Abstract

Nowadays researchers face a challenge when explaining how oligopolies operate on the market. The Cournot and Bertrand models give an elegant starting point in the consideration of the competition between oligopolies. As the number of observed oligopolies is regulated by market forces and they co-exist with small firms, more realistic description of their co-existence requires other tools, one of which is the monopolistic competition theory. In this talk, we present a possible way to model an industry that produces a differentiated good under a monopolistic competition. We claim that big firms behave strategically. As a result, they underproduce their varieties of the differentiated good, which is well known from classical descriptions of monopolies. In equilibrium, big firms earn the same profit per variety as small firms do: big firms exploit their comparative advantage in costs assumed in the model setting, whereas small firms benefit from strategical behavior of big firms even more than the big firms themselves. The balance of firms' profits prevents firms from the expansion, coalescence, and fragmentation. Depending on parameters, the monopolistic competition in the model ends up with two quantitatively different stable outcomes that are characterized by the absence of big firms and by a few amount of big firms. The first outcome is known from a general theory of monopolistic competition. The second outcome describes the co-existence of big and small firms. We emphasis that if the number of big firms is small they expand to large oligopolies and control a majority of the market. In this case oligopolies have incentives to push small firms out of the market by non-market methods. This explains an economic mechanism that stays behind the lobbying.

How many oligopolies operate within monopolistically competitive industries

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1 Model

1.1 Economy

We consider a single-sector economy that produces a differentiated good. The production side is represented by single and multi-product firms. Multi-product firms operate as conglomerates of single-product firms. Associating varieties of the differentiated good, which has the mass N, with points of segment $[0, N]$, we prescribe points — that have measure 0 — to single-product firms and segments — sets of non-zero measure — to multi-product firms. The length of these segments indicate the scope of the varieties produced by multi-product firms. We also call the two type of firms as small and big. When few big firms operate on the market they are associated with oligopolies.

Discuss in the introduction? Big and small firms: price index makers and price index takers.

The labor is a single production factor. Small firms fire homogeneous workers. The production of big firms, being more complicated, requires not only workers but also managers. The number of workers and managers in the economy is exogenous. For the sake of simplicity the wages of both types of labor force are assumed to be equal and fixed to 1 as *numeraire*.

Individuals are homogeneous as consumers. They are endowed by separable unspecified utility, which is another exogenous characteristic of the economy.

1.2 Demand

An economy is populated by L consumers with income $Y = 1$. A consumer chooses the quantities Q_x of the varieties $x \in [0, N]$ in order to maximize the utility

$$U = \int_0^N u(Q_x)\, dx \to \max \tag{1}$$

under budget constrain

$$\int_0^N p_x Q_x \leq 1, \tag{2}$$

where p_x is the price for the x-th variety of the differentiated good. The first order condition implies that

$$u'(Q_x) = \lambda p_x, \tag{3}$$

where λ is the Lagrange multiplier corresponding to optimization problem (1)

We will show later that the optimal demand and general equilibrium are described in terms of

$$\sigma(Q) = -\frac{u'(Q)}{u''(Q)Q}$$

interpreted as the elasticity of substitution between varieties of the differentiated good.

A consumer's problem formulated here is standard in the monopolistic competition theory. We only note that consumers do not differentiate between varieties produced by big and small firms and act as if all firms were single-product.

1.3 Supply

1.3.1 Small firm

A small firm producing the variety x maximizes the profit

$$\pi_{S,x} = (p_{S,x} - c_{S,x})q_{S,x} - F_{S,x} \to \max \tag{4}$$

where the prices $p_{S,x}$ and the output $q_{S,x}$ are the optimization variables. In the optimum, the output $q_{S,x} = LQ_{S,x}$ equals to the aggregate demand for the variety x and the prices are

$$p_S = \frac{\sigma_S c_S}{\sigma_S - 1}, \tag{5}$$

where the index x is dropped and $\sigma_S = \sigma(Q_S)$.

1.3.2 Big firm

A big firm produces a scope of varieties (of mass N_B) and competes monopolistically with the other firms. Its profit is

$$\Pi = \int_0^{N_B} \pi_{B,x}\, dx \to \max \tag{6}$$

where

$$\pi_{B,x} = (p_{B,x} - c_{B,x})q_{B,x} - F_{B,x} \tag{7}$$

and $q_{B,x} = LQ_{B,x}$ is the aggregate demand for the variety x. As Equations (4) and (7) read, the profit per variety of a big firm is structured in the same was as the profit of a small firm.

We recall that a small firm maximizing its profit does not affect integral market characteristics. We assume that a big firm does affect. In particular, the Lagrange multiplier λ, Equation (3), depends on the range of prices chosen by a big firm[1]. In the case of the CES preferences, the Lagrnage multiplier is related to the price index of the differentiated good. Hence, big firms behave as price index makers, whereas small firms do as price index takers. Under monopolistic competition, all firms are price makers, and the difference between strategic — big — and non-strategic — small — firms are observed through their influence on the price index.

For the sake of simplicity we imply symmetric setting among each type of firms. In particular, the costs $c_{S,x}$ and $F_{S,x}$ is independent of x. The first order condition of a big firm's problem relates the price $p_{B,x}$ charged by this big firm to its variable costs $c_{B,x}$ and output Q_B. Further simplifying the optimization problem, we are looking for a symmetrical pricing policy of each big firm: the prices $p_{B,x} = p_B$ do not depend on x. Then[2] the first order condition leads to

$$\frac{p_B - c_B}{p_B} = \frac{1}{\sigma_B} + p_B N_B Q_B \frac{\sigma_B - 1}{\sigma_B}, \tag{8}$$

where $\sigma_B = \sigma(Q_B)$, see Lemma ???

Second order conditions???

[1] Technically, the maximization in (6) is performed with respect to the range of prices, i.e. with respect to a function p_x. Computing the variation of the profit we involve the Gateaux derivative. The Lagrange multiplier λ is "hidden" in the aggregate demand q_x. Its Gateaux derivative is found with a standard variation technique, see Appendix, whereas this derivative is assumed to be zero for small firms after Dixit and Stiglitz.

[2] Technically, constant functions are substituted into the first order condition written with the Gateaux derivatives; see the details in the Appendix.

