A Response of Employment Structure to Immigration under Heterogeneity of Entrepreneurial Abilities

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Abstract

We explore the impact of immigration on the labor market structure. This structure is formed by the mechanism of occupation choice between entrepreneurship and hired work. Immigrants become either entrepreneurs or employees depending on their entrepreneurial abilities. We discriminate between the rise and fall of the share of entrepreneurs under immigration by using residents' cumulative distribution function of entrepreneurial abilities. Power distributions represent a border line case: The share of entrepreneurs remains the same. We argue that entrepreneurs-residents in developed countries are better off, and those in the less developed countries are worse off. Workers always gain from the arrival of migrants.

keywords: migration; heterogeneity; monopolistic competition; entrepreneurship

JEL codes: L11, L26, J61, D59

1 Introduction

Intensive labor mobility has provoked international migration during the last few years. According to [Coleman, 2008], migration intensity will grow for, at least, the next 50 years. A state prefers to choose immigrants as permanent residents based on their social ability to settle in the state and take part in its economy. Immigration programs are skilled-biased to enforce economic development. Such programs are managed, for example, in Canada and Australia. Immigrants fill under-supplied occupational positions and strengthen competition in the labor market. In general, competition positively affects economic development but enlarges social and income inequalities. Immigration flow includes accepted applicants but, in reality, is not limited by them. Since characteristics of new comers deviate from standards pre-defined by immigration programs, the government applies costly policies to balance between economic development and social tension. The efficiency of migration policies is estimated by their expenditure and the gain from new residents. The gain is likely to go up if "the most desired" immigrants are accurately described. The description is usually linked to migrants' skills as such. We argue theoretically that this gain crucially depends on properties of residents' distributions of their skills.

Researchers, extensively discussing the above issues, outline how migration affects such aspects of economic activity, as employment structure, labor productivity, economic growth, income inequality etc. [Borjas, 1999]. Their predictions are ambiguous. Chiswick [1978] and Carliner [1980] assert that residents, in general, have higher incomes than migrants. However, for skill-extensive sectors, migrants' reward can be higher [Bell, 1997, Grant, 1999]. Accepting the income gap between immigrants and residents, researchers discuss its dynamics. La Londe and Topel [1991], Card [1990, 2005], Lubotsky [2007] affirm the convergence of immigrants' and residents' income, whereas Borjas and Ramey [1995], Borjas [1995], Yuengert [1994] argue that both type of changes are possible, introducing corresponding examples.

Scientists discuss whether immigrants improve the welfare of receiving societies. Passel and Clark [1994] conclude that migrants have been taxed during their life more than supported by social benefits; however Huddle and Network (1993) arrive at the opposite conclusion. The latter is confirmed by Smith et al. [1997], Auerbach and Oreopoulos [1999], Lee and Miller [2000]. The balance between the gain and loss coming from immigrants depends at least on their age: Younger immigrants gives receiving economies a positive balance whereas losses exceed gains in the case of aged immigrants.

The impact of low-skilled immigration on the labor structure in the country of destination is also debated. Card [1990] conclude that a sudden flow of immigrants from Cuba to the US, so called Mariel boatlift, does not affect wages of low-skilled residents. Recently Borjas [2015] have questioned this conclusion and inspired further discussion, controversial at the moment [Borjas, 2016, Peri and Yasenov, 2015].

In this paper we explore the impact of immigration on the share of entrepreneurs, diversity of differentiated good, income inequality, and welfare. We define the threshold level $\hat{\varphi}$ of entrepreneurial abilities allowing entrepreneurs to lunch a profitable firm and find the response of this $\hat{\varphi}$ to the arrival of migrants. The threshold shifts in the direction of skills exhibited by migrants (in other words, $\hat{\varphi}$ goes up/down when migrants are high/low-skilled). However the share of entrepreneurs in the economy can increase as well as decrease, depending on properties of the distribution of residents' and immigrants' entrepreneurial abilities. In particular, when the immigrants are low-skilled and the elasticity of the complement distribution function of residents' entrepreneurial abilities decreases, the share of entrepreneurs shrinks. This elasticity condition is understood as a relative lack of talented entrepreneurs in the receiving economy. On the contrary, economies have created a wide family of talented entrepreneurs who expand its entrepreneurship in response to the arrival of low-skilled immigrants. The counter-intuitiveness of this result underlies its importance. We stress that an economy can benefits from low-skilled immigrants when it is ready for their arrival. Our paper links this "readiness' to (potentially) computable characteristics of the distribution of abilities.

We come to the opposite prediction while exploring the arrival of high-skilled migrants. If the elasticity of the complement distribution function of residents' entrepreneurial abilities decreases, the share of entrepreneurs enlarges. This result is derived under the additional assumption that the average skills of old and new residents are identical. Our prediction is a bit more complicated in general cases.

Our general equilibrium model is based on the selection mechanism between entrepreneurship and hired work suggested by Lucas in his pioneer paper [Lucas, 1978], where individuals choose their occupation by comparing entrepreneurial potential earnings with wages of workers. Following Jovanovic [1994], Evans and Jovanovic [1989], Van Praag and Cramer [2001], etc., we deal with the exogenous distribution of individuals' productivity, the importance of which is highlighted by Baumol [1990]. Individuals' productivity is associated with entrepreneurial ability. The heterogeneity of entrepreneurs induces a heterogeneity of firms that compete non-strategically a lá Melitz (2003) under monopolistic competition, modeled with consumers' CES preferences as in [Dixit and Stiglitz, 1977]. We study the market size effect based on the arrival of new residents. In standard models with CES preferences, the impact of market size is negligible. We are able to analyze the market size effect under consumers' CES preferences because the arrival of immigrants with their own abilities perturbs the distribution of residents' abilities. Such specification underlines the novelty of our approach.

Selection models are widely used in new trade theory and urban economics. The baseline of research takes into account the distribution of skills as given without any perturbation of it [Behrens et al., 2012, 2013, Oyama et al., 2011, Sato et al., 2012, Nocke, 2006, Poschke, 2013, Lazear, 2004, 2005, Crozet and Trionfetti, 2013].

With our selection model, standard in the trade and urban theories, we complement Borjas vs. Peri debates [Borjas, 2015, 2016, Peri and Yasenov, 2015] pointing out at an additional characteristic that affects gains and losses from a low-skilled immigration. We claim that not only the matching between residents' and immigrants' skills but also the distribution of residents' skills affect the labor structure.

