Benchmark characteristic as a proxy of Quality under Duopoly

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Abstract

This paper studies the incentives to invest in benchmark characteristic that is relevant for objective quality only up to a certain level, unknown for consumers. Unlike some existing models with a fixed proportion of informed consumers, we assume that information acquisition is a possible consumer's choice and comes at a cost.

If the cost of investment is small enough compared to magnitude of the upgrade and consumers' cost of discovering the objective quality, there are always inefficient pooling equilibria with either incentives for pricing irrelevant characteristic or no incentives for a useful characteristic update. But due to rival's incentives to "denounce" the lie about relevance of a useless characteristic, duopolistic competition also makes sustainable an efficient separating equilibrium that reproduces the full information outcome without distortions. Moreover, both duopolistic firms enjoys highest ex ante profits in this efficient separating equilibrium. However, this separating equilibrium disappears if the upgrade of benchmark characteristic is drastic, as the rival firm prefers to benefit from increased differentiation.

Keywords: quality competition, signalling, benchmark characteristic, consumer uncertainty

JEL Classification Codes;: D42, D43, D82, L15

Introduction

Very often, technological products (like digital cameras for instance) may be so complex that it is quite difficult for a non-professional consumer to fully ascertain their value or their performance.

Usually, in these cases, when deciding which product to choose, consumers have to apply some "rule-of-a-thumb" to simplify their decisions by focusing on only one particular characteristic of a product (megapixel definition for digital cameras). It is then very common that such characteristic of a product becomes the benchmark indicator of quality. At the same time, there have been a number of situations where markets of technological goods witnessed intense competition on a particular characteristic of a product, going far beyond consumer's objective needs.

1 Some illustrative examples of excessive competition

A spectacular example of excessive competition on benchmark characteristic could be illustrated by the so-called "Megapixel Race": the digital camera producers keep increasing the pixel definition of their cameras' image sensors (commonly measured in megapixels) even though after a certain stage, it actually worsens the quality of shots. For the early digital cameras, the megapixel count was indeed the most important characteristic. In order to deliver high-quality journal-like prints of 4x6 inches (10x15 cm, standard size), the camera's image sensor should have at least 2.2 Megapixels (see Clark, 2008). Most buyers of digital cameras aren't likely to print bigger pictures often, but even 8x10 inches (around A4 format) high-quality prints don't require more than 7.2 Megapixels. At the same time, these figures - the sensor's megapixel count - are only one of the necessary conditions for obtaining prints of a certain quality for a given size. The camera's optics can also be (and usually is) a limiting factor: a poor lens just won't convey enough resolution to the sensor, and then the sensor's potential just won't be fully exploited. Another factor that deteriorates dramatically the quality of a picture is the digital noise. It primarily depends on the physical size of the sensor's pixels. The less is the physical size of a pixel, the less quantity of light (photons) would fall on each pixel, which leads to higher noise (see Clark, 2011). There are different algorithms of image processing that treat the noise problem, but always at expense of image sharpness. Moreover, an important characteristic of a digital camera is its dynamic range, which also depends on the physical size of a pixel (as explained in Wetzel and Frosini, 1999). Cameras with pixels of smaller size would have lower dynamic range, and consequently they would be much more likely to produce pictures with blown out highlights and poorly detailed dark shadows. From a producer's point of view, it is much easier to "cram" more pixels into a sensor of certain dimensions than to increase the physical size of the sensor, or improve the technology of each pixel. But, other things being equal, the fewer pixels are crammed into a sensor of a fixed size, the bigger is the size of each pixel and therefore the better is the quality of the pictures. One might be able to make big prints from a small image sensor with, say, 8Megapixels, but the quality will be extremely poor. Much better results would have been obtained from a sensor of same dimensions with fewer megapixels.

The megapixels race is rather an extreme case when producer's decision to focus excessively on a quantitative benchmark characteristic of a technological product may damage the product's objective quality. In less dramatic examples, the producers just over-invest in development of a characteristic which gives zero marginal increase to consumers' "objective" utility. This is the case for the 'Battle of Blades', initiated by Gilette in 1971 when it first introduced on the market a two-blade razor, followed by the commercialization of the 'Mach3' triple-blade razor in 1998. In 2003, Schick/Wilkinson (which is Gilette's key rival) responded to Mach3 with the four-blade razor 'Quattro'. At this time, media were joking that in a while there would be 5-blade razors (which seemed ridiculous yet in 2004). However, today both Gilette and Schick have 5-blade razors in their product lines – 'Gilette Fusion' and 'Schick Hydro 5'.

At the same time, the rival firms might want to "denounce" irrelevance of benchmark characteristic and its overpricing by rivals. In July 2001 Apple made a series of comparisons of performance of its processors with lower clockspeed than the direct Intel Pentium competitors, which undermined the long-lasting "MHz myth". A "gentlemen's agreement" of German car manufacturers makes them limit cars' top speed at 250 km/h, limiting competition on this characteristic.

1.1 Literature on quality, product characteristics and competition

The fundamental economic theoretical literature taking product quality into account was first given by Lancaster (1966, 1971) in his "New Economic Theory of Consumer Behavior". The key assumption of this theory (shared by numerous further contributions to the field) is that consumers don't derive a direct utility from the good; instead, they value the different characteristics associated with the product. Thus, any product is perceived as just a bundle of characteristics sold together at a certain price. One widely accepted contribution by Lancaster (1979) is the distinction between "vertical differentiation" and "horizontal differentiation", as well as the introduction of these two terms. In general, the difference between vertical and horizontal differentiation is that only under the former the customers agree on the ranking between the products. The basic result on oligopolistic competition from Gabszewicz and Thisse (1979), as well as from Shaked and Sutton (1982), with vertical differentiation and one characteristic, is that oligopolists may benefit from relaxing price competition through vertical differentiation. Interestingly enough, models of vertical differentiation extended to the case of goods with multiple characteristics like in Vandenbosch and Weinberg (1995); they find that oligopolists would maximize the differentiation on only one characteristic. Generally, firms first simultaneously make their quality decisions and then simultaneously set their prices. In such framework, more competition leads to higher social welfare because firms' market power is always overridden by increased consumers' variety. One more important feature of these early models is the fact that the competitor who offers the highest quality product gets the higher profit. This effect is known as the "high quality advantage". The fact that profits may be increasing with quality, like in Waughty (1996) or Choi and Shin (1992), implies that each competitor would like to play the role of the high-quality firm and thus be willing to make some investments to obtain it.

However, when the assumption of consumers' perfect information on quality is relaxed, competition may sometimes lead to lower social welfare. One cause exposed in Anderson and Renault (2000) can be that consumers may incur excessive information gathering costs (increasing with the number of goods in the market) to find the product that matches best with their preferences. This leads firms to increase their prices since they may better discriminate informed consumers by exploiting their willingness-to-pay. Another reason can be that competition gives firms incentives to develop more complex products, making it difficult for consumers to compare them and choose the best one according to their will as in Gabaix and Laibson (2003) or Sauer (2011). It results into higher mark-ups for firms and thus lower social welfare. Moreover, when products are complex and when some attribute's price may be misevaluated by consumers, Gabaix and Laibson (2006) show that firms have no incentives to disclose the price of shrouded attributes because it would reduce their market power by making consumers choose more wisely among competitors.

The idea of price as a signal of uncertain quality dates back to Bagwell and Riordan (1991) (in the monopoly context). Daughety and Reinganum (2007) show existence of separating equilibria under duopoly where each firm is only informed of its own quality. In our model the signalling game is more complex, as both duopolistic producers know each other's qualities. Janssen, Roy (2008) study occurence of separating equilibria in oligopoly.

The closest paper to our research is by Bester, Delmuth (2013), who study price signalling of quality under duopoly. In their model all consumers are sure about incumbent's quality but a share of consumers is uninformed about the entrant's quality. They find that for a fully efficient separating equilibrium it is necessary to have a sufficient share of informed consumers. We argue that even under monopoly such separating equilibrium would be sustainable under monopoly, provided the exogenously given share of informed consumers is high enough. In contrast, in our article efficient separating equilibrium only arises due to rival's existence. Instead of a share of uniformed consumers as ingredient of "quality uncertainty", in our model all consumers are initially uninformed and have to exert a costly effort to acquire information. Although no consumer exerts effort in the separating equilibrium, the cost of effort should be low enough for its sustainability.

The goal of our article is to find a rationale for firms to engage into excessive competition on a benchmark characteristic leading to wasteful investments (those that don't improve the global product performance) and to find conditions provoking such inefficiency. In a model where two firms choose the quality of the good they sell and compete in prices, we show that if consumers are not fully sure of their needs and may only discover this information after a costly effort, they may choose to rely exclusively on one characteristic for benchmarking, permitting the high-quality firm increase profits by a wasteful investment in higher (but useless) characteristics. This results into higher prices (and profits) at the detriment of consumers' and social welfare.

The article is organized as follows: section 2 describes consumers and firms' objectives and the timing of the game, section 3 solves the competition game under full information while sections 5 and 6 continues the analysis with asymmetric information case, looking for pooling and separating equilibria respectively.