1.3.3 Pricing policies of big and small firms

Equations (5) and (8) describe a short run behavior of a big firm. In a long run, firms have to take into account the behavior of competitors. Now we discuss principles of the co-existence of big and small firms in equilibrium. First, an economic force has to stay behind the existence of big firms. We do not model this process, taking it as it is, but assume that big firms have a comparative advantage in costs: $c_B < c_S$, $c_B/F_B < c_S/F_S$. We set the simplest dependence of the variable costs on the firm mass. Namely, the costs of an arbitrary big firm (whose mass is larger than 0) immediately switch from c_B to c_S and from F_B to F_S[3].

According to Equations (5) and (8), the pricing policy of big and small firms are different. Mathematics implies that small firms decrease their profit mimicking the pricing of big firms. However, why big firms do not mimic the pricing of small firms? If big firms did they would change the price index inappropriately and, as a result, reduce the profits. In other words, the strategy of big firms is more monopolistic than that of small firms. Endowed by a larger market power, big firms produce less and charge higher price than single-product firms would do. The wider the scope of varieties produced by a big firm, the stronger the underproduction per variety is. This underproduction favors not only big firms but also their competitors.

We note that under identical costs, big and small firms should decide upon their price and output identically $p_S = p_B$. Then the conglomerates of small firms are unstable. Any part of a conglomerate can be separated without any impact on individual agents and the whole economy.

1.4 Balances

1.4.1 Free entry

Expansion for entry deterrence. In general, a comparative advantage in cost gives big firms a larger profit per variety. Consumers' love for variety creates incentives for big firms to "launch" new varieties and expand, widening their scopes. Small firms also have a certain market power. With the expansion of big firms and the growth of their prices, the market power of small firms increases. This creates an economic force that could decrease the profit

[3]If the mass of a firm is 0, its costs are c_B and F_B; if the mass is positive then they are c_S and F_S; the values of c_S and F_S do not depend on the positive firm mass.

of big firm under its further expansion. Nevertheless, big firms have incentives to expand even if further expansion decreases its profit. Indeed, assume that the production of a new variety is still profitable. This attracts new small firms to enter the market. The appearance of a new small firm harms a big firm more than its own expansion, since the expansion allows to compensate a loss in demand by “picking up” a positive profit that comes from launching a new variety.

Zero profit condition. The process of firms' formation is beyond the scene. Our model is static. Taking the structure of firms as it is we assume that big firms have expanded in order to deter the entrance of small firms. Then the free entry condition implied here states that the profit of both types of firms is zero: $\pi_{B,x} = \pi_{S,x} = 0$. We also assume that big firms are identical, and n denotes their number.

1.4.2 Labor market clearance

We assume that the shares θ_W and θ_M of workers and managers in the economy are given. Since managers are employed only by big firms it follows that the fixed costs F_B coincide with the number of managers in a big firm, and the total number of managers $\theta_M L$ is equal to

$$\theta_M L = n N_B F_B. \tag{9}$$

Recall that the second type of employees are individuals who are employed by small firms (as managers and as production workers) and by the production sectors of big firms. Then the budget constrain re-written with equilibrium variables turns to

$$\sigma_S F_B N_S + \frac{F_B N_B n}{\frac{1}{2\sigma_B} + \sqrt{\frac{F_B N_B}{L}\frac{\sigma_B - 1}{\sigma_B} + \frac{1}{4\sigma_B^2}}} = L. \tag{10}$$

The square root, which enters equation (10) is well defined if $F_B N_B < L$. We note that Equation (10) is equivalent to the balance of money.

2 Equilibrium

2.1 Definition

The variables θ_M, θ_W, L, $u(\cdot)$, c_S, c_B, F_S, F_B are exogenous in the model. We aim at finding the other characteristics of the economy. The set of the identical prices $\hat{p}_S = \hat{p}_{S,x}$ and $\hat{p}_B = \hat{p}_{B,x}$, outputs $\hat{Q} = \hat{Q}_x$ (all of them are independent of x), the mass of small firms $\hat{N}_S$, the mass $\hat{N}_B$ of each big firm, and the number $\hat{n}$ of big firms is called equilibrium if the following conditions hold.

First, the demand $\hat{Q}$ solves a consumer's optimization problem (1), (2) with $N = \hat{N}$, $p_x = \hat{p}_S$ if the variety x is produced by a small firm and $p_x = \hat{p}_B$ otherwise.

Second, given $N = \hat{N}$, the output q_x is related to the prices p_x for this variety by solving a consumer's problem and considered as a function of prices in a firm's optimization problem (4). The price $\hat{p}_{S,x}$ solves a small firm's optimization problem (4), into which the prices enters directly and indirectly through q_x. The prices $\hat{p}_{B,x}$ solves a big firm's optimization problem (6).

Third, balances (9) and (10) hold.

Forth, all big firms have the same mass $\hat{N}_B$.

According to this definition, the equilibrium variables solve the system of equations (3), (5), (8) (9), and (10).

2.2 Existence and uniqueness

The existence of the equilibrium requires the following assumption.

Assumption 1.

$$\sigma(Q) > 1, \tag{11}$$

$$\sigma'(Q) < 0, \quad \mathcal{E}_\sigma > -2. \tag{12}$$

We highlight a special case of preferences given by the utility function

$$u(Q) = q^\rho, \quad \rho \in (0, 1).$$

They are characterized by a constant elasticity of substitution (CES) equalled to $\sigma(Q) = 1-\rho =$ const.

Proposition 1. *Let Assumption 1 hold. Then equilibrium exists. Under CES preferences, equilibrium is unique.*

In equilibrium, the outputs q_S and q_B of small and big firms respectively are equal to

$$q_S = \frac{F_S(\sigma_S - 1)}{c_S}, \tag{13}$$

$$q_B = \frac{F_B}{c_B} \cdot \left(\sigma_B - \frac{1}{2} - \frac{1}{2}\sqrt{1 + 4\sigma_B(\sigma_B - 1)m}\right) \cdot \frac{2}{1 + \sqrt{1 + 4\sigma_B(\sigma_B - 1)m}}, \tag{14}$$

where $m = N_B F_B / L$ is interpreted as the number of a big firm's managers normalized by the total mass of individuals; see Lemma 8. In Lemma 8 we establish that m solves equation

$$m = \left(1 - \frac{(\sigma_S - 1)\sigma_B}{\sigma_S(\sigma_B - 1)} \cdot \frac{c_B}{c_S} \frac{u'(Q_S)}{u'(Q_B)}\right) \left(1 - \frac{\sigma_S - 1}{\sigma_S} \frac{c_B}{c_S} \frac{u'(Q_S)}{u'(Q_B)}\right). \tag{15}$$

We have discussed the mechanism of the underproduction by big firms in section 1.3.3. Based on explicit Equations (13) and (35), which the outputs q_S and q_B satisfy to, we establish the underproduction rigorously in Lemma 11.