The rest of the paper is structured as follows. The next section describes the modeled economy in general. In section 3, we formalize behavior of agents within the economy, define an equilibrium, and prove its existence and uniqueness. Section 4 introduces the main part of the paper; it focuses on the influence of migration on the equilibrium. Section 5 analyzes the impact of the immigration on consumer welfare. The last section concludes our findings. Technical proofs and lemmas are postponed to the Appendix.

2 Economy

We consider a single-sector economy with L individuals that form a set Ξ. The individuals differ by their ability φ to engage in entrepreneurship. The distribution of the abilities is described by a continuous probability density $\gamma(\varphi)$ or cumulative distribution function $\Gamma(\varphi)$. The support of the distribution lies on an interval $(\varphi_0, \varphi_\infty)$, where φ_0 is positive and φ_∞ can be ∞. Then for any $\varphi \in (\varphi_0, \varphi_\infty)$ the mass of individuals with the ability φ is equal to $\gamma(\varphi)L$.

Labor is a unique production factor in the economy. Each individual $\xi \in \Xi$ is endowed by a unit of labor, that is supplied to the market inelastically. She decides upon salaried employment and entrepreneurship. As a worker, the individual is paid a wage w. As an entrepreneur, she launches a firm and receives the operational profit. She chooses the activity that gives her a higher income.

We assume that each firm produces a single variety of the differentiated good under monopolistic competition, using technologies with constant returns to scale. The entrepreneurial ability φ_ξ of the firm's owner ξ represents its productivity: The inverse productivity is proportional to the marginal cost.

In the equilibrium, individuals are split endogenously into entrepreneurs and salaried workers, denoted by Ξ_E and Ξ_W respectively. The two groups are separated by agents with an equilibrium threshold of entrepreneurial abilities $\hat{\varphi}$ who are indifferent between the entrepreneurship and salaried jobs. Their choice regarding employment activity is negligible for the economy. We note that the set Ξ_E of entrepreneurs corresponds to the set of varieties produced in the economy: Each entrepreneur $\xi \in \Xi_E$ is associated with a single-product firm. For the sake of simplicity, we use the notation Ξ_E not only for the set of entrepreneurs but also for the set of varieties of the differentiated good.

3 Basic Model

Supply and demand. We assume that the preferences of all consumers are given by utility function with a constant elasticity of substitution (CES) σ. Individual $\xi \in \Xi$ with an income Y_ξ maximizes her utility

$$U = \left(\int_{\Xi_E} Q_j^{\frac{\sigma-1}{\sigma}}(\xi) \mathrm{d}j \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

subject to a budget constrain to determine her optimal demands $Q_j(\xi)$ for all varieties $j \in \Xi_E$. Consumers altogether form an aggregate demand

$$q_j = \int_{\Xi} Q_j(\xi)\, d\xi.$$

This aggregate demand is a function of prices p_j charged by firms for their varieties j and a market indicator represented by the price index P. Each firm $j \in \Xi_E$ maximizes its operational profit π_j, clearing the market of its variety:

$$\pi_j = (p_j - w/\varphi_j)q_j. \tag{2}$$

If entrepreneurs have identical abilities φ then they produce the same amount of their varieties, price them equally, and get the same profit $\pi(\varphi)$. From now on $\pi(\varphi)$ denotes the optimal profit of an arbitrary firm, whose owner has the entrepreneurial ability φ. The optimal profit $\pi(\varphi)$ evaluated in the Appendix (Lemma 3) depends positively on the owner's entrepreneurial ability φ and negatively on the average productivity $\tilde{\varphi}$ among entrepreneurs[1].

Selection between entrepreneurship and salaried employment. An individual with entrepreneurial ability φ becomes an entrepreneur if she is able to obtain higher income running a firm than working at a salaried job with the wage w: $\pi(\varphi) > w$. By the continuity of $\pi(\varphi)$, there is a threshold level $\hat{\varphi}$ of abilities such that the indifference condition:

$$\pi(\hat{\varphi}) = w \tag{3}$$

is satisfied. The individuals with this $\hat{\varphi}$ are indifferent between entrepreneurship and salaried jobs. Equation (3) closes the model. We argue (see Appendix, Lemma 4) that Equation (3), in terms of the primitives of the model, links the share of entrepreneurs in the economy $\hat{\Gamma}$,

$$\hat{\Gamma} = \int_{\varphi_0}^{\hat{\varphi}} \gamma(\varphi)\, d\varphi,$$

and average entrepreneurs' ability $\tilde{\varphi}$,

$$\tilde{\varphi}^{\sigma-1} = \frac{1}{1-\hat{\Gamma}} \int_{\hat{\varphi}}^{\varphi_\infty} \varphi^{\sigma-1}\gamma(\varphi)\, d\varphi, \tag{4}$$

to the threshold ability $\hat{\varphi}$ in the following way:

$$\hat{\varphi}^{\sigma-1} = \frac{(\sigma-1)(1-\hat{\Gamma})}{\hat{\Gamma}} \tilde{\varphi}^{\sigma-1}. \tag{5}$$

Equilibrium. The triple $\left(\{Q_j(\xi)\}_{j\in\Xi_E,\xi\in\Xi}, \{p_j\}_{j\in\Xi_E}, \hat{\varphi}\right)$ that consists of the set of individuals' consumptions, the set of firms' prices, and the threshold of abilities respectively is called an *equilibrium* if $Q_j(\xi)$ is the optimal demand of consumer $\xi \in \Xi$ for the variety $j \in \Xi_E$ under the set of prices $\{p_j\}_{j\in\Xi_E}$, each p_j solves the optimization problem of firm $j \in \Xi_E$ under the subset $\{Q_j(\xi)\}_{\xi\in\Xi}$ of individuals' demands for the variety j, and, finally, $\hat{\varphi}$ is determined by the indifference condition.

Proposition 1. *The equilibrium exists, and it is unique.*

Equation (5) gives an intuitively tractable prediction regarding the diversity of the differentiated product. Namely, the smaller consumers' relative love for variety $1/(\sigma-1)$ is, the less diversified the differentiated product becomes ($\hat{\varphi}$ goes up in Equation (5) with a growth of σ).

4 Migration

Migrants are characterized by their own distribution of the entrepreneurial abilities, which is, in general, differs from the distribution $\Gamma(\cdot)$ characterized the residents. We consider separately cases of low-skilled and high-skilled migrants, whose abilities are below and, respectively, above the cutoff $\hat{\varphi}$. Migrants' abilities are distributed with

[1]Formal definition of $\tilde{\varphi}$ is given later by Equation (4)

a probability distribution function. Without loss of generality, we reduce this probability density to a delta-function taken at the mean φ_m of the distribution. As a result, the distribution of entrepreneurial abilities in the economy is perturbed at the value φ_m. $\Gamma_{\text{new}}(\cdot)$ and $\gamma_{\text{new}}(\cdot)$ denote the cumulative distribution function and, respectively, probability density of abilities observed after the arrival of migrants. Propositions of this section are formulated in terms of elasticities. We denote $\mathcal{E}_f(x) = f'(x)x/f$ as the elasticity of the function f at a point x.