2 Model set-up

2.1 Consumers' preferences over characteristics and quality

In this paper we want to study the particular issue of differentiation by characteristic: it is quite common that after a certain level, an increase in product's characteristic stops to bring additional satisfaction to consumers. Consumers don't derive additional utility directly from the fact that the benchmark characteristic is higher. They may care only about the "real quality" of the product μ_i which isn't directly observable. In order to reflect the fact that we now focus on a combination of attributes of the product, and not on the benchmark " quality" that is always desirable, we define a threshold level $\hat{q}$ after which any increase of benchmark characteristic is useless. Before this level, without loss of generality, the good's " real quality" is defined to coincide with the benchmark characteristic. Therefore, the relevant quality is defined by:

$$s_i = min\{q_i, \hat{q}\}, \quad where\, i = \{1, 2\} \tag{1}$$

In other words, a greater value of q_i just means that the product has a higher benchmark characteristic (which could be a camera's megapixel count, a computer's CPU speed, a car's top speed, etc). Before a certain level q, the benchmark characteristic q_i indeed determines the good's quality s_i: there is no doubt that a 5 megapixel camera is better than a camera with only 2 megapixels. Nevertheless, after the level q, further increases in such a characteristic aren't important for consumers' ordinary and commonly use any more. Probably, 16 megapixels aren't even slightly better than 10 megapixels for a typical consumer, printing at most A4 format photos (or, to give another example, a 6 blades razor isn't better than a 3 blades razor for a close shave).

We assume that characteristic is always fully relevant before the level of initial technology $q = 1$. For simplicity we model $\hat{q}$ as a binary variable: we assume that it could be equal to this level of initial technology ($\hat{q} = 1$), or to some higher level $\hat{q} = \bar{\bar{q}} > 1$. In the first case any upgrade of characteristic above 1 is irrelevant for objective quality. In the rest of the paper we denote this situation as "state F" (for "false" relationship between characteristic aand quality). In the second case an increase of characteristic to any level below $\bar{\bar{q}}$ would be fully translated to an increase of bjective quality. This realisation would be referred to "state R" (for "real" relationship between characteristic aand quality).

The demand is composed like in Mussa and Rosen (1978), by an infinite number of consumers characterized by their valuation of quality θ, such that their utility from buying good of quality s_i at price p_i (with $i = \{1, 2\}$) is given by:

$$U(s_i, \theta) = \theta s_i - p_i \quad = \tag{2}$$

$$= \begin{cases} \theta q_i - p_i & if\ q_i < \hat{q} \\ \theta \hat{q} - p_i & if\ q_i \geq \hat{q} \end{cases} \quad (3)$$

Consumers also have the outside option of not buying any product; then they get zero utility.

Thus, the piecewise form of demand (3), deduced from incorporating our definition of " relevant quality" (2) into the standard Mussa-Rosen utility function (1), reflects the new issue that arises when studying the differentiation by characteristic: it is quite common that after a certain level an increase in product's characteristic stops to bring additional utility to consumers.

We assume that there is mass 1 of consumers with taste for quality uniformly distributed: $\theta \backsim u[0;1]$. Each consumer chooses the product that brings him the highest expected utility $U(s_i, \theta)$ given the set of available information; he can also refrain from buying any product and then get zero utility.

2.2 Asymmetric information

2.3 Duopolistic competition on characteristic

After a preliminary stage, the duopolists Firm 1 and Firm 2[1] compete on characteristic and prices in a two-stage subgame Q (see Figure 1).

The firms can freely choose the characteristic q_i for their goods (denoted G1 and G2 respectively), up to some level given by their technology. F1 pretends to be " high quality producer" and is less restricted in its choice of characteristic; F2 provides a more basic good.

At the preliminary stage, F1 has a possibility to make an investment. If F1 doesn't make the investment at the preliminary stage, the maximum levels of characteristic $\bar{q}_i$ that both firms would be able to choose later in the subgame Q are the same: $\bar{q}_1 = \bar{q}_2 = \bar{q} = 1$. There is only one option of investment: it would permit F1 to increase its maximum possible level of characteristic by δ, up to $\bar{q}_1 = 1 + \delta$.

In other words, F2 can always choose its characteristic only from the interval $q_2 \,\epsilon\, [0;1)$. F1 would be able to choose the characteristic $q_1 \,\epsilon\, [0;1]$ if it doesn't make the investment and $q_1 \,\epsilon\, [0;1+\delta]$ if it makes the investment.

It is assumed that the investment costs I to F1 in case $\hat{q} > 1$ and costs nothing otherwise. This simplifying assumption[2] means that only an upgrade is costly unless this upgrade is completely irrelevant for objective quality.

[1] from here on denoted as F1 and F2, respectively

[2] The paper's results would also hold if we assume instead that investment is always costly, but at least slightly more costly when an upgrade of characteristic still matters for objective quality

The values of parameters δ and I are assumed to be relatively small positive, and the cost of investment I is low enough compared to δ, so that the investment isn't an *apriori* unprofitable option.

We assume the following timing for firms' decisions:

1. F1 decides to invest in order to reach a maximal benchmark characteristics of $1+\delta$, whereas firm 2 can at most reach up to 1 (not inclusively). Investment costs I if at the existing level of technology $q=1$ characteristic still matters for objective quality, and zero otherwise
2. F1 chooses its characteristic q_1 and then F2 chooses its characteristic q_2 ;
3. F1 and F2 simultaneously set their prices: p_1 and p_2;
4. Consumers make their choice.

The game subgame Q (encompassing steps 2, 3 and 4) played by the duopolists is analogous to one specified by Choi and Shin (1992), with the only difference coming from the distinction between the product's real quality s_i(which ultimately matters for consumer's utility but is unobservable) and the product's observable benchmark characteristics q_i. Consumers observe the specifications of the two products available on the market. Their respective prices and maximize their utility function given in (3). The payoffs are then realized.

This timing of the game is summarized in Figure 1.

Figure 1: Timing of the game.

Competition in prices and characteristics choice

We now turn to the resolution of the game, using a classic backward induction method, for two cases . Section 3 describes the outcome of the game under full information, where consumers are certain about their own needs and know which product to choose according to their normal (common and standard) use. Section 4 explores the game under incomplete information. There consumers initially don't have precise understanding of their objective need in the characteristic but have an opportunity to learn it after exerting a costly effort. Both firms know the real link between characteristic and quality and might try to signal it to consumers through their actions.

3 The full information case: informed consumers

As in the previous part we start our analysis with the complete information case, where the threshold $\hat{q}$ is common knowledge and consumers don't appreciate any excess of characteristic above this "fully sufficient" level.

Let's assume first $q_2 < q_1$. We would then show that under our assumption $\varepsilon > 0$ this condition is always satisfied in the equilibrium of Subgame Q, so that the role of high quality producer is always assigned to F1.

As we have stated, the consumer has a choice between the following options:

- s_1^{OBS}: buy good 1, with observable objective quality $\mu_1 = min(q_1, \hat{q})$ and price p_1
- s_2: buy good 2, with observable objective quality $\mu_2 = q_2$ and price p_2
- s_0: not buy any good

The utility of consumer of type θ according to the strategy he chooses is:

The utility of consumer from buying good 1 according to his taste for quality is

$$U[s_0] = 0 \tag{4}$$

$$U[s_2] = \theta q_2 - p_2 \tag{5}$$

$$U[s_1^{CI}] = \theta s_1 - p_1 \tag{6}$$

While the consumer indifferent between buying the low-quality good and nothing is characterized by:

$$U(s_2) = 0 \Leftrightarrow \theta_{02} = \frac{p_2}{q_2} \tag{7}$$

The consumer indifferent between buying the good of quality s_1 and $s_2 = q_2$ is defined by:

$$U(s_1^{CI}) = U(s_2) \Leftrightarrow \widetilde{\theta}_{12} = \frac{p_1 - p_2}{s_1 - q_2} \tag{8}$$

Demands are thus given by:

$$D_1(s_1, q_2, p_1, p_2) = 1 - \widetilde{\theta}_{12} = 1 - \frac{p_1 - p_2}{s_1 - q_2} \tag{9}$$

And the resulting profits for firm 1 and 2 are defined by:

$$\pi_1 = p_1 D_1(s_1, q_2, p_1, p_2) = p_2(1 - \frac{p_1 - p_2}{s_1 - q_2}) \tag{10}$$

Maximizing profits with respect to prices leads to:

$$p_1(s_1, q_2) = \frac{2s_1(s_1 - q_2)}{4s_1 - q_2}; \quad p_2(s_1, q_2) = \frac{q_2(s_1 - q_2)}{4s_1 - q_2} \tag{11}$$

$$p_1(s_1, q_2) = \frac{2s_1(s_1 - q_2)}{4s_1 - q_2}; \quad p_2(s_1, q_2) = \frac{q_2(s_1 - q_2)}{4s_1 - q_2} \tag{12}$$

and thus:

$$\pi_1(s_1, q_2) = \frac{4s_1^2(s_1 - q_2)}{(4s_1 - q_2)^2}; \quad \pi_2(s_1, q_2) = \frac{q_2 s_1(s_1 - q_2)}{(4s_1 - q_2)^2} \tag{13}$$