The introduction of big firms complicates a standard rigorous analysis of the equilibrium equations. In particular, instead of single Equation (13), which gives the output of small firms, the (closed) system of two Equations (14), (15) is required to expose the equilibrium characteristic of big firms. We establish the existence of equilibrium exploring this system.

2.3 Number of big firms

According to (9), the number of big firms is given by $n = \theta_M m^{-1}$. Since the number of firms n is at least 1, the quantity m should be small. The latter holds if the both brackets in (15) are close to zero; in particular, the ratio c_B/c_S is close to 1. We associate the comparative advantage of big firms with the parameter ε defined as $\varepsilon = 1 - c_B/c_S$. If ε is small, an approximate solution of Equation (15) can be found through the expansion of its right hand side into series. This leads to an approximate formula for the number of firms.

We need another assumption to estimate the number of firms. The function $r_f(\varkappa) = -f''(\varkappa)\varkappa/f'(\varkappa)$ is assigned to an arbitrary function $f(\varkappa)$. According to [1], r_u is interpreted as love for variety. This $r_u(Q)$ is inverse to the elasticity of substitution $\sigma(Q)$.

Assumption 2. *Let $r_u(Q)$ and $r_{u'}(Q)$ are increasing and respectively decreasing functions of Q and*

$$\sigma(Q) > 4/3, \tag{16}$$

$$r_{u'}(Q_B) > 2r_u(Q_B). \tag{17}$$

Proposition 2. *Under CES preferences and small ε, the number of firms is*

$$n \approx \theta_M \left(\sigma \sqrt{\frac{\sigma - 1}{2}} \left(1 - \left(\frac{c_B}{c_S}\right)^{1 - \frac{1}{\sigma}}\right)^{-1/2} - \frac{\sigma}{2} + 1\right). \tag{18}$$

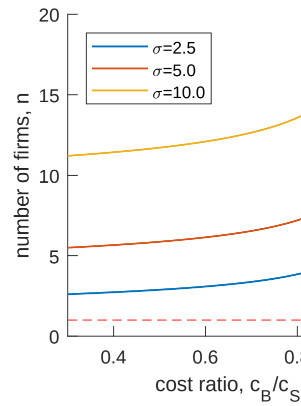

Figure 1: The number of firms found with Equation (18) and three values of σ; the horizontal dashed line indicates 1.

Under an unspecified utility satisfying Assumptions 1 and 2, the number of firms is

$$n \approx 2^{-1/2}\theta_M \big(1 - r_u(Q_B)\big)^{-1/2} r_u^{-3/2}(Q_B)\big(r_{u'}(Q_B) - 2r_u(Q_B)\big)\left(1 - \frac{c_B}{c_S}\right)^{-1/2}. \tag{19}$$

Figure 1 illustrates Equation (18). This Figure gives evidence that the model realizes the economy with a few number of oligopolistic firms. We stress that the number of big firms increases as the square root of $(1 - c_B/c_S)$, as the latter tends to 0. This growth is relatively small; then the market with a few oligopolistic firms can be observed with a wide range of the comparative advantage.

The approximation (19) obtained for the economy with an unspecified utility is less accurate than (18), since only the main term with respect to $1/\varepsilon$ is found but the second term, a constant, is skipped. We note that the $(1-c_B/c_S)^{-1/2}$-growth of n is a general characteristic of the model economy.

2.4 Comparative statics

Proposition 3. *Let Assumptions 1 and 2 hold. Then the approximation* (19) *to the number of big firms is increasing in L.*

A shock in the market size affects big firms in a natural way. With a growth of individuals, the demand for the product diversity enlarges. This shrinks the market power of each firm. It implies that a big firm is more restricted on larger markets when exploiting its comparative advantage. In other words, the comparative advantage in costs looses its significance on larger markets. Therefore, responding to a positive shock in the market size, big firms decrease their scopes. The share of their varieties becomes smaller. As the number of managers is assumed to be independent of L, Equation (9) implies that the number of big firms is lesser on larger markets.

3 Conclusion

A Appendix

Lemma 1. *The solution of optimization problem* (6) *is given by*

$$(p_x - c_x)\frac{\delta q_x(p)}{\delta p} + q(p_x) = 0. \tag{20}$$

Proof. Recall, the Gateaux derivative of the function F, $F : X \to Y$ is defined in two steps. First,

$$\delta F = \lim_{t\to 0} \frac{d}{dt}\frac{F(x+th) - F(x)}{t}.$$

If

$$\delta F = G(x)h,$$

then the mapping $G(x)$ is called the Gateaux derivative:

$$\frac{\delta F}{\delta x} = G.$$

The variation of the profit Π:

$$\delta\Pi = \lim_{t\to 0}\frac{1}{t}\int_0^{N_1}\big((p_x + th_x)q_x(p_x + th_x) - c_x q_x(p + th_x)) - p_x q_x + c_x q_x\big)\,dx$$

$$\delta\Pi = \lim_{t\to 0}\frac{1}{t}\int_0^{N_1}\big(p_x\cdot(q_x(p_x+th_x) - q_x(p_x)) + th_x\cdot q(p_x+th_x) - c_x(q_x(p_x+th_x) - q_x(p_x))\big)\,dx = \int_0^{N_1}\left(p_x\frac{\delta q_x(p)}{\delta p}h_x + h_x q(p_x) - c_x\frac{\delta q_x(p)}{\delta p}h_x\right)dx = \int_0^{N_1}\left((p_x - c_x)\frac{\delta q_x(p)}{\delta p} + q(p_x)h_x\right)dx.$$

Since the variation of the profit is zero for any appropriate (???) function h_x, it follows that

$$(p_x - c_x)\frac{\delta q_x(p)}{\delta p} + q(p_x) = 0.$$

Then the first order condition for the firm optimization problem follows from the last equation.