4.1 Low-skilled migrants

We assume that migrants are characterized by their abilities $\varphi_m < \hat{\varphi}$. Therefore they cannot become entrepreneurs in the country of residence. Their arrival decreases the average level of the entrepreneurial abilities in the economy and increases the demand for the differentiated product. The latter underlies a fall of the cutoff $\hat{\varphi}$ and an increase of the differentiated product's diversity. A rigorous formulation is given by the following Proposition.

Proposition 2. *When low-skilled migrants ($\varphi_m < \varphi$) enter the economy, the cutoff $\hat{\varphi}$ goes down.*

We stress that the direction of changes in cutoff $\hat{\varphi}$ does *not* specify whether the share of entrepreneurs enlarges or shrinks. Indeed, the arrival of migrants is not only followed by the appearance of new entrepreneurs ($\hat{\varphi} \to \hat{\varphi}_{\text{new}} < \hat{\varphi}$). but also by a growth of the total number of population. Therefore the change in the share of entrepreneurs is *apriori* ambiguous. The following Proposition determines when this share goes up or down.

Proposition 3. *The share of the entrepreneurs goes up if and only if the elasticity $\mathcal{E}_{1-\Gamma}(\varphi)$ decreases*[2]*.*

According to Proposition 3, all feasible distributions are split into two classes with Pareto distributions, which have constant elasticities $\mathcal{E}_{1-\Gamma}(\varphi)$, representing a border line. This specification of the border line is linked to the CES preferences of consumers.

We interpret two classes of the distributions introducing a group $\mathcal{T}(\varphi)$ of talented entrepreneurs. The participants of this group are endowed by abilities that are at least φ. Therefore the number of the participants is equal to $(1-\Gamma(\varphi))L$. The group $\mathcal{T}(\varphi)$ can enlarge, continuously admitting new members with frontier abilities. The number $\gamma(\varphi)Ld\varphi$ of new members from the interval $[\varphi - d\varphi, \varphi)$ constitutes the share $\gamma(\varphi)d\varphi/(1-\Gamma(\varphi))$ in the whole group $\mathcal{T}(\varphi)$. Thus, the elasticity $\mathcal{E}_{|\mathcal{T}|}(\varphi)$ of the number $|\mathcal{T}(\varphi)|$ of the participants of the group $\mathcal{T}(\varphi)$ with respect to φ is equal to $\mathcal{E}_{1-\Gamma}(\varphi)$. We argue that if $\mathcal{E}_{1-\Gamma}(\varphi)$ decreases, the expansion of talented individuals occurs with an increasing intensity. In this case, a decline of the census φ of the group $\mathcal{T}(\varphi)$ makes the number $|\mathcal{T}(\varphi)|$ of talented individuals fall drastically. Such economies incur a decrease of the number of entrepreneurs when low-skilled migrants arrive.

4.2 High-skilled migrants

Now we explore the case of migrants that demonstrate higher entrepreneurial abilities than the individual with the cutoff level $\hat{\varphi}$ of abilities who is indifferent between entrepreneurship and salaried jobs. Formally, it means that $\varphi_m > \hat{\varphi}$, where φ_m, as before, characterizes the average entrepreneurial ability of migrants. These new coming individuals become entrepreneurs because their entrepreneurial abilities are quite high. Their arrival increases the cutoff level of entrepreneurial abilities, shifting $\hat{\varphi}$ to the direction of φ_m. As a result, some residents are pushed out from entrepreneurship to salaried jobs. This is proved in Lemma 11 formulated in the Appendix.

We argue that high-skilled migration triggers three effects. First, the economy enlarges with the arrival of high-skilled migrants. Second, running firms, these migrants increase the number of the entrepreneurs in the

[2]mathematically correct statement involves non-strict inequalities

economy. Third, a part of entrepreneurs-residents changes their activity from entrepreneurship to salaried jobs. They altogether can ambiguously affect the employment structure. The direction of the integral effect depends on consumers' preferences, the structure of entrepreneurial abilities, and the level of migrants' skills. The latter is described by the variable k,

$$k = \frac{\int_{\hat{\varphi}}^{\varphi_\infty} \varphi^{\sigma-1}\gamma(\varphi)\, d\varphi}{(1-\hat{\Gamma})\varphi_m^{\sigma-1}}, \tag{6}$$

which measures the correspondence of immigrants' entrepreneurial abilities to the average level of entrepreneurs.

Proposition 4. *The share of the entrepreneurs increases if and only if the elasticity of CDF $\Gamma(\cdot)$ is big, or migrants are rather talented (k is small), or consumers are diversity-lovers (σ is also small).*

The Proof immediately follows from the technical result formulated and proved in Lemma 13 (see the Appendix).

Proposition 4 formulates qualitative conditions that guarantee the enlargement as well as shrink of the entrepreneurial sector. These conditions are sufficient but not necessary. The formulation of necessary conditions requires specifications of $\Gamma(\varphi)$. The following Proposition completely resolves the ambiguity of the integral effect triggered by the appearance of high-skilled migrants, under a Pareto distribution of abilities:

$$\Gamma(\varphi) = 1 - \left(\frac{\varphi_0}{\varphi}\right)^t, \quad \gamma(\varphi) = \frac{t\varphi_0^t}{\varphi^{t+1}}, \quad \varphi_0 > 0, \tag{7}$$

where $t > \sigma - 1$.

Proposition 5. *Let entrepreneurial abilities follow the Pareto distribution defined by* (7). *If $k < 1$, then the share of entrepreneurs increases: $\Delta\Gamma > 0$. On the contrary, if $k > 1$, the share of entrepreneurs decreases: $\Delta\Gamma < 0$.*

The Proof is given in the Appendix.

Exhibiting the effects mentioned above, Proposition 5 states that the refuse from entrepreneurship by residents affects the share of the entrepreneurs stronger than the entry of high-skilled migrants, if migrants' skills are high indeed ($k > 1$). The following mechanism underlies Proposition 5. High-skilled new entrepreneurs price their products lower than their competitors do on average. Consumers switch their demand to these more attractive products of entrepreneurs-migrants. Firms run by insufficiently skilled residents become non-profitable and quit. Former entrepreneurs move to salaried jobs, and the share of workers increases.

