NATIONAL RESEARCH UNIVERSITY

Department of theoretical economic

DECISION MAKING UNDER RISK: TV SHOW ANALYSIS

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Decision criteria:

Lottery A is chosen over B when $Eu_A > Eu_B$,

$$Eu_j = \sum_{i=1}^{N^j} p_i^j u\left(x_i^j + W\right), j = A, B$$

Where W reflects the individual initial wealth, p_i^j is an objective probability of outcome $x_i^j$and $u\left(x_i^j\right)$ is a value of Bernoulli utility function $u(x)$ at point $x = x_i^j$

PROSPECT THEORY

Kahneman & Tversky approach

Utility function $u(x)$ → Value function $v(x)$

Probability p_i → Decision weight $w(p_i)$

Initial wealth W → Reference point RP

Decision criteria:

Prospect A is chosen over B when

$$\sum_{i=1}^{N^A} w(p_i^A)v(x_i^A) > \sum_{i=1}^{N^B} w(p_i^B)v(x_i^B)$$

Kahneman, Daniel & Tversky, Amos, 1979. "Prospect Theory: An Analysis of Decision under Risk," Econometrica, Econometric Society, vol. 47(2), pages 263-291

DECISION WEIGHTS

First generation prospect theory

People tend to overweight most probable and underweight least probable outcomes

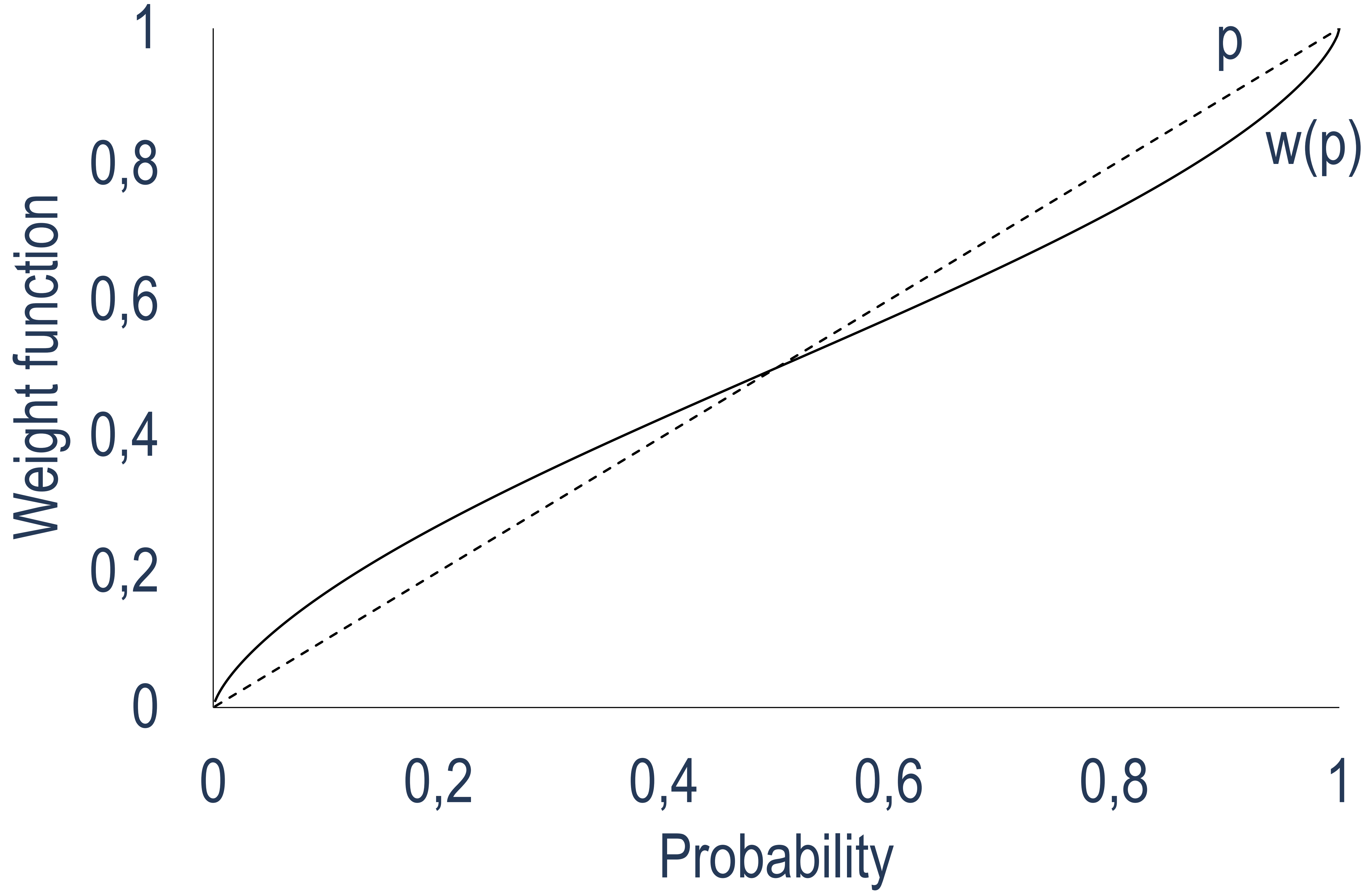
Possible weight functions:

$$w(p_i) = \frac{{p_i}^{(1-\gamma_1)}}{\left(p_i^{(1-\gamma_1)} + (1-p_i)^{(1-\gamma_1)}\right)^{1/(1-\gamma_1)}}$$

$$w(p_i) = \frac{(1-\gamma_2){p_i}^{(1-\gamma_1)}}{(1-\gamma_2)p_i + (1-p_i)^{(1-\gamma_1)}}$$

$$w(p_i) = \exp\left(-(1-\gamma_2)(-\ln p_i)^{(1-\gamma_1)}\right)$$

Problems: stochastic dominance violation, interpretability – weights are not probabilities



Diminishing sensitivity to changes in probabilities moving further away points $p = 0$ and $p = 1$ causes S-shaped form of weight function

DECISION WEIGHTS

Rank-dependent expected utility approach

Cumulative probabilities instead of individual probabilities

$$x_1 \leq \cdots \leq x_N$$

$$w(p_i) = \widetilde{w}\left(\sum_{i=2}^{N} p_i\right) - \widetilde{w}\left(\sum_{i=1}^{N} p_i\right)$$

Advantage: allows to avoid violation of stochastic dominance

Problem: possible transitivity violation

Possible functions $\widetilde{w}(.)$:

$$\widetilde{w}(p_i) = p_i^{(1-\gamma_1)}$$

$$\widetilde{w}(p_i) = \frac{p_i^{(1-\gamma_1)}}{\left(p_i^{(1-\gamma_1)} + (1-p_i)^{(1-\gamma_1)}\right)^{1/(1-\gamma_1)}}$$