□

Lemma 2. *Let*

$$\int_0^N u(Q_x)\,dx \to \max \tag{21}$$

$$\int_0^N p_x Q_x \le Y \tag{22}$$

be an optimization problem of a consumer with income Y *who faces prices* p_x*;* N *is the number of the varieties. Then its first order condition is given by the following two equations:*

$$\lambda = \frac{1}{Y}\int_0^N u'(Q_x)Q_x\,dx, \tag{23}$$

Equation (3)*, and the budget constrain* (2) *written as the equality.*

Proof. The first order condition of the maximization problem leads to (3). We substitute Equation (3) to the budget (2) understood as the equality:

$$\frac{1}{\lambda}\int_0^N u'(Q_x)Q_x = Y.$$

The latter is equivalent to Equation (23). □

Lemma 3. *The Gateaux derivative of* λ *determined by Equation* (23) *with respect to the prices* $p(x)$ *for varieties* $x \in [0, N_1]$ *is the linear operator*

$$\frac{\delta\lambda}{\delta p}h = \int_0^{N_1} K(p_x)h_x\,dx,$$

where

$$K(p_x) = \frac{1}{Y}\big(u''(Q_x(p_x))Q_x(p_x) + u'(Q_x(p_x))\big)\frac{\delta Q_x}{\delta p}h_x.$$

Proof.

$$\delta_p\lambda(p_x,h_x) = \lim_{t\to 0}\frac{1}{t}\frac{\lambda(p+th)-\lambda(p)}{t} =$$
$$\lim_{t\to 0}\frac{1}{t}\frac{1}{Y}\int_0^{N_1}\big(u'(Q_x(p_x+th_x))Q_x(p_x+th_x) - u'(Q_x(p_x))Q_x(p_x)\big)\,dx,$$

where the function h_x is zero outside the interval $[0, N_1]$. Simplifying, we get:

$$\delta_p\lambda(p_x,h_x) = \lim_{t\to 0}\frac{1}{t}\frac{1}{Y}\int_0^{N_1}\big(u'(Q_x(p_x+th_x)) - u'(Q_x(p_x))\big)Q_x(p_x+th_x) +$$
$$u'(Q_x(p_x))\big(Q_x(p_x+th_x) - Q_x(p_x)\big)\,dx.$$

$$\delta_p\lambda(p_x,h_x)=\frac{1}{Y}\int_0^{N_1}\Big(u''(Q_x(p_x))\lim_{t\to 0}\frac{1}{t}\big(Q_x(p_x+th_x)-Q_x(p_x)\big)Q_x(p_x)+$$
$$u'(Q_x(p_x))\lim_{t\to 0}\frac{1}{t}(Q_x(p_x+th_x)-Q_x(p_x))\Big)\,dx.$$

$$\delta_p\lambda(p_x,h_x)=\frac{1}{Y}\int_0^{N_1}\big(u''(Q_x(p_x))Q_x(p_x)+u'(Q_x(p_x))\big)\frac{\delta Q_x}{\delta p}h_x\,dx.$$

Let

$$K(p_x)=\frac{1}{Y}\big(u''(Q_x(p_x))Q_x(p_x)+u'(Q_x(p_x))\big)\frac{\delta Q_x}{\delta p}h_x.$$

Then the Gateaux derivative is the linear operator that maps a function h to $\mathbf{R}$, as stated in the Equation:

$$\frac{\delta\lambda}{\delta p}h=\int_0^{N_1}K(p_x)h_x\,dx.$$

□

Lemma 4. *Let*

$$I_2=\int_0^{N_1}u''(Q[p])Q[p]\frac{\delta Q}{\delta p}h\,dx.$$

Then

$$\left(1-\frac{1}{Y}\int_0^{N_1}Q[p]\,dx\right)I_2=\int_0^{N_1}\lambda hQ[p]\,dx+\frac{1}{Y}\int_0^{N_1}Q[p]\,dx\int_0^{N_1}u'(q[p])\frac{\delta Q}{\delta p}h\,dx. \tag{24}$$

Proof. According to Lemma 2,

$$u'(Q_x)=\lambda p_x. \tag{25}$$

We are going to vary this equation with respect to the prices p_x charged by a single big firm and, therefore, defined on $[0,N_1]$. Initially, we substitute to this equation p_x+th_x for p_x and drop the dependence on x:

$$u'(Q[p+th])=\lambda[p+th](p+th),$$

where quantities inside the square brackets indicate the (functional) variables of the outer functions. Adding and subtracting $\lambda[p](p+th)$, we have:

$$u'(Q[p+th])=(\lambda[p+th]-\lambda[p])(p+th)+\lambda[p]p+\lambda[p]th, \tag{26}$$

Subtracting (25) from (26) we get:

$$u''(Q[p])\delta Q[p]th=\lambda[p]th+(\delta\lambda[p])(p+th).$$

By using Lemma 3, we tend t to zero:

$$u''(Q)\frac{\delta Q}{\delta p}h = \lambda h + \frac{p}{Y}\int_0^{N_1}\left(u''(Q(p))Q(p) + u'(Q(p))\right)\frac{\delta Q}{\delta p}h\,dx,$$

where all functionals are defined in p. Integrating the both parts of the last inequality over the interval $[0, N_1]$, we have

$$I_2 = \int_0^{N_1}\lambda h Q[p]\,dx + \frac{p}{Y}\int_0^{N_1}Q[p]\,dx\left(I_2 + \int_0^{N_1}u'(q[p])\frac{\delta Q}{\delta p}h\,dx\right).$$

□

Lemma 5. *Let N_1 be the total mass of varieties produced by a single big firm. We assume that this firm prices all its varieties symmetrically: $p_x = p = \text{const}$. Then*

$$\frac{p-c}{p} = \frac{1}{\sigma} + \frac{pN_1Q}{Y}\frac{\sigma-1}{\sigma}. \tag{27}$$

$$p = \frac{\sigma}{\sigma-1}\cdot\frac{c}{1-\alpha}, \tag{28}$$

$$\pi_b = \frac{1-\alpha+\alpha\sigma}{(\sigma-1)(1-\alpha)}cq - F_B, \tag{29}$$

where $\alpha = pN_1Q/Y$.