The border line between a growth and decline of the share of entrepreneurs is characterized by $k = 1$. This equality means that the entrepreneurial abilities of migrants are identical to the average entrepreneurial ability among residents who currently run firms. We show that these specific abilities of migrants balance the appearance of new entrepreneurs and the exit from entrepreneurship of less skilled residents, keeping the employment structure unchanged.

The exceptional case $k = 1$ is explored here with the unspecified distribution of entrepreneurial abilities.

Proposition 6. *Let the entrepreneurial abilities of migrants coincide with the average abilities of the entrepreneurs-residents: $k = 1$. We claim that the share of the entrepreneurs goes up if and only if the elasticity $\mathcal{E}_{1-\Gamma}(\varphi)$ increases*[3].

Proposition 6 adds necessary conditions to sufficient conditions given by Proposition 4 in specific case $k = 1$.

[3]mathematically correct statement involves non-strict inequalities

5 Welfare

We turn to the welfare analysis. The appearance of migrants raises the demand for the differentiated good. Responding to this rise the market enlarges the supply side. The integral effect of these changes on the structure of individual consumption depends on the role of immigrants in the economy (entrepreneurs or workers). The following two Propositions address the two cases consequently.

Proposition 7. *Let the level of immigrants' abilities be less than the cutoff: $\varphi_m < \hat{\varphi}$. Then the utility of each resident increases.*

Proof follows from Lemma 16 and 17.

Corollary 1. *The welfare of the economy also increases.*

The following intuition lies behind Proposition 7. Entrepreneurs become better off because of an increasing demand. A portion of workers becomes entrepreneurs, since a lower threshold of entrepreneurship allows them to change fields. Their utility increases with income. When new firms operate on the market, individuals diversify their consumptions satisfying their love for variety. An increasing diversity of consumption drops the price index and pushes the utility of workers up.

Proposition 8. *Let the level of immigrants' abilities be greater than the cutoff: $\varphi_m > \hat{\varphi}$. Then, first the utility of each worker increases. Second, the utility of each entrepreneur increases if and only if $\mathcal{E}_\Gamma(\hat{\varphi}) > (\sigma-1)(\sigma-2)$. Third, the integral profit of firms increases such that its ratio to the integral wages is constant.*

Proof follows from Lemmata 19, 20, and 22.

When skilled migrants enter the economy ($\varphi_m > \hat{\varphi}$) they become entrepreneurs. Repeating the above arguments we find out that the price index also decreases. Therefore the utility of workers increases. Changes in the utility of entrepreneurs are more complicated. The value of the parameter σ underlies the two different outcomes. If σ is big, then the diversity of the differentiated product is small. This induces a small number of firms and entrepreneurs on the market. Therefore the share of immigrants among entrepreneurs is noticeable. The integral profit of entrepreneurs increases but the share of residents-entrepreneurs is small to benefit from this rise of the profit. On the contrary, with small values of σ entrepreneurs-residents are able to benefit from the rise of the integral profit.

We describe the direction of changes in the inequality between entrepreneurs with a given ability and the least skilled entrepreneur (who has the threshold ability). The inequality increases when the threshold $\hat{\varphi}$ decreases and vice versa (Lemmata 18 and 21). Indeed, the impact from immigration is decomposed into the gain from the drop of the price index affected individuals equally and changes in the profit. The latter is 1 for the threshold entrepreneur and $\left(\varphi/\hat{\varphi}\right)^{\sigma-1}$ for an entrepreneur with the ability φ. We note that different entrepreneurs correspond to a fixed quantile of all entrepreneurs before and after the arrival of immigrants because the share of entrepreneurs varies. Therefore standard measures of inequalities alike differences in quantiles or the Ginni coefficient follow changes in the inverse threshold $\varphi/\hat{\varphi}$ only for a limited class of `cdfs` Γ that generate quite a small shift in abilities' quantiles when immigrants arrive. An independent study can consider a complete description of inequality between different groups of population under immigration as a challenge.

6 Conclusion

We explore the impact of migration on macroeconomic variables, welfare, and inequality. We find out that immigrants affect positively the welfare of the economy. At the individual level workers are also always better off.

But the changes in the utility of entrepreneurs-residents can be negative, if immigrants have large entrepreneurial skills and the demand for varieties of the differentiated good is quite elastic. In this case the differentiated good is not diversified enough. The latter can be associated with low developed economies. In the other cases, entrepreneurs-residents are better off.

We give a complete classification of the impact of immigration on the labor structure, which is the ratio of the number of entrepreneurs to the number of workers. This share responds ambiguously to the arrival of migrants. The classification is based on the shape of the distribution of residents' abilities and the skills of immigrants.

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Appendix

Lemma 1. *Let a consumer $\xi \in \Xi$ with the income Y_ξ maximize her utility* (1) *subject to the budget constrain*

$$\int_{\Xi_E} p_j Q_j(\xi)\, dj \le Y_\xi.$$

Then her demand $Q_j(\xi)$ for the variety j is equal to

$$Q_j(\xi) = p_j^{-\sigma} Y_\xi P^{\sigma-1}, \tag{8}$$

where

$$P = \left(\int_{\Xi_E} p_j^{1-\sigma}\, dj \right)^{\frac{1}{1-\sigma}}.$$

The Proof of Lemma 1 is derived by using a standard technique.

Lemma 2. *We consider a firm j launched by an individual with the entrepreneurial ability φ_j. This firm maximizes its profit* (2)*, given optimal demands* (8)*. Then the optimal price and profit of the firm are determined by the entrepreneurial ability φ_j. In particular,*

$$\pi(\varphi_j) = \frac{\varphi_j^{\sigma-1}}{\int_{\Xi_E} \varphi_{j'}^{\sigma-1}\, dj'} \frac{Y}{\sigma}, \tag{9}$$

where $Y = \int_\Xi Y_\xi \mathrm{d}\xi$ is the aggregate income in the economy and $N = |\Xi_E|$ is the number (mass) of firms.

Proof. Firm j faces the following aggregate demand for its variety:

$$q_j = p_j^{-\sigma} P^{\sigma-1} Y, \tag{10}$$

where

$$P^{1-\sigma} = \int_{\Xi_E} p_j^{1-\sigma}\, dj.$$

is the price index for the differentiated product. Maximizing its profit, the firm j charges the price

$$p_j = \frac{\sigma w}{(\sigma-1)\varphi_j}. \tag{11}$$

for its good. Then the price index satisfies the following equation:

$$P^{\sigma-1} = \left(\frac{\sigma-1}{\sigma w} \right)^{\sigma-1} \frac{1}{\int_{\Xi_E} \varphi_j^{\sigma-1}\, dj}. \tag{12}$$

Substituting (10), (11), and (12) into (2), we get the statement of the Lemma. □

We stress that Equation (9) defines the system of equations with respect to all profits $\pi(\varphi_j)$, $j \in \Xi_E$, because firms' profits constitute a part of the total income Y of individuals. The following Lemma solves this system.