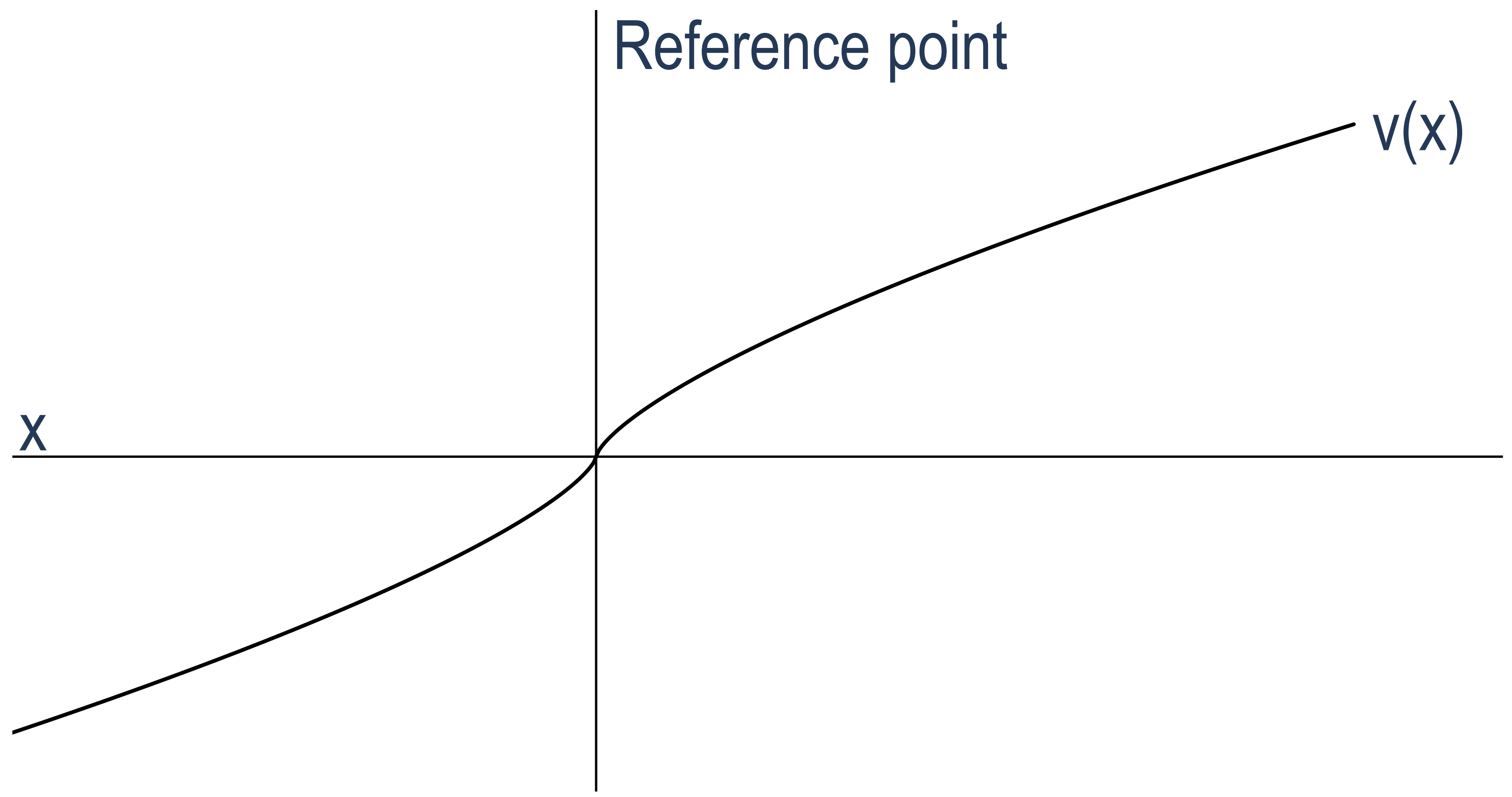
$$\widetilde{w}(p_i) = \frac{(1-\gamma_2)p_i^{(1-\gamma_1)}}{(1-\gamma_2)p_i + (1-p_i)^{(1-\gamma_1)}}$$

$$\widetilde{w}(p_i) = \exp\left(-(1-\gamma_2)(-\ln p_i)^{(1-\gamma_1)}\right)$$

Quiggin, J. (1982), ‘A theory of anticipated utility’, Journal of Economic Behavior and Organization 3(4), 323–43

PROSPECT THEORY APPROACH

Value function and reference point



Different behavior depending on reference point:

$$v(x) = \begin{cases} -(1-\theta_2)(RP-x)^{1-\theta_1}, x \leq RP \\ (RP-x)^{1-\theta_1}, x > RP \end{cases}$$

Reference point determinants

Static: wealth, expectations, prizes achieved by other individuals for the same tasks

Dynamic: may depend on the game structure

$$RP = \underbrace{\tau_1}_{\text{Static part}} + \underbrace{\tau_2 \sum_{i=1}^{N} g_i x_i}_{\text{Dynamic part}}$$

Particularly, g_i may be:

- Objective probabilities p_i
- Subjective probabilities π_i
- Decision weights w_i