Proof. Let $p(x) = \text{const}$, $Q(x) = \text{const}$, $h(x) = 1$. Then

$$u''(Q)Q\frac{\delta Q}{\delta p} = \lambda Q + \frac{p}{Y}N_1Q\left(u''(Q)Q\frac{\delta Q}{\delta p} + u'(Q)\frac{\delta Q}{\delta p}\right).$$

$$\left(u''(Q) - \frac{pN_1}{Y}u''(Q)Q - \frac{pN_1}{Y}u'(Q)\right)\frac{\delta Q}{\delta p} = \lambda. \tag{30}$$

From (20) and equation $q = N_1Q$ it follows that $\delta Q/\delta p = -Q/(p-c)$. Combining this observation and Equation (3) with (30), we have

$$-\frac{Q}{p-c}\left(u''(Q) - \frac{pN_1}{Y}u''(Q)Q - \frac{pN_1}{Y}u'(Q)\right) = \frac{u'(Q)}{p}.$$

Put, $\sigma(Q) = -u'(Q)/(u''(Q)Q)$. Then

$$\frac{p}{p-c}\left(1 - \frac{pN_1Q}{Y} + \frac{pN_1Q\sigma(Q)}{Y}\right) = \sigma(Q)$$

This equation leads to (8). □

Lemma 6. *We consider a profit Π of a big firm as a function of prices p_B. These prices does not depend on the variety type. The variation of Π with respect to the prices is changed to the partial derivative. Let Assumption 1 hold. Then $\partial^2\Pi/\partial p_B^2 < 0$ at a point that satisfies the first order condition* (8)*.*

Proof. As derivatives substiutes variations, the second derivative of the profit is given by

$$\Pi'' = N_B L \frac{\partial Q}{\partial p}\left(2 + (p-c)\frac{\partial^2 Q/\partial p^2}{\partial Q/\partial p}\right). \tag{31}$$

We find the second derivative of the demand computing the derivatives of the both hand sides of Equation (3) and using the derivative of λ given by Lemma 3:

$$\frac{\partial Q}{\partial p} = -\frac{\sigma Q}{p}\frac{1}{1+NpQ(\sigma-1)}. \tag{32}$$

The alternative way of getting this equation is to simplify (24) with $p = \text{const}$ and $h = 1$. Then taking the logarithm and computing the derivative of the both hand sides of the obtained equation, we have:

$$\frac{\partial^2 Q/\partial p^2}{\partial Q/\partial p^2} = -\left(\frac{\sigma'}{\sigma} + \frac{1}{Q}\right)\frac{\partial Q}{\partial p} + \frac{1}{p} + \frac{N}{1+NpQ(\sigma-1)}\left(Q(\sigma-1) + p(\sigma-1)\frac{\partial Q}{\partial p} + pQ\sigma'\frac{\partial Q}{\partial p}\right). \tag{33}$$

Substituting (33) into (31), we have:

$$\Pi'' = 2 - (p-c)\left(\frac{\sigma'}{\sigma} + \frac{1}{Q}\right)\frac{\partial Q}{\partial p} + \frac{(p-c)Np(\sigma-1+Q\sigma')}{1+NpQ(\sigma-1)}\frac{\partial Q}{\partial p} + \frac{p-c}{p} + \frac{(p-c)NQ(\sigma-1)}{1+NpQ(\sigma-1)}.$$

Taking into account (27), we sum up the last two terms in the right hand side and get:

$$\Pi'' = 2 - (p-c)\left(\frac{\sigma'}{\sigma} + \frac{1}{Q}\right)\frac{\partial Q}{\partial p} + \frac{(p-c)Np(\sigma-1+Q\sigma')}{1+NpQ(\sigma-1)}\frac{\partial Q}{\partial p} + \frac{1+2NpQ(\sigma-1)}{\sigma}.$$

With a big firm's first order condition

$$\frac{\partial Q}{\partial p} = -\frac{Q}{p-c},$$

we simplifty Π'' to

$$\Pi'' = 2 + \left(\frac{\sigma'}{\sigma} + \frac{1}{Q}\right)Q - \frac{Np(\sigma-1+Q\sigma')Q}{1+NpQ(\sigma-1)} + \frac{1+2NpQ(\sigma-1)}{\sigma}.$$

We group the terms in the following way:

$$\Pi'' = \left(2 + \frac{\sigma' Q}{\sigma}\right) + \left(1 - \frac{Np(\sigma-1)Q}{1+NpQ(\sigma-1)} + \frac{1+2NpQ(\sigma-1)}{\sigma}\right) + \left(\frac{(-\sigma')Q^2}{1+NpQ(\sigma-1)}\right).$$

The condition (12) provides that the first and third brackets are positive. Establishing that the second bracket is positive, we put $t = NpQ(\sigma-1)$ and prove the inequality

$$\frac{2t^2 - (\sigma-3)t + 1}{(1+t)\sigma} + 1 > 0.$$

Since $\sigma > 1$, it is enough to prove that $2t^2 + 3t + 2 > 0$. The latter is evident. □

Lemma 7. *The output of a small firm under the free entry ($\pi_S = 0$) is given by Equation* (13).

The proof of the Lemma is well known.

The following lemma first gives technical issues obtained with general preferences. Further in this lemma we obtain the number of big firms with CES preferences.

Lemma 8. *Under zero profit condition for a big firm, its prices and outputs are given by the following equations*

$$p_B = c_B \sigma_B \frac{2}{2\sigma_B - 1 - \sqrt{1 + 4\sigma_B(\sigma_B - 1)m}}, \tag{34}$$

$$Q_B = \frac{F_B(\sigma_B - 1)}{Lc_B} \cdot \frac{\sigma_B - \frac{1}{2} - \frac{1}{2}\sqrt{1 + 4\sigma_B(\sigma_B - 1)m}}{\sigma_B - 1} \cdot \frac{2}{1 + \sqrt{1 + 4\sigma_B(\sigma_B - 1)m}}, \tag{35}$$

where m solves Equation (15).

Proof. Equalizing the profit (7) of a big firm to zero we get

$$Q_B = \frac{F_B}{p_B L}\left(1 - \frac{c_B}{p_B}\right)^{-1} = \frac{F_B}{(p_B - c_B)L} \tag{36}$$

Substituting (36) into (28), we get the equation

$$p_B\left(1 - \frac{p_B F_B N_B}{p_B - c_B}\right) = \frac{\sigma_B c_B}{\sigma_B - 1},$$

which is quadratic with respect to p_B. Dividing the both hand sides by p_B and transforming, we obtain

$$1 - \frac{\sigma_B}{\sigma_B - 1}\frac{c_B}{p_B} = \frac{p_B F_B N_B}{(p_B - c_B)L};$$

and

$$\left(1 - \frac{c_B}{p_B}\right)\left(1 - \frac{\sigma_B}{\sigma_B - 1}\frac{c_B}{p_B}\right) = \frac{F_B N_B}{L}. \tag{37}$$

The solution of this equation with respect to p_B is given by

$$p_B = \frac{c_B}{1 - \frac{1}{2\sigma_B} - \sqrt{\frac{F_B N_B}{L}\frac{\sigma_B - 1}{\sigma_B} + \frac{1}{4\sigma_B^2}}}.$$

The last equation is equivalent to (34). Substituting (34) to (36), we have

$$Q_B = \frac{F_B}{c_B L}\left(\frac{2\sigma_B}{2\sigma_B - 1 - \sqrt{1 + 4\sigma_B(\sigma_B - 1)m}} - 1\right)^{-1}. \tag{38}$$

Equations (35) and (38) are equivalent.