Lemma 3. *The optimal profit of a firm launched by an individual with the entrepreneurial ability φ is*

$$\pi(\varphi) = \frac{w}{\sigma - 1} \frac{\varphi^{\sigma-1}}{\tilde{\varphi}^{\sigma-1}} \frac{\hat{\Gamma}}{1 - \hat{\Gamma}}, \tag{13}$$

where the average entrepreneurs' productivity $\tilde{\varphi}$ is defined in (4).

Proof. The aggregate income written in the right hand side of Equation (9) consists of the wages of salaries workers and the incomes of entrepreneurs. The latter coincide with the profits of firms. The total income of salaried workers is equal to $L(1 - \hat{\Gamma})w$. The total income of entrepreneurs is obtained as the sum of all profits. Therefore, Equation (9) is transformed into

$$\pi_j = \frac{\sigma (P\varphi_j)^{\sigma-1}}{w^{\sigma-1}} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \left((1 - \hat{\Gamma})Lw + \int_J \pi_j \mathrm{d}j\right). \tag{14}$$

Formula (14) represents a system of equations with respect to the profits π_j, where $j \in J$ belongs to a continuous set. We denote $\Pi = \int_J \pi_j \, dj$ the aggregate profit. Integrating (14) with respect to the variable j, we have the following equation with respect to Π:

$$\Pi = \frac{\sigma P^{\sigma-1} I}{w^{\sigma-1}} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \left((1 - \hat{\Gamma})Lw + \Pi\right), \tag{15}$$

where

$$I = \int_J \varphi_j^{\sigma-1} \, dj.$$

Solving Equation (15) with respect to the aggregate profit Π and substituting the result into Equation (14), we obtain:

$$\pi(\varphi_j) = \frac{w}{\sigma - 1} \frac{\varphi_j^{\sigma-1} \hat{\Gamma}}{\int_{\Xi_E} \varphi_{j'}^{\sigma-1} \, dj'}. \tag{16}$$

The number of firms organized by individuals with the entrepreneurial abilities φ is $L\varphi\gamma(\varphi)$. Therefore

$$\int_{\Xi_E} \varphi_{j'}^{\sigma-1} \, dj' = \int_{\hat{\varphi}}^{\varphi_\infty} \varphi^{\sigma-1} \gamma(\varphi) \, d\varphi,$$

and Equation (16) and (13) are equivalent. $\square$

Lemma 4. *Let the average abilities $\tilde{\varphi}$ of* entrepreneurs *be defined by* (4)*. Then Equation* (5) *is valid.*

Proof. Substituting $\hat{\varphi}$ for φ into Equation (13), we obtain the profit of the firm run by individuals that are indifferent between entrepreneurship and salaried jobs. Then Equation (3) is turned to (5) after elementary algebra. $\square$

Lemma 5. *The price index P defined in Lemma 1 is given by the following equation:*

$$P^{\sigma-1} = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \frac{1}{L\hat{\Gamma}\hat{\varphi}^{\sigma-1}}.$$

Proof. Substituting the optimal prices into the definition of the price index, one get the statement of the Lemma. $\square$

Proof of Proposition 1. The left hand side increases in $\hat{\varphi}$ from φ_0 to φ_∞. The right hand side is a positive decreasing function of $\hat{\varphi}$. Therefore the two hand sides have a unique intersection. $\square$

The new probability density of individuals' entrepreneurial abilities is given by

$$\gamma_{\text{new}}(\varphi) = \begin{cases} \frac{L}{L+\Delta L}\gamma(\varphi) + \frac{\Delta L}{L+\Delta L}\gamma_m(\varphi) & \text{if } \varphi \neq \varphi_m, \\ \frac{L}{L+\Delta L}\delta(\varphi) & \text{if } \varphi = \varphi_m, \end{cases} \tag{17}$$

where $\delta(\cdot)$ is the Dirac delta-function.

Lemma 6. *Let $\varphi_m < \hat{\varphi}$. Then*

$$\big(\sigma\hat{\gamma}\hat{\varphi} + \hat{\Gamma}(\sigma-1)\big)\frac{\Delta\varphi}{\varphi} = -\frac{\Delta L}{L}. \tag{18}$$

In particular,

$$\mathcal{E}_{\hat{\varphi}}(L) = -\frac{1}{(\sigma-1)\hat{\Gamma}} \cdot \frac{1}{\frac{\sigma}{\sigma-1}\mathcal{E}_{\Gamma}(\hat{\varphi}) + 1} < 0.$$

Proof. We put

$$I = \int_{\hat{\varphi}}^{\varphi_\infty} \varphi^{\sigma-1}\gamma(\varphi)\, d\varphi.$$

Then the indifference condition (5) is equivalent to

$$\hat{\varphi}^{\sigma-1} = \frac{(\sigma-1)I}{\hat{\Gamma}} \tag{19}$$

We write this equation for the new probability density $\gamma_{\text{new}}(\cdot)$ and expand all terms into series up to the second order terms. First,

$$\hat{\Gamma}_{\text{new}} = \hat{\Gamma}\left(1 - \frac{\Delta L}{L}\right) + \hat{\gamma}\Delta\varphi + \frac{\Delta L}{L}.$$

Second, let

$$I_{\text{new}} = \int_{\hat{\varphi}}^{\varphi_\infty} \varphi^{\sigma-1}\gamma_{\text{new}}(\varphi)\, d\varphi.$$

Then

$$\int_{\hat{\varphi}_{\text{new}}}^{\varphi_\infty} \varphi^{\sigma-1}\gamma_{\text{new}}(\varphi)\, d\varphi = \int_{\hat{\varphi}}^{\varphi_\infty} \varphi^{\sigma-1}\frac{L}{L+\Delta L}\gamma(\varphi)\, d\varphi - \int_{\hat{\varphi}}^{\hat{\varphi}_{\text{new}}} \varphi^{\sigma-1}\frac{L}{L+\Delta L}\gamma(\varphi)\, d\varphi =$$
$$I\left(1 - \frac{\Delta L}{L}\right) - \hat{\gamma}\hat{\varphi}^{\sigma-1}\Delta\varphi.$$