SUBJECTIVE PROBABILITIES

Probabilities driven by beliefs in luck

Individual believes that he is

Lucky ☺	**Neither 😐**	**Unlucky ☹**
desired (undesired) outcomes are subjectively more (less) likely	subjective and objective probabilities overlap	desired (undesired) outcomes are subjectively less (more) likely

Formalization

Let subjective probabilities parameter $\lambda \in R$ be individual's level of luck.

$$P\left(X_{\lambda_2} \geq x\right) \geq P\left(X_{\lambda_1} \geq x\right) \Leftrightarrow \lambda_2 \geq \lambda_1 \ \forall x \in R$$

$$(\forall x \in R: P(X = x) = P(X_\lambda = x) \Leftrightarrow \lambda = 0)$$

Stochastic dominance condition: More lucky person subjective probability to get more then x not less then the same subjective probability for unlucky person

Objectivity condition: individual subjective beliefs are objective if and only if he believes that he has zero level of luck

Dolgikh S. I. The influence of subjective beliefs in luck on the decision-making under risk: TV show analysis. *Applied Econometrics*, 2019, v. 56, pp. 74–98.

SUBJECTIVE PROBABILITIES

Beliefs in luck dynamics

Individual

Believes in lucky streak	Neither	Subjects to gamblers fallacy
desired (undesired) outcomes indicate that he is more luck that used to think	Nothing may affect his luck	desired (undesired) outcomes indicate that he becomes less (more) lucky since he is wasted (gain) some luck

Formalization

Consider individual's beliefs in luck at period 0 and at period 1 which are $\lambda^{(0)}$ and $\lambda^{(1)}$ correspondingly and x is his outcome at period 0 (the same reevaluation logic holds for any two consequent periods)

$$\lambda^{(1)}(x) = \lambda^{(0)} + \tilde{\lambda}\underbrace{\left(P\left(X_{\lambda^{(0)}} \leq x\right) - 0.5\right)}_{\text{Events related to extreme values cause greater luck reevaluation}} \forall x \in R$$

- If $\tilde{\lambda} > 0$ then **lucky streak**
- If $\tilde{\lambda} < 0$ then **gamblers fallacy**
- If $\tilde{\lambda} = 0$ then **neither**

Dolgikh S. I. The influence of subjective beliefs in luck on the decision-making under risk: TV show analysis. *Applied Econometrics*, 2019, v. 56, pp. 74–98.

DEAL OR NO DEAL

Rules

Before game starts several amounts of money are laid out in the identical briefcases.

Game stages:

- Contestant starts to open briefcases. Sums in the unopened briefcases are eliminated from the game.
- After a few briefcases are opened contestant faces proposition to stop the game in exchange for fixed sum of money. If contestant agrees, game stops.
- If contestant rejects the offer, he/she continues to open briefcases until the next offer.
- If contestant rejects all propositions, he/she wins sum of money from the very last briefcase.

THE MODEL

Decision making process

Decision making process:

$$d_{jr} = sv_{jr} - cv_{jr} + \varepsilon_{jr}$$

$$cv_{jr} = \sum_{i=1}^{N_{r+1}} g_i v\left(b_i^j\right)$$

$$sv_{jr} = v\left(b_r^j\right)$$

where b_i^j, $i = 1, \dots, N_{r+1}$ — possible bank offer in round $r+1$ for player j ;

b_r^j — current bank offer in round r ;

ε_{jr} — random error reflecting the idea of limited computation abilities.

SPECIFICATION

Utility (value) functions

Utility function:

$$u(x) = \frac{(x+W)^{1-\theta_1}}{1-\theta_1}$$

Value function:

$$v(x) = \begin{cases} -(1-\theta_2)(RP-x)^{1-\theta_1}, x \leq RP \\ (RP-x)^{1-\theta_1}, x > RP \end{cases}$$

W – initial wealth; RP – reference point;
θ_1, θ_2 – estimated parameters

SPECIFICATION

Decision weights and subjective probabilities

Decision weights

Cumulative approach:

$$x_1 \leq \cdots \leq x_N$$

$$w(p_i) = \widetilde{w}\left(\sum_{i=2}^{N} p_i\right) - \widetilde{w}\left(\sum_{i=1}^{N} p_i\right)$$