Now we derive Equation (15). We combine Equations (3) faced by small and big firms to:

$$\frac{u'(Q_B)}{u'(Q_S)} = \frac{p_B}{p_S}.$$

Then expressing p_B from the last equation and using Equation (5), we obtain

$$p_B = \frac{u'(Q_B)}{u'(Q_S)} \frac{c_S \sigma_S}{\sigma_S - 1}.$$

With this p_B, Equation (37) is transformed to (15). □

Lemma 9. *Let $\sigma(Q)$ be a decreasing function. Then the equation*

$$Q_B = \frac{F_B}{c_B L} \left(\frac{1}{1 - \frac{1}{2\sigma_B} - \sqrt{\frac{1}{4\sigma_B^2} + \left(1 - \frac{1}{\sigma_B}\right) m}} - 1 \right)^{-1} \tag{39}$$

considered with respect to Q_B for any fixed $m \in [0, 1)$ has a unique solution. This solution $Q_B = Q_B(m)$ is a decreasing function of m.

Proof. We put $r(Q) = 1/\sigma(Q)$ and re-write Equation (39) as

$$Q_B = \frac{F_B}{c_B L} \left(\frac{1}{1 - \frac{1}{2} r_B - \sqrt{\frac{1}{4} r_B^2 + (1 - r_B) m}} - 1 \right)^{-1}.$$

We define an auxiliary function $h(r) = \frac{1}{2} r + \sqrt{\frac{1}{4} r^2 + (1 - r) m}$, where $m \in [0, 1)$ is a parameter, and establish its growth in r. Computing

$$h'(r) = \frac{1}{2} + \frac{\frac{1}{2r} - m}{2\sqrt{\frac{1}{4} r^2 + (1 - r) m}},$$

we conclude that the inequality $h'(r) > 0$ is equivalent to the inequality

$$1 > \frac{m - r/2}{\sqrt{\frac{1}{4} r^2 + (1 - r) m}}.$$

If $2m - r < 0$ then the last inequality holds. If $2m - r \leq 0$, the last inequality can be written as

$$\frac{1}{4} r^2 + (1 - r) m > m^2 - mr + \frac{1}{4} r^2 \quad \text{or} \quad m > m^2,$$

which is evident. Since $\sigma' < 0$ it follows that $r' > 0$ and $h(r(Q))$ is an increasing function of Q. Then the right hand side (rhs) of Equation (39) is a decreasing function of Q_B, whereas the left hand side (lhs) is an increasing function. Since the lhs varies from 0 to $+\infty$ and the rhs is positive, their unique intersection exists. Investigating the rhs explicitly, we claim that it decreases in m. Then the solution $Q_B(m)$ decreases as a function of m. □

Lemma 10. *We assume that Q_B is defined as the solution of Equation* (39)*. Let $\sigma' < 0$. Then Equation* (15) *has a solution m and the difference of the right and left hand sides of* (15) *changes its sign from plus to minus at the minimal m^* that solves Equation* (15)*. If the solution m unique it also has this property.*

Proof. We plan to show that the rhs of (15) increases in m. The function $u'(Q_B)$ is positive and decreasing. Therefore, the second bracket in (15) is decreasing in Q_B. The first bracket is also decreasing because $u'(Q_B)(\sigma(Q_B) - 1)/\sigma(Q_B) = u'(Q_B)(1 - 1/\sigma(Q_B))$ decreases in Q_B. Thus, the rhs decreases in Q_B but increases in m.

The lhs of (15) increases from 0 to $+\infty$. If $m = 0$ then the equations determining Q_B and Q_S coincides and, as a result, $Q_B = Q_S$. Then $rhs(0) = (1 - c_B/c_S)^2 \in (0, 1)$. Since the rhs is bounded by 1 from above, the intersection of the left and right hand sides exists. Let m^* be the minimal m that satisfies (15). Then the difference of the right and left hand sides of (15) changes its sign from plus to minus at m^*. □

Lemma 11. *Let $\hat{Q}_B$ solves the equation $Q = F_B(\sigma(Q) - 1)/(c_B L)$; $\hat{q}_B = \hat{Q}_B L$. Then $q_B < q_S$ in equilibrium.*

Proof. A simple algebra gives evidence that the product of the second and the thrid fraction in the right hand side of Equation (35) is less than 1. As a result, from (35) it follows that $Q_B < F_B(\sigma_B - 1)/(c_B L)$. According to Assumption 1, the right hand side of the obtained inequality is decreasing in Q. Therefore, $Q_B < \hat{Q}_B$. □

Lemma 12. *Let T_1 and T_2 denote the second and third fractions in Equation* (35)*:*

$$T_1 = \frac{\sigma_B - \frac{1}{2} - \frac{1}{2}\sqrt{1 + 4\sigma_B(\sigma_B - 1)m}}{\sigma_B - 1} \qquad T_2 = \frac{2}{1 + \sqrt{1 + 4\sigma_B(\sigma_B - 1)m}} \tag{40}$$

Then the expansion of the product $T_1 T_2$ into series up the m^3-term is given by

$$T_1 T_2 \approx 1 - \sigma_B^2 m + 2\sigma_B^3(\sigma_B - 1)m^2 - \sigma_B^3(\sigma_B - 1)^2(4\sigma_B + 1)m^3. \tag{41}$$

Proof. We consequently expand the square root, fraction T_1, and fraction T_2 into series. The square root is:

$$\sqrt{1 + 4\sigma_B(\sigma_B - 1)m} \approx 1 + 2\sigma_B(\sigma_B - 1)m - 2\sigma_B^2(\sigma_B - 1)^2 m^2 + 4\sigma_B^3(\sigma_B - 1)^3 m^3.$$

The factor T_1:

$$T_1 \approx 1 - \sigma_B m + \sigma_B^2(\sigma_B - 1)m^2 - 2\sigma_B^3(\sigma_B - 1)^2 m^3.$$