Third,

$$\hat{\varphi}_{\text{new}}^{\sigma-1} = \hat{\varphi}^{\sigma-1} + (\sigma-1)\hat{\varphi}^{\sigma-2}\Delta\varphi.$$

Substituting those expansion into Equation (19), we have

$$\hat{\Gamma}\hat{\varphi}^{\sigma-1}\left(1 - \frac{\Delta L}{L}\right) + \hat{\gamma}\hat{\varphi}^{\sigma-1}\Delta\varphi + \hat{\varphi}^{\sigma-1}\frac{\Delta L}{L} + (\sigma-1)\hat{\Gamma}\hat{\varphi}^{\sigma-2}\Delta\varphi = (\sigma-1)I\left(1 - \frac{\Delta L}{L}\right) - (\sigma-1)\hat{\gamma}\hat{\varphi}^{\sigma-1}\Delta\varphi.$$

This equation together with (19) turns to the statement of the Lemma. □

Proof of Proposition 2. The proof directly follows from Lemma 6. □

Lemma 7. *Let $\varphi_m < \hat{\varphi}$. Then*

$$\frac{\Delta\Gamma}{1-\hat{\Gamma}} = \left(1 - \frac{\hat{\gamma}\hat{\varphi}}{(1-\hat{\Gamma})(\sigma\hat{\gamma}\hat{\varphi} + (\sigma-1)\hat{\Gamma}}\right)\frac{\Delta L}{L}. \tag{20}$$

Proof. We repeat the expansion of $\hat{\Gamma}_{\text{new}}$ into series undertaken during the Proof of Lemma 6:

$$\hat{\Gamma}_{\text{new}} = \hat{\Gamma}\left(1 - \frac{\Delta L}{L}\right) + \hat{\gamma}\Delta\varphi + \frac{\Delta L}{L}.$$

Combining this with Equation (18), we obtain

$$\frac{\Delta\Gamma}{1-\Gamma} = \left(1 - \frac{\hat{\gamma}\hat{\varphi}}{(1-\hat{\Gamma})(\sigma\hat{\gamma}\hat{\varphi} + (\sigma-1)\hat{\Gamma})}\right)\frac{\Delta L}{L}. \tag{21}$$

Re-arranging terms in (21), we obtain the statement of the Lemma. □

Lemma 8. *Let $\varphi_m < \hat{\varphi}$. Then*

$$\Delta\Gamma \gtrless 0 \quad \Longleftrightarrow \quad \mathcal{E}_\Gamma(\hat{\varphi})\big(1 - \sigma(1-\hat{\Gamma})\big) \lesseqgtr (\sigma-1)(1-\hat{\Gamma}) \tag{22}$$

Proof. Equation (20) determines $\Delta\Gamma$. Re-arranging terms in the inequality

$$1 - \frac{\hat{\gamma}\hat{\varphi}}{(1-\hat{\Gamma})(\sigma\hat{\gamma}\hat{\varphi} + (\sigma-1)\hat{\Gamma}} > 0,$$

we obtain (22). □

Lemma 9. *Let*

$$\lim_{\varphi\to+\infty} \varphi^{\sigma-1}(1-\Gamma(\varphi)) = 0$$

Then the indifference condition is equivalent to

$$\hat{\Gamma} = \frac{\sigma-1}{\sigma}\left(1 + \frac{\sigma-1}{\hat{\varphi}^{\sigma-1}} I_2\right), \tag{23}$$

where

$$I_2 = \int_{\hat{\varphi}}^{\varphi\infty} \varphi^{\sigma-2}(1-\Gamma(\varphi))\,d\varphi.$$

Proof. The expression $\tilde{\varphi}(1-\hat{\Gamma})$ from the indifference condition is integrated by parts in the following way:

$$\tilde{\varphi}(1-\hat{\Gamma}) = -\int_{\hat{\varphi}}^{\varphi_\infty} \varphi^{\sigma-1} d(1-\Gamma(\varphi)) = \hat{\varphi}^{\sigma-1}(1-\hat{\Gamma}) + (\sigma-1)I_2, \tag{24}$$

since the integrated term $\varphi^{\sigma-1}(1-\Gamma(\varphi))$ is equal to zero at φ_∞. Substituting $\tilde{\varphi}(1-\hat{\Gamma})$ given by Equation (24) into (5), we get Equation (23). □

Lemma 10. *Let $\varphi_m < \hat{\varphi}$. Then*

$$\Delta\Gamma \gtrless 0 \quad \Longleftrightarrow \quad \frac{\hat{\gamma}(\sigma-1)}{\hat{\varphi}^{\sigma-2}} I_2 \gtrless \hat{\Gamma}(1-\hat{\Gamma}),$$

where I_2 is the same as in Lemma 9.

Proof. The right inequality given by (22) is equivalent to

$$\frac{\hat{\gamma}\hat{\varphi}}{\sigma-1}\big(1 - \sigma(1-\hat{\Gamma})\big) \gtrless \hat{\Gamma}(1-\hat{\Gamma}). \tag{25}$$

We use Equation (23) to evaluate the left hand side of Inequality (25) ($\hat{\Gamma}$ in the right hand side is not changed). As a result, the statement of the Lemma is obtained. □

Proof of Proposition 3. We re-write the integral I_2 defined in Lemma 9:

$$I_2 = \frac{1-\hat{\Gamma}}{\hat{\gamma}\hat{\varphi}} \int_{\hat{\varphi}}^{\varphi_\infty} \frac{\mathcal{E}_{1-\Gamma}(\hat{\varphi})}{\mathcal{E}_{1-\Gamma}(\varphi)} \varphi^{\sigma-1}\gamma(\varphi)\, d\varphi.$$

If the elasticity $\mathcal{E}_{1-\Gamma}(\varphi)$ is an increasing function, then

$$\frac{\mathcal{E}_{1-\Gamma}(\hat{\varphi})}{\mathcal{E}_{1-\Gamma}(\varphi)} < 1 \quad \text{and} \quad I_2 < \frac{1-\hat{\Gamma}}{\hat{\gamma}\hat{\varphi}} I.$$

With I expressed from the indifference condition (5), the latter inequality is transformed into

$$\frac{\hat{\gamma}(\sigma-1)}{\hat{\varphi}^{\sigma-2}} I_2 < \hat{\Gamma}(1-\hat{\Gamma}). \tag{26}$$

According to Lemma 10, Inequality (26) means that $\Delta\Gamma < 0$. Under a decreasing elasticity $\mathcal{E}_{1-\Gamma}(\varphi)$, applying the same technique, we end up with the opposite conclusion: $\Delta\Gamma > 0$. □