At step 2, both firms choose simultaneously their quality, q_2 and s_1 . Firm 2 would choose the characteristic of its product q_2 that maximizes its profit$\pi_2(s_1, q_2)$:

$$\frac{\delta \pi_2(s_1, q_2)}{\delta q_2} = 0 \Leftrightarrow q_2 = \frac{4}{7} s_1 \tag{14}$$

The profit of Firm 1 is an increasing function of its objective quality:

$$\frac{d\pi_1(s_1, q_2)}{ds_1} = \frac{4s_1((2s_1 - \sqrt{2}q_2)^2 + (4\sqrt{2} - 3)s_1 q_2)}{(4s_1 - q_2)^2} > 0, \ \forall q_2 < s_1 \tag{15}$$

So, we can conclude that Firm 1 would always prefer to set the objective quality of its good at the maximum possible level. The equilibrium of Subgame Q is then

$$\begin{gathered} s_1 = \hat{q}; \ q_2 = \frac{4}{7}\hat{q}\,; p_1 = \frac{\hat{q}}{4}; \ p_2 = \frac{\hat{q}}{14}; \\ D_1 = \frac{7}{12}; \ D_2 = \frac{7}{24}; \ \pi_1 = \frac{7\hat{q}}{48}; \ \pi_2 = \frac{\hat{q}}{48} \end{gathered} \tag{16}$$

Here, Firm 1 gets higher profit as it gets the role of “high quality producer”. Firm 2 also would have liked to get this role, but as it has to choose quality after its rival, it prefers to differentiate itself with $q_2 < s_1^* = 1$. Thus in our game the condition $q_2 < q_1$ is always satisfied.

As the demand of Firm 1 is dependent on the true value of$\hat{q}$, the price p_1 reflects the objective quality and varies across states. The investment is always made in state R, in order to increase the objective quality by δ. The equilibrium of the whole game is thus:

$$F1: \quad \begin{cases} \left(costless\; investment,\; q_1 =\in [1; 1+\delta],\; p_1 = \frac{1}{4}\right);\; \pi_1 = \frac{7}{48}; & if\; F \\ \left(invest,\; q_1 = 1+\delta,\; p_1 = \frac{1+\delta}{4}\right);\; \pi_1 = \frac{71+\delta}{48} - I & if\; R \end{cases};$$

$$F2: \quad \begin{cases} \left(q_2 = \frac{4}{7}\,,\; p_2 = \frac{1}{14}\right);\; \pi_2 = \frac{1}{48} & if\; F \\ \left(q_2 = \frac{4}{7}(1+\delta),\; p_2 = \frac{1+\delta}{14}\right);\; \pi_2 = \frac{1+\delta}{48} & if\; R \end{cases};$$

$$Consumers: \quad \begin{cases} buy\; G1 & if\;\; \theta > \theta_{12} \\ buy\; G2 & if\;\; \theta_{02} < \theta < \theta_{12} \\ buy\; G1 & if\;\; \theta_{02} < \theta < \theta_{12} \end{cases} \tag{17}$$

We now turn to the most interesting case of assymetric information, where the consumers don't observe $\hat{q}$.

4 The asymmetric information case: uncertainty on consumers' needs

The rest of the article focuses on the asymmetric information case under duopoly.The next sections consider the asymmetric information case in the monopoly setting.

Here the consumers don't know the exact level of $\hat{q}$, although they are aware that after some unknown level, the product's characteristics become useless. So, the consumers now understand that there may be a discrepancy between the "real" quality of a product s and its benchmark characteristic q. We also suppose that consumers have a possibility of exerting a costly effort (with cost f) to learn the objective quality of a product.

Formally, we assume that $\hat{q}$ may take one of the two possible values: $\hat{q} = 1$ with probability $\frac{1}{2}$ or $\hat{q} = \bar{\bar{q}} > 1+\delta$ with probability $\frac{1}{2}$. As before we will denote these realisations as states F (for “false”) and R (for “real”), respectively. It means that either the "initial technology" leaves space for substantial upgrade (with probability 50%), either the "initial technology" already permits to achieve the fully sufficient amount of characteristics (with probability 50%). The consumers are aware of these two options and know their apriori probabilities, but don't observe which of the two states has been realized. A consumer who chooses to exert the costly effort learns the state of nature. Apart from the uncertainty on realisation of state of nature consumers perfectly now the rules of the game; they also know the values of exogenous parameters I, f and δ.

Unlike the consumers, the two firms knows from the beginning whether the upgrade of characteristic is relevant or not[3]. Also, as F2 is only able to choose $q_2 \epsilon [0;1)$, consumers have no uncertainty about the objective quality of G2.

This asymmetry of information leads to a signalling game. When making their decision consumers also observe the price and characteristic. Fully rational consumers would try to use this additional information to solve the game backwards and update their beliefs about relevance of characteristic.

We use the solution concept of Perfect Bayesian Equilibrium for the analysis of asymmetric inforation game. A PBE is constituted by each player's equilibrium strategy and a system of consumers' posterior beliefs about state of nature. Consumers' posterior beliefs should be consistent with Bayesian updating of apriori beliefs given monopoly's equilibrium strategy. Consumers' equilibrium strategy should maximize their expected utility given their posterior beliefs and monopoly's equilibrium strategy.

As F2 is only able to choose $q_2 \epsilon [0;1)$ consumers have no uncertainty about the objective quality of G2.

5 Separating vs pooling equilibria.

Potentially, any pure strategy Perfect Bayesian Equilibrium of our signalling game could be classified either as a pooling or a separating one.

In separating PBE the firms' actions would be different across states. F1 would only make consumers pay higher for upgrade of charcteristic above the level of initial technology in state R[4]. In any separating equilibrium the consumers would be able to infer the state of nature just by observing prices and characteristics of the two firms. In particular, when facing some pairs of price and characteristic they would believe that the state of nature is R; we denote this set of pairs of price and characteristic as $\{(p_1, p_2, q_1, q_2)\}^R$. For other market outcomes consumers would infer that state of nature is F; so the "separating" system of beliefs always has the form:

$$\begin{cases} believe\, "R" & if\, observe\, (p_1, p_2, q_1, q_2) \in \{(p_1, p_2, q_1, q_2)\}^R \\ believe\, "F" & if\, observe\, (p_1, p_2, q_1, q_2) \notin \{(p_1, p_2, q_1, q_2)\}^R \end{cases} \tag{18}$$

In pooling PBE the firms would make the same decisions on investment, characteristics and price across states R and F. Then the consumers would be totally unable to infer the state of

[3] in other words, the monopoly observes directly the realisation of state of Nature

[4] Obviously, it never makes sense for a firm to invest in state F and not invest in state R; so we exclude this type of separating candidate equilibria as apriori unsustainable

nature by interpreting the fims' obsservable actions. Consequently, in any pooling equilibrium consumers' posterior beliefs are same as their apriori beliefs:

$$believe\ state\ is\ "R"\ with\ probability\ \frac{1}{2}\ and\ "F"\ with\ probability\ \frac{1}{2} \tag{19}$$

5.1 Pooling equilibrium without investment

Under asymmetric information for duopoly case there always exists a pooling equilibrium where no investment is made and the game outcome is the same as in (??).

To see this, assume that consumers expect such pooling equilibrium, with characteristic and price $q_1^{FI,F}$, $p_1^{FI,F}$, $q_2^{FI,F}$, $p_2^{FI,F}$. Then, consider the following strategy of each consumer: "choose as under full information in state F when observe $q_1^{FI,F}$, $p_1^{FI,F}$, $q_2^{FI,F}$, $p_2^{FI,F}$, refrain from buying otherwise".

Under such "skeptic" beliefs of consumers the best response for Firms 1 and 2 is to set $q_1 = 1$; $q_2 = \frac{4}{7}$; $p_1 = \frac{1}{4}$; $p_2 = \frac{1}{14}$. This is their only option that brings positive profit. Consequently, this implies no investment at the preliminary stage. On the other hand, when faced to $q_1 = 1$; $q_2 = \frac{4}{7}$; $p_1 = \frac{1}{4}$; $p_2 = \frac{1}{14}$. the consumers have no uncertainty about the objective quality, and therefore their best response is to choose between buying and refraining as in the full information case.

5.2 Pooling equilibrium with investment: properties and existence.

Take a consumer who expects a pooling equilibrium. This means he expects to face the same equilibrium prices and characteristics of the two goods across states R and F. We call these prices and characteristics $p_1^{pooling}$, $p_2^{pooling}$, $q_1^{pooling}$, $q_2^{pooling}$. If a consumer expects a pooling equilibrium and observes this market outcome he would be unable to update his apriori beliefs by inferring the state of nature. The only information he would posess for decision-making is the apriori probability of state R, m. The choice on product and effort would then be done by comparing the ex ante expected payoffs. We also imposed the following out-of-equilibrium beliefs: if a consumer expects a pooling equilibrium but observes a market outcome different from s $p_1^{pooling}$, $p_2^{pooling}$, $q_1^{pooling}$, $q_2^{pooling}$he would refrain from buying any good.