RD1: $\widetilde{w}(p_i) = {p_i}^{(1-\gamma_1)}$

RD2: $\widetilde{w}(p_i) = \frac{{p_i}^{(1-\gamma_1)}}{\left(p_i^{(1-\gamma_1)}+(1-p_i)^{(1-\gamma_1)}\right)^{1/(1-\gamma_1)}}$

RD3: $\widetilde{w}(p_i) = \frac{(1-\gamma_2){p_i}^{(1-\gamma_1)}}{(1-\gamma_2)p_i + (1-p_i)^{(1-\gamma_1)}}$

RD4: $\widetilde{w}(p_i) = \exp\left(-(1-\gamma_2)(-\ln p_i)^{(1-\gamma_1)}\right)$

Subjective probabilities (model with luck)

LUCK1: $\pi(x_{ki}) = \frac{x_{ki}^{\lambda}}{\sum_{j=1}^{N} x_{kj}^{\lambda}}$

LUCK2: $\pi(x_i) = \frac{x_{ki}^{\lambda(k)}}{\sum_{j=1}^{N} x_{kj}^{(k)}}$,

$$\lambda^{(k)}(y_{ki}) = \lambda^{(k-1)} + \tilde{\lambda}\left(\sum_{z:z\leq y_{ki}} \frac{z^{-\lambda}}{\sum_{j=1}^{N} y_{kj}^{-\lambda}} - 0.5\right)$$

k – the round,

x_{ik} – possible bank offer (greater – better),

y_{ik} – possible briefcase to be opened (greater – worse)

z – notation for the briefcases lower then y_{ik}

$\gamma_1, \gamma_2, \lambda, \tilde{\lambda}$ – estimated parameters

SPECIFICATION

Reference point

$$RP = \tau_1 + \tau_2 \sum_{i=1}^{N} g_i x_i$$

Specification for g_i:

NM: objective probabilities p_i

RD1-RD4: respective decision weights w_i respectively

LUCK1-LUCK2: subjective probabilities π_i

τ_1, τ_2 – estimated parameters

ESTIMATION RESULTS

Decision weights, no reference point

Parameters	NM	RD1	RD2	RD3	RD4	LUCK1	LUCK2
Utility (value) function parameters							
θ_1	0.71065*** (0.17131)	1.17819*** (0.38220)	0.95118*** (0.17688)	1.24519*** (0.39365)	1.24050*** (0.38908)	0.88132*** (0.22805)	0.22598 (0.46747)
Probability (weight) function parameters							
γ_1/λ		-0.29346 (0.20699)	0.35667*** (0.08496)	0.22471 (0.20801)	0.27326** (0.13005)	0.02612 (0.02391)	0.01702*** (0.00556)
$\gamma_1/\tilde{\lambda}$				0.38005** (0.17557)	-0.24829 (0.22697)		-0.04171** (0.01653)
Calculation limitation parameter							
σ	0.33724*** (0.04904)	0.45642*** (0.11780)	0.29253*** (0.03563)	0.38862*** (0.11767)	0.38779*** (0.11596)	0.34293*** (0.05069)	0.38653*** (0.05214)
Log-likelihood	-50.41055	-49.22687	-49.06713	-48.35136	-48.37300	-50.04792	-47.14572
AIC	104.8211	104.45374	104.13425	104.70272	104.74599	106.09585	102.29144
Prediction, %	92.98893	92.98893	93.726937	93.726937	93.726937	93.726937	92.98893
Obs.	271	271	271	271	271	271	271