Lemma 11. *Let $\varphi_m > \hat{\varphi}$, $\Delta\varphi = \hat{\varphi}_{new} - \hat{\varphi}$. Then*

$$\big(\sigma\hat{\gamma}\hat{\varphi} + (\sigma-1)\hat{\Gamma}\big)\frac{\Delta\varphi}{\hat{\varphi}} = (\sigma-1)\frac{\varphi_m^{\sigma-1}}{\hat{\varphi}^{\sigma-1}}\frac{\Delta L}{L}. \tag{27}$$

In particular,

$$\mathcal{E}_{\hat{\varphi}}(L) = \frac{1}{\hat{\Gamma}}\left(\frac{\varphi_m}{\hat{\varphi}}\right)^{\sigma-1} \frac{1}{\frac{\sigma}{\sigma-1}\mathcal{E}_{\Gamma}(\hat{\varphi}) + 1} > 0. \tag{28}$$

Proof. We expand three factors of (19) into series up to the first order terms. First,

$$\hat{\Gamma}_{\text{new}} = \hat{\Gamma}\left(1 - \frac{\Delta L}{L}\right) + \hat{\gamma}\Delta\varphi$$

Second, let

$$I = \int_{\hat{\varphi}}^{\varphi_\infty} \varphi^{\sigma-1}\gamma(\varphi)\, d\varphi.$$

Then

$$\int_{\hat{\varphi}_{\text{new}}}^{\infty} \varphi^{\sigma-1}\gamma_{\text{new}}(\varphi)\, d\varphi = \int_{\hat{\varphi}}^{\infty} \varphi^{\sigma-1}\frac{L}{L+\Delta L}\gamma(\varphi)\, d\varphi - \int_{\hat{\varphi}}^{\hat{\varphi}_{\text{new}}} \varphi^{\sigma-1}\frac{L}{L+\Delta L}\gamma(\varphi)\, d\varphi + \varphi_m^{\sigma-1}\frac{\Delta L}{L} =$$
$$I\left(1 - \frac{\Delta L}{L}\right) - \hat{\gamma}\hat{\varphi}^{\sigma-1}\Delta\varphi + \varphi_m^{\sigma-1}\frac{\Delta L}{L}.$$

Third,

$$\hat{\varphi}_{\text{new}}^{\sigma-1} = \hat{\varphi}^{\sigma-1} + (\sigma-1)\hat{\varphi}^{\sigma-2}\Delta\varphi.$$

Substituting those expansion into Equation (19), we have

$$\hat{\Gamma}\hat{\varphi}^{\sigma-1}\left(1 - \frac{\Delta L}{L}\right) + \hat{\gamma}\hat{\varphi}^{\sigma-1}\Delta\varphi + (\sigma-1)\hat{\Gamma}\hat{\varphi}^{\sigma-2}\Delta\varphi = (\sigma-1)I\left(1 - \frac{\Delta L}{L}\right) - (\sigma-1)\hat{\gamma}\hat{\varphi}^{\sigma-1}\Delta\varphi + (\sigma-1)\varphi_m^{\sigma-1}\frac{\Delta L}{L}.$$

This equation together with (19) turns to Equation (27). Equation (28) follows directly from (27). □

Lemma 12. *Let $\varphi_m > \hat{\varphi}$. Then*

$$\frac{\Delta\Gamma}{\hat{\Gamma}} = \left(-1 + \frac{\hat{\gamma}\hat{\varphi}}{k(1-\hat{\Gamma})(\sigma\hat{\gamma}\hat{\varphi} + (\sigma-1)\hat{\Gamma}}\right)\frac{\Delta L}{L}. \tag{29}$$

Proof. We repeat the expansion of $\hat{\Gamma}_{\text{new}}$ into series undertaken during the Proof of Lemma 11:

$$\hat{\Gamma}_{\text{new}} = \hat{\Gamma}\left(1 - \frac{\Delta L}{L}\right) + \hat{\gamma}\Delta\varphi.$$

Combining this with Equation (27) we obtain

$$\frac{\Delta \Gamma}{\Gamma} = \left(-1 + \frac{\hat{\gamma}\hat{\varphi}}{\hat{\Gamma}}\left(\frac{\varphi_m}{\hat{\varphi}}\right)^{\sigma-1}(\sigma-1)\frac{1}{\sigma\hat{\gamma}\hat{\varphi} + (\sigma-1)\hat{\Gamma}}\right)\frac{\Delta L}{L}. \tag{30}$$

By using indifference condition (5) and the definition of k given by (6), we transform:

$$\left(\frac{\varphi_m}{\hat{\varphi}}\right)^{\sigma-1}(\sigma-1) = \frac{\hat{\Gamma}\varphi_m^{\sigma-1}}{\int_{\hat{\varphi}}^{\varphi_\infty}\varphi^{\sigma-1}\gamma(\varphi)\,d\varphi} = \frac{\hat{\Gamma}}{k(1-\hat{\Gamma})}.$$

Substitution this Equation into Equation (30) leads to the statement of the Lemma. □

Lemma 13. *Let $\varphi_m > \hat{\varphi}$. Then*

$$\mathcal{E}_\Gamma(L) \gtreqless 0 \iff \mathcal{E}_\Gamma(\hat{\varphi})\big(1 - \sigma k(1-\hat{\Gamma})\big) \gtreqless k(1-\hat{\Gamma})(\sigma-1). \tag{31}$$

Proof. Equation (29) determines the elasticity of Γ with respect to L. Its sign coincides with the sign of the bracket written in the righ hand side of Equation (29). Transforming this bracket, we come to the statement of the Lemma. □

Lemma 14. *Let the distribution of the abilities be power (given by Equation (7)). Then*

$$\hat{\Gamma} = \frac{t(\sigma-1)}{t\sigma - (\sigma-1)}.$$

Proof. Under Pareto distribution given by (7) Cutoff Equation (5) is transformed into

$$\hat{\varphi}^{\sigma-1} = \frac{\sigma-1}{\hat{\Gamma}} \cdot \frac{t(1-\hat{\Gamma})\hat{\varphi}^{\sigma-1}}{t-(\sigma-1)}.$$

This equation is equivalent to the statement of the Lemma. □

Proof of Proposition 5. Power distribution (7) is characterized by the elasticity $\mathcal{E}_\Gamma(\hat{\varphi}) = t(1-\hat{\Gamma})/\hat{\Gamma}$. Substituting $\hat{\Gamma}$ found in Lemma 14 into Statement (31), we conclude that

$$\mathcal{E}_\Gamma(L) \gtreqless 0 \iff t(t\sigma - \sigma + 1 - \sigma k t + \sigma k(\sigma-1)) \gtreqless kt(\sigma-1)^2.$$

The right inequality is equivalent to

$$(1-k)\big((\sigma-1)^2 + \sigma(t-(\sigma-1))\big) \gtreqless 0.$$

Since $t > \sigma - 1$, it follows that the sign of the left hand side of the last inequality follows the sign of $1-k$. The Proposition is proved. □

Proof of Proposition 6 is similar to the Proof of Proposition 3.