5.2.1 Demand functions under pooling beliefs

Throughout this subsection we assume that consumers expect the incomplete information game to lead to a pooling equilibrium. This wil permit us to describe the properties of such pooling outcomes; later we will check if they are sustainable as equilibria.

In other words, here consumers are sure that the investment would be made by Firm 1 regardless of its relevance for objective quality, and the prices and specifications chosen by the two firms would also be the same across states F and R. This means that consumers won't be able to infer the state of nature by observing the market outcome, and won't be able to update their ex ante beliefs.

As in the full information case, consumers have the opportunity to choose strategies $s_0$and s_2 (because there is no uncertainty about the objective quality of good 2). Again, they would bring to a consumer of type θ utility defined in ??and ??, respectively:

$$U[s_0] = 0$$

$$U[s_2] = \theta q_2 - p_2$$

The strategy $s_1$is different here, compared to the previous section. Consumer who doesn't exert the effort and buys good 1 is now faced to uncertainty. He knows that with probability m, the additional advance of characteristic of good 1 is still relevant, so that the objective quality of good 1 is $\mu_1 = q_1$. Then buying good 1would bring utility $U[s_1|\hat{q}=\bar{\bar{q}}] = \theta q_1 - p_1$. On the other hand, the consumer knows that with probability $1-m$, any excess of characteristic above the initial technology is completely useless. The objective quality of good 1 is then $\mu_1 = 1$, and buying this good would bring utility$U[s_1|\hat{q}=1] = \theta \cdot 1 - p_1$.The ex ante expected utility of consumer of type θ from buying good 1 without exerting the effort is therefore

$$\begin{aligned} EU[s_1] = mU[s_1|\hat{q}=\bar{\bar{q}}] + (1-m)U[s_1|\hat{q}=1] = \\ = m(\theta q_1 - p_1) + (1-m)(\theta \cdot 1 - p_1) \end{aligned} \quad (20)$$

An additional class of strategies are those that require effort before making the buying decision. If the consumer exerts the effort while $\hat{q} = \bar{\bar{q}}$ (which happens with probability m), he would learn that $\mu_1 = q_1$. Then, buying good 1 would bring him utility

$$U[s_{e1}|\hat{q} = \bar{\bar{q}}] = \theta q_1 - p_1 - f \quad (21)$$

and buying good 2 would bring utility

$$U[s_{e2}|\hat{q} = \bar{\bar{q}}] = \theta q_2 - p_2 - f \quad (22)$$

It can now be shown that exerting the effort and then "ignoring" the signal is always a dominated strategy for any consumer.Comparing (21)and (22) implies that if consumer has exerted effort while $\hat{q} = \bar{\bar{q}}$, he would choose to buy good 1 if $\theta \geq \frac{p_1 - p_2}{q_1 - q_2}$.Otherwise he would choose good 2. On the other hand, if the consumer exerts the effort while $\hat{q} = 1$(which happens with probability $1-m$), he would learn that $\mu_1 = q_1$. He would then get the following utility from buying good 1:

$$U[s_{e1}|\hat{q} = \bar{\bar{q}}] = \theta q_1 - p_1 - f \quad (23)$$

and from buying good 2:

$$U[s_{e1}|\hat{q} = \bar{\bar{q}}] = \theta q_1 - p_1 - f \tag{24}$$

Comparing 23and 24implies that if consumer has exerted effort while $\hat{q} = 1$, he would choose to buy good 1 if $\theta \geq \frac{p_1 - p_2}{1 - q_2}$.Otherwise he would choose good 2.

At the same time, for the extreme values of taste parameter consumer doesn't need to acquire information. He would alsways prefer to buy good 1 if $\theta \geq \frac{p_1 - p_2}{1 - q_2}$, in both of the state of nature. Similarly,consumers with

would always prefer to choose good 2, in both states of nature. As the effort is costly, it is always a dominated strategy for consumers with $\theta \geq \frac{p_1 - p_2}{1 - q_2}$and $\theta < \frac{p_1 - p_2}{q_1 - q_2}$as the information on state of nature doesn't matter for their decisions.

So, effort could only be a potentially rational strategy for "intermediate" consumers with $\frac{p_1 - p_2}{q_1 - q_2} < \theta < \frac{p_1 - p_2}{1 - q_2}$. As was shown above, these consumers prefer to buy good 1 if they learn $\hat{q} = \bar{\bar{q}}$ and good 2 if they learn that $\hat{q} = 1$. So, we can conclude that if a consumer chooses to exert the effort, he would use the signal he recieves: he would buy good 1 if efort tells$\hat{q}$=$\bar{\bar{q}}$]and good 2 if it tells $\hat{q} = 1$. Hence, the other effort-based strategies that ignore the signal (like, exert the effort and buy good 1 in both states of nature) can be excluded from further analysis, as they are never optimal for consumer.

We would denote the strategy based on effort that uses the additional information obtained as "s_{ew}" , for "effort, buy wisely". This strategy might be used by consumers with taste $\frac{p_1 - p_2}{q_1 - q_2} < \theta < \frac{p_1 - p_2}{1 - q_2}$.

The the expected utility of this strategy is :

$$EU[s_{ew}|\theta] = m(\theta q_1 - p_1) + (1 - m)(\theta q_2 - p_2) - f \tag{25}$$

Equations (21)-(**??**) allow us to describe the ranking of expected utilities of the three strategies, according to consumer's value for quality:

Proposition 6 (Proof in appendix): under asymmetric information, a consumer prefers to play:

$$a)\ s_1 \succeq s_{ew} \ \Leftrightarrow \ \theta \geq \theta_{1e} = \frac{(p_1 - p_2)(1 - m) - f}{(1 - m)(1 - q_2)} \tag{26}$$

$$b)\ s_2 \succeq s_{ew} \ \Leftrightarrow \ \theta \leq \theta_{2e} = \frac{m(p_1 - p_2) + f}{m(q_1 - q_2)} \tag{27}$$

$$c)\ s_1 \succeq s_2 \ \Leftrightarrow \ \theta \geq \theta_{12} = \frac{(p_1 - p_2)}{mq_1 + 1 - m - q_2} \tag{28}$$

$$d)\, s_2 \succeq s_0 \;\Leftrightarrow\; \theta \geq \theta_{02} = \frac{p_2}{q_2}; s_1 \prec s_2 if \theta < \theta_{02} \tag{29}$$

The values $\theta_{1e}, \theta_{2e}, \theta_{12}$ found in (26)-(29) directly allow to formulate the following results:

Lemma 2(algebraic proof in Appendix): the condition $\theta_{1e} \geq \theta_{12}$ is equivalent to $\theta_{2e} \leq \theta_{12}$ and is $eff \leq \frac{m\Delta p(q_1(1-m)+m)}{mq_1+1-m-q_2}$

Lemma 3(algebraic proof in Appendix):$0 \leq \theta_{02} \leq \theta_{12} \leq 1$ and $\theta_{02} \leq \theta_{2e}$.

These two lemmas reflect the fact that consumers' preferences over strategies are transitive, so that out of the four possible strategies $\{s_e, s_1, s_2, s_0\}$ each consumer always has one that dominates all the others.

Therefore, we can conclude that there are only two possible rankings of $\theta_{2e}, \theta_{12}, \theta_{1e}$:

$$\text{either } \theta_{1e} < \theta_{12} < \theta_{12}(Configuration\ A),$$

$$\text{either } \theta_{2e} < \theta_{12} < \theta_{1e}(Configuration\ B).$$

Let's describe the behavior of consumers according to their taste for quality in each of the configurations A and B. To do so, it is enough to apply the relationships found in Lemma 1(a-d) to each possible interval of the configuration.

In configuration A, we have $0 \leq \theta_{02} \leq \theta_{1e} \leq \theta_{12} \leq \theta_{2e}$. The dominant strategies for each $\theta \epsilon [0;1]$ are determined in the following table:

Table 1: Choice of consumer θ in Configuration A

Taste for quality θ	Ranking of strategies	Optimal strategy
$0 < \theta < \theta_{02}$	$s_2 \prec s_0, s_1 \prec s_{ew}; s_1 \prec s_2; s_{2e} \prec s_2$	s_0 : not buy
$\theta_{02} < \theta < \theta_{1e}$	$s_2 \succ s_0, s_1 \prec s_{ew}; s_1 \prec s_2; s_{2e} \prec s_2$	s_2: buy good 2
$\theta_{1e} < \theta < \theta_{12}$	$s_2 \succ s_0, s_1 \succ s_{ew}; s_1 \prec s_2; s_{2ew} \prec s_2$	s_2: buy good 2
$\theta_{12} < \theta < \theta_{2e}$	$s_2 \succ s_0, s_1 \succ s_{ew}; s_1 \succ s_2; s_{2ew} \prec s_2$	s_1: buy good 1
$\theta_{2e} < \theta < 1(if\, \theta_{2e} < 1)$	$s_2 \succ s_0, s_1 \succ s_{ew}; s_1 \succ s_2; s_{2ew} \prec s_2$	s_1: buy good 1

As we see, in configuration A consumers are divided into three categories. Those with the lowest valuations refrain from buying; "intermediate" consumers buy the "basic" good 2 while consumers with the highest valuation buy the good 1 (with more advanced characteristics).