The factor T_2:

$$T_2 \approx \frac{1}{1+\sigma_B(\sigma_B-1)m-\sigma_B^2(\sigma_B-1)^2m^2+2\sigma_B^3(\sigma_B-1)^3m^3}.$$

$$T_2 = 1-\sigma_B(\sigma_B-1)m+\sigma_B^2(\sigma_B-1)^2m^2-2\sigma_B^3(\sigma_B-1)^3m^3+\sigma_B^2(\sigma_B-1)^2m^2-2\sigma_B^3(\sigma_B-1)^3m^3$$
$$= 1-\sigma_B(\sigma_B-1)m+2\sigma_B^2(\sigma_B-1)^2m^2-4\sigma_B^3(\sigma_B-1)^3m^3.$$

Then Equation (41) gives the product T_1T_2. □

Proof of Proposition 2. Under CES setting, $u'(Q_B)/u'(Q_S) = (Q_S/Q_B)^{1/\sigma}$, and we are able to expand the right hand side up to the second order term using the form:

$$\frac{u'(Q_S)}{u'(Q_B)} = 1-\left(\frac{c_B}{c_S}\right)^{\frac{\sigma-1}{\sigma}}(T_1T_2)^{\frac{1}{\sigma}}.$$

By using (41), we have

$$(T_1T_2)^{\frac{1}{\sigma}} = 1-\sigma m+2\sigma^2(\sigma-1)m^2-\sigma^2(\sigma-1)^2(4\sigma+1)m^3-\frac{\sigma-1}{2\sigma^2}\sigma^4m^2+$$
$$\frac{\sigma-1}{2\sigma^2}2\sigma^2 2\sigma^3(\sigma-1)m^3 = 1-\sigma m+\frac{3}{2}\sigma^2(\sigma-1)m^2+\sigma^2(\sigma-1)^2(2\sigma-4\sigma-1)m^3 =$$
$$1-\sigma m+\frac{3}{2}\sigma^2(\sigma-1)m^2-\sigma^2(\sigma-1)^2(2\sigma+1)m^3.$$

Now we return to Equation (15) written in the form:

$$m = \left(1-\left(\frac{c_B}{c_S}\right)^{\frac{\sigma-1}{\sigma}}(T_1T_2)^{\frac{1}{\sigma}}\right)\left(1-\frac{\sigma-1}{\sigma}\left(\frac{c_B}{c_S}\right)^{\frac{\sigma-1}{\sigma}}(T_1T_2)^{\frac{1}{\sigma}}\right) \tag{42}$$

Put,

$$\tilde{\varepsilon} = 1-\left(\frac{c_B}{c_S}\right)^{\frac{\sigma-1}{\sigma}}.$$

The first bracket in the right hand side of (42) is expanded to

$$\sigma m-\frac{3}{2}\sigma^2(\sigma-1)m^2+\sigma^2(\sigma-1)^2(2\sigma+1)m^3+\tilde{\varepsilon}-\sigma\tilde{\varepsilon}m$$

The second bracket is

$$\frac{1}{\sigma}+(\sigma-1)m-\frac{3}{2}\sigma(\sigma-1)^2m^2+\frac{\sigma-1}{\sigma}\tilde{\varepsilon}-(\sigma-1)\tilde{\varepsilon}m.$$

Then Equation (42) becomes

$$m = m-\frac{3}{2}\sigma(\sigma-1)m^2+\sigma(\sigma-1)^2(2\sigma+1)m^3+\frac{\tilde{\varepsilon}}{\sigma}-\tilde{\varepsilon}m+$$
$$\sigma(\sigma-1)m^2+\sigma(\sigma-1)^2(2\sigma+1-3\sigma)m^3+(\sigma-1)\tilde{\varepsilon}m-\frac{3}{2}\sigma^2(\sigma-1)^2m^3+(\sigma-1)\tilde{\varepsilon}m.$$

It is reduced to

$$0 = \frac{\tilde{\varepsilon}}{\sigma} - \frac{1}{2}\sigma(\sigma-1)m^2 + (2\sigma-3)\tilde{\varepsilon}m - \sigma(\sigma-1)^3m^3.$$

The obtained equation stays behind the approximation $m \approx B\sqrt{\tilde{\varepsilon}}+C\tilde{\varepsilon}$, where the factors B and C can be found equalizing the coefficients at the same powers of $\tilde{\varepsilon}$. Equalizing the coefficients multiplied by $\tilde{\varepsilon}$, we have

$$\frac{1}{\sigma} - \frac{1}{2}\sigma(\sigma-1)B^2 = 0,$$

and

$$B = \frac{1}{\sigma}\sqrt{\frac{2}{\sigma-1}}.$$

Equalizing the coefficients multiplied by $\tilde{\varepsilon}^{3/2}$, we have

$$-\frac{1}{2}\sigma(\sigma-1)2BC + (2\sigma-3)B = \sigma(\sigma-1)^3B^3 = 1$$

This turns to

$$C = \frac{\sigma-2}{\sigma^2(\sigma-1)}.$$

We conclude that

$$m \approx \frac{1}{\sigma}\sqrt{\frac{2}{\sigma-1}}\sqrt{\tilde{\varepsilon}} + \frac{\sigma-2}{\sigma^2(\sigma-1)}\tilde{\varepsilon}. \tag{43}$$

Inverting the last equation, we have:

$$m^{-1} \approx \sigma\sqrt{\frac{\sigma-1}{2}}\left(1-\left(\frac{c_B}{c_S}\right)^{1-\frac{1}{\sigma}}\right)^{-1/2} - \frac{\sigma}{2} + 1.$$

Now Equation (18) follows from (9). □

Proof of Proposition 3. We are going to solve Equation (15) We will establish that

$$R = \frac{(\sigma_S-1)\sigma_B}{\sigma_S(\sigma_B-1)} = 1 + \bar{o}(m^2).$$

and use the expansion of Equation (15) into series up to the m^2-term obtained in Lemma 2. Indeed,

$$R - 1 = \frac{\sigma_S - \sigma_B}{\sigma_B - 1} = \frac{\sigma_B'(Q_S - Q_B)}{\sigma_B - 1} = \frac{\sigma_B' Q_B}{\sigma_B - 1}\left(\frac{Q_S}{Q_B} - 1\right), \tag{44}$$

where $R_B' = R'(Q_B)$. By using (13) and (35), we develop the ratio of the demands:

$$\frac{Q_S}{Q_B} = \frac{\sigma_S - 1}{\sigma_B - 1}\frac{c_B}{c_S}\frac{1}{T_1T_2}, \tag{45}$$