Lemma 15. *The utility U of an individual with the income Y is*

$$U = \frac{Y}{P},$$

where P is the price index.

Proof. Combining definition (1) of the utility with the optimal demand and the price index given by Lemma 1, we turn to the statement of the Lemma. ◻

Lemma 16. *Let $\varphi_m < \hat{\varphi}$. Then the utility U_w of a worker increases, and*

$$\frac{\Delta U_w}{U_w} = \frac{\hat{\gamma}\hat{\varphi}}{\hat{\Gamma}} \frac{1}{(\sigma-1)\hat{\Gamma} + \sigma\hat{\gamma}\hat{\varphi}} \frac{\Delta L}{L} > 0. \tag{32}$$

Proof. According to Lemma 15,

$$\frac{\Delta U_w}{U_w} = -\frac{\Delta P}{P}.$$

Using Lemma 5 we get

$$(\sigma-1)\frac{\Delta U_w}{U_w} = \frac{\Delta L}{L} + (\sigma-1)\frac{\Delta\hat{\varphi}}{\hat{\varphi}} + \frac{\Delta\hat{\Gamma}}{\hat{\Gamma}}$$

Substituting $\Delta\hat{\varphi}$ and $\Delta\hat{\Gamma}$ found in Lemmata 6 and 7 into the last equation, we obtain the statement of the Lemma. ◻

Lemma 17. *Let $\varphi_m < \hat{\varphi}$. Then the welfare of an entrepreneur with an ability φ increases, and*

$$\frac{\Delta U_e(\varphi)}{U_e(\varphi)} = \left(\frac{\hat{\gamma}\hat{\varphi}}{\hat{\Gamma}} + \sigma - 1\right) \frac{1}{(\sigma-1)\hat{\Gamma} + \sigma\hat{\gamma}\hat{\varphi}} \frac{\Delta L}{L} > 0. \tag{33}$$

Proof. According to Lemma 15,

$$U_e(\varphi) = \pi(\varphi) U_w.$$

Lemma 3 and 4 imply that

$$\pi(\varphi) = \frac{\varphi^{\sigma-1}}{\hat{\varphi}^{\sigma-1}}.$$

Based on these two equations we have to repeat the Proof of the previous Lemma. ◻

Lemma 18. *Let $\varphi_m < \hat{\varphi}$. Then the inequality between an entrepreneur with the ability φ and the least skilled entrepreneurs increases in terms of their indirect utilities:*

$$\Delta\left(\frac{U_e(\varphi)}{U_e(\hat{\varphi})}\right) > 0.$$

In particular, the statement is valid for the most skilled entrepreneur.

Proof.

$$\Delta\left(\frac{U_e(\varphi)}{U_e(\hat{\varphi})}\right) = \Delta\big(\pi(\varphi)\big) = \Delta\left(\frac{\varphi^{\sigma-1}}{\hat{\varphi}^{\sigma-1}}\right) = \varphi^{\sigma-1}\Delta\left(\frac{1}{\hat{\varphi}^{\sigma-1}}\right).$$

According to Lemma 6, the last expression is positive. ◻

Lemma 19. *Let $\varphi_m > \hat{\varphi}$. Then the welfare U_w of a worker increases, and*

$$\frac{\Delta U_w}{U_w} = \frac{\hat{\gamma}\hat{\varphi} + (\sigma-1)\hat{\Gamma}}{k(1-\hat{\Gamma})} \frac{1}{(\sigma-1)\hat{\Gamma} + \sigma\hat{\gamma}\hat{\varphi}} \frac{\Delta L}{L} > 0. \tag{34}$$

Proof. The Proof of the Lemma repeats the Proof of Lemma 16 involving the statements of Lemma 12 and 11. ◻

Lemma 20. *Let $\varphi_m > \hat{\varphi}$. Then the change in the welfare of an entrepreneur with an ability φ is given by the following Equation:*

$$\frac{\Delta U_e(\varphi)}{U_e(\varphi)} = \left(\frac{\hat{\gamma}\hat{\varphi}}{\hat{\Gamma}} - (\sigma-1)(\sigma-2)\right) \frac{\hat{\Gamma}}{k(1-\hat{\Gamma})\big((\sigma-1)\hat{\Gamma} + \sigma\hat{\gamma}\hat{\varphi}\big)} \frac{\Delta L}{L} > 0. \tag{35}$$

Proof. The Proof of the Lemma repeats the Proof of Lemma 17 involving the statements of Lemma 12 and 11. □

Lemma 21. *Let $\varphi_m > \hat{\varphi}$. Then the inequality between an entrepreneur with the ability φ and the least skilled entrepreneurs decreases in terms of their indirect utilities:*

$$\Delta\left(\frac{U_e(\varphi)}{U_e(\hat{\varphi})}\right) < 0.$$

In particular, the statement is valid for the most skilled entrepreneur.

Proof. Using Lemmata 15, 11, and 13, we compute

$$\Delta\left(\frac{U_e(\varphi)}{U_e(\hat{\varphi})}\right) = \Delta\big(\pi(\varphi)\big) = \Delta\left(\frac{\varphi^{\sigma-1}}{\hat{\varphi}^{\sigma-1}}\right) = \varphi^{\sigma-1}\Delta\left(\frac{1}{\hat{\varphi}^{\sigma-1}}\right).$$

According to Lemma 11, the last expression is negative. □

Lemma 22. *Let $\varphi_m > \hat{\varphi}$. The integral profit of firms is*

$$\Pi = \frac{L\hat{\Gamma}}{\sigma - 1},$$

the integral wages of workers is

$$W = L\hat{\Gamma}.$$

The both quantities increase when immigrants arrive.

Proof. We argue that

$$\Pi = L\int_{\hat{\varphi}}^{\varphi_\infty}\left(\frac{\varphi}{\hat{\varphi}}\right)^{\sigma-1}\gamma(\varphi).\,d\varphi.$$

Combining this equation with Equation (5) we obtain the expression for the integral profit. The equation for the aggregate wages is evident, since all workers have identical wages equaled to 1 whereas $L\hat{\Gamma}$ is the number of workers. Lemma 12 implies that the integral profit and the integral wages increase. □

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