Moreover, we can say that all these three groups are non-empty since we know that$0 < \theta_{02}, \theta_{12} < 1$.

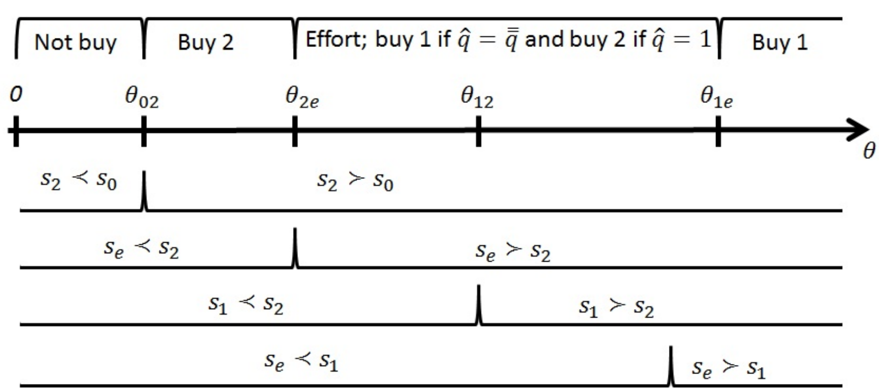
A graphical representation of consumers' optimal choice in configuration B is given next:

In configuration B, we have $0 \leq \theta_{02} \leq \theta_{2e} \leq \theta_{12} \leq \theta_{1e}$. The dominant strategies for each $\theta \epsilon [0; 1]$ are determined in the following table:

Table 2: Choice of consumer θ in Configuration B

Taste for quality θ	Ranking of strategies	Optimal strategy
$0 < \theta < \theta_{02}$	$s_2 \prec s_0, s_{2e} \prec s_2 s_1 \prec s_e, s_e \prec s_2;$	s_0 : not buy
$\theta_{02} < \theta < \theta_{2e}$	$s_2 \succ s_0, s_e \prec s_2, s_1 \prec s_e, s_1 \prec s_2$	s_2: buy good 2
$\theta_{2e} < \theta < \theta_{12}$	$s_2 \succ s_0, s_e \succ s_2, s_1 \succ s_e, s_1 \prec s_2$	s_2: buy good 2
$\theta_{12} < \theta < \theta_{1e}$	$s_2 \succ s_0, s_e \succ s_2, s_1 \succ s_e; s_1 \succ s_2$	s_e: effort, buy wisely
$\theta_{1e} < \theta < 1 (if\, \theta_{1e} < 1)$	$s_2 \succ s_0, s_e \succ s_2, s_1 \succ s_e; s_1 \succ s_2$	s_1: buy good 1

Figure 3: consumers' choice in configuration B



As in Configuration A, consumers with the lowest taste for quality don't buy any good. Those with higher valuations buy good 2; then there come the consumers who exert the effort and buy "wisely" according to what they learn. All these groups are always non-empty. Finally, if $\theta_{1e} < 1$ there would be an additional category of consumers who buy good 1. If $\theta_{1e} > 1$, ths group is empty and consumers with the highest taste for quality are still among those who exert the effort.

Now we can conclude that potentially there could be two types of equilibria of Subgame Q:

those where no effort is exerted at all (configuration A) and those where some consumers exert the effort (configuration B).

As shown in Lemma 2, the Subgame Q would end in an equilibrium of " type A" if characteristics and prices chosen by the two firms satisfy:

$$f > f_{limit} = \frac{m(p_1 - p_2)(q_1(1-m)+m)}{mq_1 + 1 - m - q_2} \tag{30}$$

Intuitively, this means that if the cost of effort higher than a certain threshold, exerting the effort would become a poor(i.e., dominated) strategy for the totality of consumers. As we could expect, this threshold is proportional to the difference of prices $(p_1 - p_2)$ and inverse proportional to the ex-ante expected difference in " objective quality", $mq_1 + (1-m)\cdot 1) - q_2$.

Plugging in the expressions $\theta_{02}, \theta_{2e}, \theta_{12}, \theta_{1e}$ and using the fact that mass 1 of consumers is distributed $\theta \backsim u[0;1]$ we can find the demand functions for goods 1 and 2:

$$\underbrace{If\, \Delta p < f \cdot \frac{m(q_1-1)+(1-q_2)}{(1-m)(q_1-m)m}}_{(config.A)} \Rightarrow \begin{cases} D_1^A = (1 - \frac{\Delta p}{mq_1+1-m-q_2}) \\ D_2^A = (\frac{\Delta p}{mq_1+1-m-q_2} - \frac{p_L}{q_L}) \end{cases}$$

$$\underbrace{If\, \Delta p > f \cdot \frac{m(q_1-1)+(1-q_2)}{(1-m)(q_1-m)m}}_{(config.B)}\ if\ F \quad \begin{cases} D_1^B|F = 1 - (p_1-p_2)(1-m) - f \\ D_2^B|F = \frac{(p_1-p_2)(1-m)-f}{(1-m)(1-q_2)} - \frac{p_2}{q_2} \end{cases}$$

$$\underbrace{f\, \Delta p > f \cdot \frac{m(q_1-1)+(1-q_2)}{(1-m)(q_1-m)m}}_{(config.B)}\ if\ R \quad \begin{cases} D_1^B|F = 1 - \frac{m(p_1-p_2)+f}{m(q_1-q_2)} \\ D_2^B|F = \frac{m(p_1-p_2)+f}{m(q_1-q_2)} - \frac{p_2}{q_2} \end{cases}$$

In the full information case, it was shown that both oligopolists get higher profits when the quality of good 1 is higher. However, oligopolists couldn't extract more profits from continuing to develop the benchmark characteristic above the level q because the consumers weren't ready to pay anymore for the excess quality they knew to be useless. In case of uncertainty, consumers don't know for sure if the good 1 has a useless excess of benchmark characteristic. Potentially, this might enable the Firm 1 to over-invest (or " continue the race") after the relevant level $\hat{q}$. The task is now to determine if such wasteful investment would be made. In what follows we assume that the realization of uncertainty is such that the default level of technology already permitted to

Consumers' decision over which good to buy relies on the difference of prices of the two differentiated goods compared to cost of effort in terms of bad matching. If the cost of effort is high enough, the consumers with high willingness-to-pay for quality fully rely on the rule-of-a-thumb: "more characteristic means better quality", while consumers with lower tastes totally reject this reasoning and just buy the cheapest good (for a given p_L).

This is exactly the case that corresponds to the phenomena described in section 1.1 where there is a common belief share among consumers that more megapixels (or more razor blades, more processor MHz, etc.) is better. The consumers don't bother spending time, forces and effort to check whether this is true or not. According to the model, their reluctance to make the effort is shown to have an objective reason: consumers find the benefits of being able to better judge on the good's benchmark quality lower than the cost of learning effort.

However, if the price differential becomes too important, consumers never take the principle "more is better" for granted anymore. Consumers with high taste for quality prefer to exert the effort and make sure that they wouldn't pay a significant premium for useless excess of characteristics. They buy the more sophisticated good only if they make sure that it is worth the candle (the price premium). Otherwise, they switch to the cheapest good. Consumers with low taste for quality buy the cheapest good unconditionally anyway.

5.2.2 Outcome of Subgame Q in configuration A.

The game would end in a pooling equilibrium (invariant across states R and F) only if it stays in configuration A. Such pooling outcome would is described in Proposition 2.

Proposition 6 (proof in the Appendix): if $f > f_{neverA} = \frac{5}{12}m(1-m)(q_1-1)$ there exists a pooling equilibrium with investment:

$$F1\ strategy: invest;\ q_1^{pooling} = 1+\delta;\ ;p_1^{pooling} = \frac{1}{4}(1+m\delta)$$
$$F2\ strategy: q_2^{pooling} = \frac{4}{7}(1+m\delta);\ p_2^{pooling} = \frac{1}{14}(1+m\delta) \tag{31}$$
$$Consumers'\ beliefs:\ believe\ states\ R\ and\ F\ have\ probabilities\ \frac{1}{2}\ regardless\ \{(p_1,p_2,q_1,q_2)\}$$

$$Consumers'\ strategy: \begin{cases} buy & if\ \theta \geq \theta_{1e} \\ effort;\ buy\ wisely & if\ \theta_{2e} < \theta < \theta_{1e} \\ buy\ G2 & if\ \theta_{02} < \theta < \theta_{2e} \\ not\ buy & if\ \theta < \theta_{02} \end{cases} \tag{32}$$

$$
\begin{aligned}
invest;\ q_1^{pooling} = 1+\delta;\ q_2^{pooling} &= \frac{4}{7}(1+m\delta);\\
p_1^{pooling} = \frac{1}{4}(1+m\delta);\ p_2^{pooling} &= \frac{1}{14}(1+m\delta);\\
\pi_1^{pooling} = \frac{7}{48}(1+m\delta) - I;\ \pi_2^{pooling} &= \frac{1}{48}(1+m\delta) \qquad (33)
\end{aligned}
$$

This pooling outcome (**??**)would only be an equilibrium if it is sustainable. However, the type A-outcome of Subgame Q under pooling beliefs isn't necessarily sustainable (still under pooling beliefs). It should satisfy the condition $\Delta p < f \cdot \frac{m(q_1-1)+(1-q_2)}{(1-m)(q_1-m)m}$, in order to stay in configuration A with no effort by any consumer. Plugging into the last inequality the characteristics and prices from(**??**) we obtain a condition, trivially necessary for existence of pooling equilibrium.