ESTIMATION RESULTS

Decision weights, reference point

Parameters	NM	RD1	RD2	RD3	RD4	LUCK1	LUCK2
Utility (value) function parameters							
θ_1	0.29016*** (0.07399)	0.34884*** (0.11223)	0.29337*** (0.06545)	0.32957*** (0.10132)	0.32671*** (0.10262)	0.29381*** (0.08525)	0.18122 (0.14785)
θ_2	-0.27422 (0.41698)	-0.57823 (0.73865)	-0.43221 (0.42957)	-0.60361 (0.64469)	-0.60254 (0.62949)	-0.31420 (0.56574)	-0.48224 (0.67676)
Probability (weight) function parameters							
γ_1/λ		-0.18397 (0.19205)	0.27586*** (0.09406)	0.18084 (0.20124)	0.19902 (0.12936)	0.00532 (0.01978)	0.01906** (0.00870)
$\gamma_1/\tilde{\lambda}$				0.23171 (0.17193)	-0.11247 (0.17998)		-0.03116** (0.01363)
Reference point parameters							
τ_1	2350.81988*** (14.50853)	2382.15903*** (55.89669)	2378.44535*** (23.18804)	2387.57552*** (55.64566)	2387.42998*** (54.00091)	2362.08838*** (122.36463)	2465.11587*** (248.96771)
Calculation limitation parameter							
σ	0.30922*** (0.05462)	0.37380 (0.10334)	0.26954*** (0.04922)	0.31907*** (0.09926)	0.31710*** (0.09978)	0.31018*** (0.05753)	0.33068*** (0.05816)
Log-likelihood	-46.98635	-46.35211	-46.0849	-45.78728	-45.80738	46.96399	-44.85574
AIC	101.97271	102.70423	102.1698	103.57456	103.61476	103.92799	101.71148
Prediction, %	93.357934	92.98893	92.619926	92.619926	92.619926	92.98893	93.357934
Obs.	271	271	271	271	271	271	271

ESTIMATION RESULTS

Decision weights, flexible reference point

Parameters	NM	RD1	RD2	RD3	RD4	LUCK1	LUCK2
Utility (value) function parameters							
θ_1	0.27644*** (0.08482)	0.33407*** (0.11647)	0.29726*** (0.07757)	0.33122*** (0.10075)	0.33178*** (0.11380)	0.28032*** (0.08307)	0.17799 (0.12040)
θ_2	-0.32836 (0.48705)	-0.68441 (0.88934)	-0.64391 (0.52725)	-0.57170 (0.63712)	-0.57733 (0.71331)	-0.37797 (0.52334)	-0.49240 (0.52315)
Probability (weight) function parameters							
γ_1/λ		-0.19946 (0.20231)	0.35610*** (0.07750)	0.33097* (0.17422)	0.31125** (0.13983)	0.00662 (0.01359)	0.01932** (0.00937)
$\gamma_1/\tilde{\lambda}$				0.25943 (0.17027)	-0.10214 (0.18677)		-0.03120** (0.01485)
Reference point parameters							
τ_1	1993.76035*** (112.11640)	1990.40256*** (116.42087)	1843.69801*** (35.88141)	1832.84866*** (168.75698)	1824.75033*** (112.05913)	2014.70510*** (46.53141)	2298.94751*** (52.45076)
τ_2	0.06759*** (0.01208)	0.06104*** (0.01348)	0.06015*** (0.01194)	0.05503*** (0.02010)	0.05857*** (0.01942)	0.06240*** (0.00298)	0.02094*** (0.00366)
Calculation limitation parameter							
σ	0.30898*** (0.06067)	0.37748*** (0.10920)	0.27029*** (0.05951)	0.29899*** (0.09677)	0.30218*** (0.10216)	0.30976*** (0.06031)	0.33014*** (0.05701)
Log-likelihood	-46.91616	-46.19612	-44.8677	-44.83815	-44.91799	-46.65439	-43.60574
AIC	103.83232	104.39224	101.73541	103.67631	103.83597	105.30878	101.21149
Prediction, %	93.357934		92.250923	92.250923	92.250923	92.98893	93.726937
Obs.	271	271	271	271	271	271	271

CONCLUSIONS

The main findings

- No statistical evidence for loss aversion effect have been found
- Decision weights and subjective probabilities seems to describe individual behavior better than objective probabilities
- Game process matters: individuals may change their reference point and beliefs in luck depending on the results they previously demonstrated
- According to the model estimation results, individuals exhibit gambler’s fallacy effect

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