegrating regressions. In this case, while the basic structure of the test is the same as in the case of regressions with stationary regressors, the limiting distribution becomes different. We derive the asymptotic distribution of the test depending on whether a linear trend is included in the regressions and/or the coefficient associated with the I(1) regressors sustain a structural change. As in the case of stationary regressions, the critical values depend on the location of the break fraction under the null hypothesis and it is inconvenient to tabulate them for all the permissible break fractions in practical analysis. Instead, as in Kurozumi and Yamamoto (2015), we conduct response surface regressions and propose obtaining the critical values of the test for the break location in a simple formula. By Monte Carlo simulations, we show that our method can control the coverage rate of the confidence set better than BLS, while the size of the confidence set based on our method is comparable to that of BLS.