5.3 Separating Equilibrium.

Lemma. In a separating PBE no consumer chooses to exert effort

Proof: in a separating equilibrium any consumer can infer the state of nature from the firms' observable actions. Consequently he doesn't face any uncertainty any more and can make his purchasing decision as under full information. As the effort is costly and there is no need for additional information, effort is a dominated action.

Under monopoly the fact that no consumer would exert effort in a separating PBE makes unsustainable any separating PBE. In state F monopoly would prefer to "mimic" state "R", so that the consumers think that upgrade of characteristic was relevant and become ready to pay for it.

Under duopoly it becomes more difficult for F1 to "lie" to consumers in this way, as the consumers could also find additional information in rival's price and characteristic.

Lemma. In any separating PBE consumer beliefs can not put full probability on state R or state F when F1 behaves as in state R $\left((p_1, q_1) \in \{(p_1, q_1)\}^R\right)$ **and F2 chooses** $(p_2, q_2) \notin \{(p_2, q_2)\}^R$

Sketch of Proof:

1) If consumers think that state is R with certainty, they behave as under full information in state R. In this case for any given prices and rival's characteristics demand for product 1 with $q_1 > 1$ is higher than for product 1 with $q_1 \leq 1$.

Assume consumers believe that state is R if $(p_1, q_1) \in \{(p_1, q_1)\}^R$, whatever action F2 takes. Then, like in the monopoly case, F1 would always prefer to mimic state R when the realisation is actually state F.

2) If consumers think that state is F with certainty, they behave as under full information in state F. In this case in state R F2 can always increase its demand compared to $(p_1, p_2, q_1, q_2)^R$ by deviating to slightly different (p_2, q_2) and making consumrs believe state is actually F. Consumers would then think that G1 is overpriced with respect to its supposed quality $q_1 = 1$, and some consumers would switch to G2. Consequently, F2 would then always deviate to $(p_2, q_2) \neq (p_2, q_2)^R$ when the state is R.

Proposition 4. When the cost of investment isn't prohibitive and the cost of effort low enough, there is a continuum of separating equilibria of form:

$$F1: \quad \begin{cases} \left(costless\, investment,\, q_1 = \in [1; 1+\delta],\, p_1 = \frac{1}{4}\right);\, \pi_1 = \frac{7}{48}; & if\, F \\ \left(invest,\, q_1 = 1+\delta,\, p_1 = \frac{1+\delta}{4}\right);\, \pi_1 = \frac{71+\delta}{48} - I & if\, R \end{cases};$$

$$F2: \quad \begin{cases} \left(q_2 = \frac{4}{7},\, p_2 = \frac{1}{14}\right);\, \pi_2 = \frac{1}{48} & if\, F \\ \left(q_2 = \frac{4}{7}(1+\delta),\, p_2 = \frac{1+\delta}{14}\right);\, \pi_2 = \frac{1+\delta}{48} & if\, R \end{cases};$$

$$Consumers' beliefs: \begin{cases} believe\ state\ is\ F & if\ (p_1, q_1)^{R,\,FI} \neq (p_1, q_1)^{R,\,FI} \\ believe\ \ R\ with\ probability\ \lambda \in [0;1] & if\ (p_1, q_1)^{R,\,FI}\ and\ (p_2, q_2) = (p_2, q_2)^d \\ believe\ state\ is\ R & if\ (p_1, q_1)^{R,\,FI}\ and\ (p_2, q_2) \neq (p_2, q_2)^d \end{cases} \tag{34}$$

$$Consumers\ strategy: \quad \begin{cases} not\ buy & if\ \ \theta < \theta_{2e} \\ effort, buy\ wisely & if\ \theta_{2e} < \theta < \theta_{1e} \\ buy\ G1 & if\ \ \theta_{1e} < \theta \end{cases}$$

where $(p_1, p_2, q_1, q_2)^{R,\,FI}$ are prices and characteristics under full information in state R, and $(p_2, q_2)^d$ some devitation prices that satisfy conditions a),b), c) below.

Sketch of Proof:

Mechanism: if F1 tries to "mimic" state R while the actual state is F, F2 should find profitable to deviate by setting a lower price/quality ratio, in order to induce effort by enough consumers. Consumers who exert effort discover the lie and switch to F2, making F1 earn less than in case of truth-telling that state is F by lower prices.

For existence of a separating equilibrium it is crucial that:

a) F2 is willing to denounce "lie" when state is F but the rival firm F1 tries to "mimic" state R by setting $(p_1, q_1)^{R,\,FI}$. Denouncement here would consist in setting $(p_2', q_2') \neq (p_2, q_2)^{R,\,FI}$, so that the consumers believe state is R only with probability λ. Thus, the following should be true:

$$\pi_2(p_1^{R,\,FI}, q_1^{R,\,FI}, p_2', q_2')|\lambda, state\ is\ ``F" > \pi_2^{R,\,FI}$$

b) F2 is unwilling to pretend there is "lie" when state is R and the rival firm F1 sets $(p_1, q_1)^{R,\,FI}$. Pretending there is lie here would consist in setting $(p_2', q_2') \neq (p_2, q_2)^{R,\,FI}$, so that the consumers believe state is R only with probability λ. Thus, the following should be true:

$$\pi_2(p_1^{R,\,FI}, q_1^{R,\,FI}, p_2', q_2')|\lambda, state\ is\ ``R" < \pi_2^{R,\,FI}$$

c) F1 is unwilling to "lie" that state is R while state is actually R, if it knows the lie would be denounced by F2:

$$\pi_1(p_1^{R,\,FI}, q_1^{R,\,FI}, p_2', q_2')|\lambda, state\ is\ ``F" < \pi_1^{F,\,FI}$$

It can be shown (proof to be added) that when the cost of effort is low enough, there exist such triplets (p_2', q_2', λ)that the three conditions above are satisfied, and a separating pure strategy PBE exists.

Comparison of equilibria under duopoly

Even under duopoly if the cost of effort is high enough, there exists a pooling equilibrium where a producer benefits from increasing a characteristic irrelevant for product's objective quality. There always also exists a "pessimistic" pooling equilibrium, where a relevant costly increase of characteristic would be rejected by a firm due to skepticism of consumers. These results are not qualitatively different from what one would obtain under monopoly.

On the other hand, an efficient separating equilibrium also becomes sustainable under duopoly, provided that consumers' cost of learning objective quality is low enough. This is because while quality leader always prefers to pretend that characteristic is relevant, his rival would prefer consumers think the opposite. A "lie" on relevance of useless characteristic can be denounced by the rival in the following way. The rival can find profitable to decrease his price to quality ratio, so that enough consumers are induced to exert the learning effort and discover the lie.

On top of the upper bound on cost of learning effort, it is crucial for sustainability of the efficient separating equilibrium that the "doubtful" upgrade of characteristic isn't very important. If this upgrade is very big, the quality-follower might also want to market a "doubtful" characteristic and benefit of consumers' persuasion that this characteristic is relevant with some probability. This logic could explain development of irrelevant upgrades even in competitive markets.

Interestingly, one could explain the Gentlemen's Agreement of German car manufacturers to limit their top speeds at 250 km/h as an attempt to limit the upgrades of this characteristic in order to stay in a separating equilibrium. Then it would be an example of Pareto-efficient self-regulation of the industry.

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Appendix.

Proof of Proposition 2.References

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If we stay in configuration (A) where$f > f_{limit} = \frac{p}{2}$ (this condition ((**??**)) needs to be checked ex-post for $p = p^{pooling}$), the demand for monopoly's good consists of consumers with $\theta \geq \theta_{0b}$. The demand is thus

$$D^A(p,q) = 1 - \theta_{0b} = 1 - \frac{2p}{1+q} \tag{35}$$

The profit is

$$\pi(p,q) = pD^A(p,q) = p - \frac{2p^2}{1+q} \tag{36}$$

Maximizing profit with respect to price leads to:

$$p^A(q) = \frac{1}{4}(1+q); \tag{37}$$

With prices set as $p = p^A(q)$, the profits of subgame Q are:

$$\pi^A(q) = \frac{1}{8}(1+q); \tag{38}$$

This is an increasing function of characteristic, so when the cost of investment isn't too high and the investment would be made. Then $q = 1 + \delta$, and the market outcome is:

$$\begin{aligned} invest;\ \ q^{pooling} &= 1 + \delta; \\ p^{pooling} &= \frac{1}{2}(1 + \frac{\delta}{2});\ D^{pooling} = \frac{1}{2} \\ \pi^{pooling} &= \frac{1}{4}(1 + \frac{\delta}{2}) - I; \end{aligned} \tag{39}$$

$$Consumers: \begin{cases} buy\ if\ \theta \geq \theta_{0b};\ not\ buy\ if\ \theta < \theta_{0b} & if\ observe\ \left(p^{pooling}, q^{pooling}\right); \\ not\ buy & if\ observe\ any\ other\ \ (p,q) \end{cases}$$

This pooling outcome ((39)) would only be an equilibrium if it is sustainable. However, the equilibrium of configuration A under pooling beliefs isn't necessarily the equilibrium of the unrestricted game (still under pooling beliefs). It should satisfy the condition $f > f_{limit} = \frac{p}{2}$, in order to stay in configuration A with no effort by any consumer. Plugging into the last inequality the characteristics and prices from(**??**) we obtain a necessary condition for existence of pooling equilibrium:

$$f > f_{limit} = \frac{1}{4}\left(1 + \frac{\delta}{2}\right) \tag{40}$$

Comparing it with the option of no investment (that brings profit of $\frac{1}{4}$) we can find the second necessary condition. The investment would be performed in both states of nature if $I \leq \frac{\delta}{8}$. If $I > \frac{\delta}{8}$a pooling equilibrium with investment wouldn't exist.