where T_1 and T_2 are introduced in Equation (40). The expansion of T_1 and T_2 are given in Lemma (12). We take those expansions up to the m^2-terms. Then

$$\frac{1}{T_1} = 1 + \sigma_B m - \sigma_B^2(\sigma_B - 1)m^2 + \sigma_B^2 m^2 = 1 + \sigma_B m - \sigma_B^2(\sigma_B - 2)m^2$$

and

$$\frac{1}{T_1 T_2} = 1 + \sigma_B^2 m - \sigma_B^3(\sigma_B - 2)m^2.$$

Substituting this expansion into Equation (45) and using $\varepsilon = 1 - c_B/c_S$, we have

$$\frac{Q_S}{Q_B} = \left(1 + \frac{\sigma_S - \sigma_B}{\sigma_B - 1}\right)(1 - \varepsilon)\big(1 + \sigma_B^2 m - \sigma_B^3(\sigma_B - 2)m^2\big)$$

and

$$\frac{Q_S}{Q_B} - 1 = \sigma_B^2 m - \sigma_B^3(\sigma_B - 2)m^2 - \varepsilon + \frac{\sigma_S - \sigma_B}{\sigma_B - 1}.$$

Substituting (44) into the last equation, we obtain

$$\frac{Q_S}{Q_B} - 1 = \sigma_B^2 m - \sigma_B^3(\sigma_B - 2)m^2 - \varepsilon + \frac{\sigma_B' Q_B}{\sigma_B - 1}\left(\frac{Q_S}{Q_B} - 1\right). \tag{46}$$

Later we prove that

$$\frac{\sigma_B' Q_B}{\sigma_B - 1} = \bar{o}(m^2). \tag{47}$$

Therefore, we use it and neglect this fraction, simplifying (46) into

$$\frac{Q_S}{Q_B} - 1 = \sigma_B^2 m - \sigma_B^3(\sigma_B - 2)m^2 - \varepsilon \tag{48}$$

Now we expand the ratio of the derivatives of u into series:

$$\frac{u'(Q_s)}{u'(Q_B)} \approx 1 - A_1 m + A_2 m^2,$$

where the coefficients A_1 and A_2 have to be found. Since

$$\frac{u'(Q_S)}{u'(Q_B)} = 1 + \frac{1}{\sigma_B}\left(1 - \frac{Q_S}{Q_B}\right) + \frac{u'''(Q_B)Q_B^2}{2u'(Q_B)}\left(1 - \frac{Q_S}{Q_B}\right)^2,$$

from (48) it follows that

$$\frac{u'(Q_S)}{u'(Q_B)} = 1 - \sigma_B m + \sigma_B^2(\sigma_B - 2)m^2 + \frac{\varepsilon}{\sigma_B} + \frac{u'''(Q_B)Q_B^2}{2u'(Q_B)}\left(1 - \frac{Q_S}{Q_B}\right)^2,$$

We conclude that $A_1 = \sigma_B$, $A_2 m^2 = \sigma_B^2(\sigma_B - 2)m^2 + \varepsilon/\sigma_B + \frac{u'''(Q_B)Q_B^2}{2u'(Q_B)}\sigma_B^4 m^2$.

We return to Equation (15):

$$m = (\sigma m - A_2 m^2 + \varepsilon)\left(\frac{1}{\sigma} + (\sigma - 1)m\right)$$

$$\frac{\varepsilon}{\sigma} = \frac{A_2}{\sigma_B} m^2 - \sigma_B(\sigma_B - 1)m^2$$
$$\varepsilon = A_2 m^2 - \sigma_B^2(\sigma_B - 1)m^2.$$
$$\varepsilon = \sigma_B^2(\sigma_B - 2)m^2 + \frac{\varepsilon}{\sigma_B} + \frac{\sigma_B^4 u'''(Q_B)Q_B^2}{2u'(Q_B)} - \sigma_B^2(\sigma_B - 1)m^2.$$
$$m^2 = \frac{\sigma_B - 1}{\sigma_B^3} \frac{2\varepsilon}{\mathcal{E}u'(Q_B)\mathcal{E}u''(Q_B)\sigma_B^2 - 2}.$$
$$r_f(t) = -\mathcal{E}f'(t)$$
$$m^2 = \frac{1 - r_u(Q_B)}{r_u^4(Q_B)} \frac{2\varepsilon}{\frac{r_{u'}(Q_B)}{r_u(Q_B)} - 2}$$

The existence condition:

$$\frac{r_{u'}(Q_B)}{r_u(Q_B)} - 2 > 0$$

If r_u increases and $r_{u'}$ decreases or r_u decreases and $r_{u'}$ increases then the factor in front of ε is decreasing with respect to Q.

CHECK: Q_B decreases in L. Then the coefficient at ε is increasing in L, and m is also increasing in L. □

Lemma 13. *The Equation* (15) *is reduced to*

$$m^2 = (1 - r_u(Q_B))r_u^3(Q_B)\frac{2\varepsilon}{r_{u'}(Q_B) - 2r_u(Q_B)} \tag{49}$$
$$Q_B = \frac{F_B(\sigma_B - 1)}{c_B L}(1 - \sigma_B^2 m + 2\sigma_B^2(\sigma_B - 1)m^2) \tag{50}$$

up to $\bar{o}(\varepsilon)$.

Proof. Equation (50) follows from Equation (40). Equation (49) was proved in Lemma 3. □

Lemma 14. *Let* $\sigma(Q) > 4/3$, $\sigma'(Q) < 0$, *and* $r_{u'}(Q_B) > 2r_u(Q_B)$. *Then the approximate value of* m *given by the system* (49), (35) *decreases with respect to* L.

Proof. An explicit differentiation yields that the auxiliary function $h(r) = (1-r)r^3$ is increasing, if $r > 3/4$. Then the right hand side of Equation (49) increases in Q_B.

Let the population increases from L_1 to $L_2 > L_1$. We turn to Equation (35) with the "old" $m_1 = m(L_1)$ but new $L = L_2$. Due to the change in L, the right hand size of (35) decreases and the solution of this equation also decreases: $Q_B(m_1, L_2) < Q_B(m_1, L_1)$. Substituting this $Q_B(m_1, L_2)$ into Equation (49) we find that its right hand side (rhs), as an increasing function in Q_B, has decreased and $m_1 > rhs\big(Q_B(m_1, L_2)\big)$. Since $0 < rhs\big(Q_B(0, L_2)\big)$, there is a new solution m_2 located to the left of m_1. □

References

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