Proof of Proposition 3.

$$M: \quad \begin{cases} \begin{cases} (invest,\, q_I^{semi},\, p_I^{semi}) & with\, probability\, k = \frac{\delta}{4\,I} - 1 \\ (not\, invest,\, q = 1,\, p = \frac{1}{2}) & with\, probability\, 1-k \end{cases} & if\, F \\ (invest,\, q_I^{semi},\, p_I^{semi}) & if\, R \end{cases} ;$$

$$Consumers: \quad \begin{cases} buy\ \ iff\ \ \theta > \frac{p_I^{semi}}{q_I^{semi}} & if\, M\, chose\, (invest,\, q_I^{semi},\, p_I^{semi}) \\ buy\ \ iff\ \ \theta > p & if\, M\, played\, differently \end{cases} \tag{41}$$

Where $1 < q_{.I}^{semi} < 1 + \delta$ and $p_I^{semi} = \frac{k + q_I^{semi} + \sqrt{\left(k + q_I^{semi}\right)\left(q_I^{semi} - 1 - 4f(k+1)\right)}}{2(k+1)}$ or $p_I^{semi} = \frac{k + q_I^{semi} - \sqrt{\left(k + q_I^{semi}\right)\left(q_I^{semi} - 1 - 4f(k+1)\right)}}{2(k+1)}$.

Proof:

If the monopoly has chosen to make no investment, there would be no uncertainty and therefore the outcome would be like in state F under full information: $q_{n.I}^{semi} = 1$; $p_{n.I}^{semi} = \frac{1}{2}$;. $\pi_{n.I}^{semi} = \frac{1}{4}$ ((**??**)).

If the monopoly has chosen to make the investment it could set some characteristic and price $\left(q_I^{semi},\, p_I^{semi}\right)$. Ex ante consumers know the probability of observing these two market outcomes:

$$prob\left[\left(q_{n.I}^{semi};\, p_{n.I}^{semi}\right)\right] = prob\,[F] \times prob\,[not\, invest \mid F] = (1-m)\,(1-k) \tag{42}$$

$$prob\left[\left(q_{.I}^{semi};\, p_{.I}^{semi}\right)\right] = prob\,[F] \times prob\,[\, invest \mid F] + prob\,[R] = k + m(1-k) \tag{43}$$

Using the Bayes formula the consumers can infer:

$$prob\left[F \mid \left(q_I^{semi};\, p_I^{semi}\right)\right] = \frac{prob\left[\left(q_I^{semi};\, p_I^{semi}\right) \mid F\right] \times prob\left[F\right]}{prob\left[\left(q_I^{semi};\, p_I^{semi}\right)\right]} = \frac{k(1-m)}{k+m(1-k)} \tag{44}$$

$$prob\left[R \mid \left(q_I^{semi};\, p_I^{semi}\right)\right] = \frac{prob\left[\left(q_I^{semi};\, p_I^{semi}\right) \mid R\right] \times prob\left[R\right]}{prob\left[\left(q_I^{semi};\, p_I^{semi}\right)\right]} = \frac{m}{k+m(1-k)} \tag{45}$$

Consumers can choose between the same strategies s_0, s_b, s_{ew} as before. But when faced to $\left(q_I^{semi};\, p_I^{semi}\right)$, the expected payoffs of the two last strategies take into account the Bayesian probabilities:

$$EU[s_b] = \frac{m}{k+m(1-k)}U[s_b|\mathrm{R}] + \frac{k(1-m)}{k+m(1-k)}U[s_b|\mathrm{F}] = \frac{k\theta - kp + \theta q - p}{k+1} \tag{46}$$

$$EU[s_{ew}] = \frac{m}{k+m(1-k)}U[s_{ew}|\mathrm{R}] + \frac{k(1-m)}{k+m(1-k)}U[s_{ew}|\mathrm{F}] = \frac{\theta q - p}{k+1} - f \tag{47}$$

As usual, $EU[s_0] = 0$.

Taking the same steps as in subsection 5.4.1 we can find that

$$a)\; s_b \succeq s_{ew} \;\Leftrightarrow\; \theta \geq \theta_{eb}^{semi} = \frac{kp - f(1+k)}{k} \tag{48}$$

$$b)\; s_0 \succeq s_{ew} \;\Leftrightarrow\; \theta \leq \theta_{0e}^{semi} = \frac{p + f(k+1)}{q} \tag{49}$$

$$c)\; s_b \succeq s_0 \;\Leftrightarrow\; \theta \geq \theta_{0b}^{semi} = \frac{p(k+1)}{(k+q)} \tag{50}$$

Note that with $k = 1$ these thresholds of taste parameter pin down to (**??**)-(**??**), respectively. Also note that again consumers' choice is transitive: $\theta_{0e}^{semi} < 0 \Leftrightarrow \theta_{0e}^{semi} > \theta_{0b}^{semi}$

As in section 5.4, for semi-separating equilibrium it is necessary that no consumer exerts the effort when faced to uncertainty: see Lemma 2 for proof. This necessary condition can be written as

$$\theta_{0e}^{semi} < 0 \Leftrightarrow f > \frac{k\, p_I^{semi}}{1+k} \tag{51}$$

This permits to find demand function for the case consumers expect semi-separating equilibrium and observe investment: only those with $\theta > \theta_{0b}^{semi}$ would buy, hence

$$D^{semi-separating} \mid \left(q_I^{semi},\, p_I^{semi}\right) = 1 - \theta_{0b}^{semi} = 1 - \frac{p_I^{semi}(k+1)}{(k + q_I^{semi})} \tag{52}$$

The monopoly profit function is after investment is

$$\pi_I^{semi} = p_I^{semi} - \frac{(p_I^{semi})^2(k+1)}{(k+q_I^{semi})} - I \tag{53}$$

For M to use mixed strategy in state F it should be that $\pi_I^{semi} = \pi_{n.I}^{semi} = \frac{1}{4}$.

$$p_I^{semi} - \frac{(p_I^{semi})^2(k+1)}{(k+q_I^{semi})} - I = \frac{1}{4} \tag{54}$$

Solving for p_I^{semi} gives:

$$p_I^{semi} = \frac{k + q_I^{semi} \pm \sqrt{\left(k + q_I^{semi}\right)\left(q_I^{semi} - 1 - 4I\,(k+1)\right)}}{2(k+1)} \tag{55}$$

The necessary condition on cost of investment for mixing is:

$$I < \frac{1}{4}\,\frac{q_I^{semi} - 1}{k+1} \tag{56}$$

This ensures that the determinant $\left(k + q_I^{semi}\right)\left(q_I^{semi} - 1 - 4I\,(k+1)\right)$is nonnegative.

Proposition 5.

Under uncertain information, a consumer prefers to play:

1. strategy s_1 over s_e if his taste for quality θ is higher than $\theta_{1e} = \frac{\Delta p(1-m)-f}{(1-m)(1-q_2)}$. If his taste for quality is lower than this level, he prefers strategy s_e over s_1.

Proof:

$s_1 \succeq s_e \Leftrightarrow EU[s_1|\theta] \geq EU[s_e|\theta]$

$\theta m q_1 - p_1 + (1-m)\theta \geq m\theta q_1 - mp_1 + (1-m)\theta q_2 - (1-m)\theta p_2 - eff?$

$$\theta \cdot (\overbrace{mq_1}^{>0} + \overbrace{(1-m)}^{>0}\overbrace{(1-q_2)}^{>0} \geq (1-m)(p_1-p_2) - eff$$

under our assumptions $0 < m < 1, q_2 < 1$ this is means:

$s_1 \succeq s_e \Leftrightarrow \theta \geq \theta_{1e} = \frac{\Delta p(1-m)-eff}{(1-m)(1-q_2)}$

Similarly, $s_1 \prec s_e \Leftrightarrow \theta < \theta_{1e}$.

1. strategy s_2 over s_e if his taste for quality θ is lower than $\theta_{2e} = \frac{m(p_1-p_2)+eff}{m(q_1-q_2)}$. If his taste for quality is higher than this level, he prefers strategy s_e over s_2.

Proof:

$s_2 \succeq s_e 2 \Leftrightarrow EU[s_2|\theta] \geq EU[s_e|\theta]$

$\theta q_2 - p_2 \geq m\theta q_1 - mp_1 + (1-m)\theta q_2 - (1-m)\theta p_2 - eff \Leftrightarrow$

$eff + mp_1 + p_2 - mp_2 - p_2 \geq \theta \cdot (mq_1 - q_2 + (1-m)q_2) \Leftrightarrow$

$$\Leftrightarrow eff + m(p_1-p_2) \geq \theta \cdot \overbrace{m}^{>0}\overbrace{(q_1-q_2)}^{>0}$$

under our assumptions $0 < m, q_2 < q_1$ this is means:

$s_2 \succeq s_e \Leftrightarrow \theta \leq \theta_{2e} = \frac{m(p_1-p_2)+eff}{m(q_1-q_2)}$

Similarly, $s_2 \succ s_e \Leftrightarrow \theta > \theta_{2e}$.

1. strategy s_1 over s_2 if his taste for quality θ is higher than $\theta_{12} = \frac{(p_1-p_2)}{mq_1+1-m-q_2}$. If his taste for quality is lower than this level, he prefers strategy s_2 over s_1.

Proof:

$s_1?s_2?EU[s_1|\theta] \geq EU[s_2|\theta]?$

$?\theta mq_1 - p_1 + (1-m)\theta \geq \theta q_2 - p_2 \Leftrightarrow$

$$\Leftrightarrow \theta \cdot \overbrace{(mq_1 + 1 - m - q_2)}^{=(m(q_1-1)+(1-q2))>0} \geq p_1 - p_2$$

under our assumptions $0 < m < 1, q_2 < 1$ this implies:

$s_1?s_2?\theta \geq \theta_{12} = \frac{(p_1-p_2)}{mq_1+1-m-q_2}$

Similarly, $s_1?s_2?\theta < \theta_{12}$.

1. strategy s_2 over s_0 if his taste for quality θ is higher than$\theta_{02} = \frac{p_2}{q_2}$. If his taste for quality is lower than this level, he prefers strategy s_0 over s_2.

Proof:

$s_2?s_0?EU[s_2|\theta] \geq EU[s_0]?\theta q_2 - p_2 \geq 0?\theta \geq \theta_{02} = \frac{p_2}{q_2}$.

Similarly, $s_1?s_2 if \theta < \theta_{02}$.

Lemma 2: The conditions $\theta_{1e} \geq \theta_{12}$ and $\theta_{2e} \leq \theta_{12}$ are equivalent and are $eff \leq \frac{m\Delta p(q_1(1-m)+m)}{mq_1+1-m-q_2}$

Proof: from Lemma 1-c we have

$\theta_{1e} \geq \theta_{12}?\frac{\Delta p(1-m)-eff}{(1-m)(1-q_2)} \geq \frac{\Delta p}{mq_1+1-m-q_2}$

$\frac{eff}{(1-m)(1-q_2)} \leq \frac{\Delta p}{(1-m)(1-q_2)} - \frac{\Delta p}{mq_1+1-m-q_2}$

$\frac{eff}{(1-m)(1-q_2)} \leq \frac{\Delta p(1-m)(m(q_1-1)-q_2)-\Delta p(1-m)(1-q_2)}{(1-m)(1-q_2)(mq_1+1-m-q_2)}$

$\frac{eff}{(1-m)(1-q_2)} \le \frac{\Delta p(m(q_1-1)-1)}{(1-q_2)(mq_1+1-m-q_2)}$

$eff \le \frac{(1-m)\Delta p(m(q_1-1)-1)}{mq_1+1-m-q_2}$

$eff \le \frac{m\Delta p(q_1(1-m)+m)}{mq_1+1-m-q_2} = a$

Similarly, from Lemma 1-b we have

$\theta_{2e} \le \theta_{12} \Leftrightarrow \frac{\Delta p}{mq_1+1-m-q_2} \ge \frac{m\Delta p+eff}{m(q_1-q_2)}$

$\frac{eff}{m(q_1-q_2)} \le \frac{m\Delta p(q_1-q_2)-m\Delta p(m(q_1-1)-q_2)}{mq_1+1-m-q_2}$

$\frac{eff}{m(q_1-q_2)} \le \frac{\Delta p(q_1-mq_1+m)}{(q_1-q_2)(mq_1+1-m-q_2)}$

$eff \le \frac{m\Delta p(q_1(1-m)+m)}{mq_1+1-m-q_2} = b;$

We see that $a \equiv b$, so the condition $\theta_{1e} \ge \theta_{12}$ is equivalent to $\theta_{2e} \le \theta_{12}$ and is $eff \le \frac{m\Delta p(q_1(1-m)+m)}{mq_1+1-m-q_2}$.

Lemma 2'. The conditions $\theta_{1e} < \theta_{12}$ and $\theta_{2e} > \theta_{12}$ are equivalent and are $eff > \frac{m\Delta p(q_1(1-m)+m)}{mq_1+1-m-q_2}$

Proposition 6 (A-type candidate equilibrium):

The discussion in section 4.2 permits us to conclude that if the cost of effort is relatively high (i.e., $f > f_{neverA}$) there exists a pooling equilibrium of the game. The strategy of each consumer would be: "when faced to $q_1^{pooling}$, $q_2^{pooling}$; $p_1^{pooling}$, $p_2^{pooling}$ take the individual decision by reasoning in expected terms and using the apriori probability of state R, m[5]. Otherwise don't buy any product".

If we stay in configuration (A) where $\Delta p < f \cdot \frac{m(q_1-1)+(1-q_2)}{(1-m)(q_1-m)m}$ (this condition needs to be checked ex-post), profits of firms 1 and 2 are given by:

$$\pi_1 = p_1 D_1^A(q_1, q_2, p_1, p_2) = p_1(1 - \frac{\Delta p}{mq_1 + 1 - m - q_2}) \tag{57}$$

[5] i.e., choose from the strategies s_0,s_1,s_2, s_{ew} the one that dominates the others in terms of ex ante expected payoff, as in **??**-29

Maximizing profits with respect to prices leads to:

$$p_1(q_1,q_2) = \frac{2(-2m^2q_1 - mq_1q_2 + m^2q_1^2 + 2mq_1 - 2m - q_2 + 1 + mq_2 + m^2)}{4mq_1 - q_2 + 4 - 4m}; \qquad (58)$$

$$p_2(q_1,q_2) = \frac{(-q_2 + mq_1 + 1 - m)q_2}{4mq_1 - q_2 + 4 - 4m} \qquad (59)$$

With prices set as $p_1(q_1,q_2), p_1(q_1,q_2)$, the profits are:

$$\pi_1(q_1,q_2) = \\ = \frac{4(mq_1 + 1 - m)(-2m^2q_1 - mq_1q_2 + m^2q_1^2 + 2mq_1 - 2m - q_2 + 1 + mq_2 + m^2)}{(4mq_1 - q_2 + 4 - 4m)^2} \qquad (60)$$

$$\pi_2(q_1,q_2) = \frac{(-2m^2q_1 - mq_1q_2 + m^2q_1^2 + 2mq_1 - 2m - q_2 + 1 + mq_2 + m^2)q_2}{(4mq_1 - q_2 + 4 - 4m)^2} \qquad (61)$$

At step 2, both firms choose simultaneously their quality, q_2 and q_1

$$\frac{\delta\pi_1(q_1,q_2)}{\delta q_1} = 4\overbrace{(m(q_1-1)+1)}^{>0} \cdot \\ \cdot \frac{\overbrace{(4m(q_1(1-m)+1) + 2m^2(1+q_1^2) + mq_1q_2 + q_2 + 2)}^{>0}}{\underbrace{(4mq_1 - q_2 + 4 - 4m)^3}_{=4E(\mu_1)-q_2>0}} > 0 \qquad (62)$$

As in the full information case we see that $\frac{\delta\pi_1(q_1,q_2)}{\delta q_1} > 0$, so that Firm 1 is interested to set its characteristic q_1 as high as possible .

Maximizing $\pi_2(q_1,q_2)$ with respect to q_2 gives

$$q_2 = \frac{4}{7}(mq_1 + 1 - m) = \frac{4}{7}E(\mu_1) \qquad (63)$$

Thus, if condition (**??**) holds, the A-type candidate equilibrium is:

$$q_{1A} = 1 + \delta; q_{2A} = \frac{4}{7}(mq_{1A} + 1 - m); p_{1A} = \frac{1}{4}(mq_{1A} + 1 - m); p_{2A} = \frac{1}{14}(mq_{1A} + 1 - m); \\ D_{1A} = \frac{7}{12}(mq_{1A} + 1 - m); D_{2A} = \frac{7}{24}(mq_{1A} + 1 - m); \\ \pi_{1A} = \frac{7}{48}(mq_{1A} + 1 - m);\ \pi_{2A} = \frac{1}{48}(mq_{1A} + 1 - m) \qquad (64)$$