Team Tournaments: Organizers vs. Freeriders

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Introduction

Tournament theory was introduced by Lazear and Rosen (1981) as a way to to analyze motivation of workers when their effort is unobservable. They show that large spread in compensation should increase worker's productivity. The implications of tournament theory were tested in many fields. Ehrenberg and Bognanno (1990) analyze if prize structure in golf follows the tournament theory. Sports provide a good context for empirical testing of tournament theory, so there are a lot of studies including car racing (Depken and Wilson, 2004), bowling (Bognanno, 1990), marathons (Frick and Prinz, 2007), horseriding (Fernie and Metcalf, 1999; Lynch and Zax, 1998), tennis (Gilsdorf and Sukhatme, 2008) foot races (Maloney and McCormick, 2000) and Ironman triathlon (Prinz, 1999).

However, tournament theory was also tested in different contexts. Choi and Gulati (2004) studied how compete for increasingly prestigious courts with the ultimate prize being the U.S. Supreme Court. Knoeber and Thurman (1994) analyze how contract growers vie to supply broiler chickens to Perdue and Tyson. Tournament theory also explains compensation structures of top managers inside the companies (Messersmith et al., 2011).

In our paper we try to incorporate team motivation into traditional rank order elimination tournaments. Following Rosen's studies, the literature on tournament theory is concentrated mostly on the individual tournaments. However, there is a number of competition of both business and sports context, in which the competitors are teams rather than individuals. Coates and Parshakov (2016) tested tournament theory in video game tournaments and conclude that tournament organizers choose different prize spreads in individual and team competitions. That might be a result of potential freerider's problem. Tournament organizer's incentive is to maximize the spectacular value of an event. In that sense, the organizer intention is to maximize the effort of all contestants.

However, this is more complicated task in team competition. Since inside the team there might be an asymmetry of information about the effort of each team member, there are incentives to shirk the responsibilities. For example, Gould and Winter (2009) suggest a model where individual team members may increase or decrease their effort in response to increased effort by teammates. So, tournament organizer's task is to find a way of maximizing the effort of each team member. This task is more difficult, than the individual tournament organizer task to maximize the competition for the first prize.

The model

We propose that during the tournament teams and team members in each tournament stage decide on the level of effort to expand. Firstly, the whole team decides on the optimal effort level maximazing value of competiting accounting for all future stages. Secondly, given the optimal effort level of team, each team member decides on personal effort level. Without any commitment free-riders problem arises and equilibrium personal effort level from the 2nd step becomes zero. The solution of the problem is to derive condition, that leads to new equlibrium without free-riders.

Design of the game

On the first step of the repeated game let's consider a tournament with a paired-comparison structure as in Rosen (1986). There are 2^N teams, that go through N stages. Define s as a number of stages, that remain to the end of the tournament. Each eliminated team gets award W_s at the stage, where s rounds are remained to be played. Award raise from the 1st stage to the final one. Consequently, $\Delta W_s \geq 0$ for each tournament round.

On each round, 2 teams compete. Their level of effort is x and y for team I and J respectively. The effort of the whole team can be presented as a sum of personal efforts of team members $x = x_1 + x_1 + \ldots + x_m$, where m is a number of team members. Moreover, team I and J is characterized by the average ability of players to the competition γ_i and γ_j respectively. Probability of team I to win against team J, with s stage remained, is as follows

$$P_i = \frac{\gamma_i x}{\gamma_i x + \gamma_j y}. \tag{1}$$

Consequently, the chance of winning increase due to increase in effort level, given natural abilities of both teams and effort level of the competitor team.

Each team competes by choosing a level of effort x maximizing their value V_s on each stage. The value of the stage can be derived as a sum of 3 main parts. First part is the expected profit EV_{s-1} of all remain stages, disount is equal to zero. The team gets this profit only if wins, consequently with probability $P_{i,s}$. The second part is award W_{s+1} for the case of losing on the current stage accounting for a probability of losing as $1 - P_{i,s}$. The last part is team costs $c(x)$. For simplicity we assume that costs for different stages have the same functional form $c(x_s) = x_s^2$. So fundamental equation of this part of the game is

$$V_s = \max_{x_s}\{P_s EV_{s-1} + [1 - P]W_{s+1} - x_s^2\}. \tag{2}$$

On the second step of the repeated game, members of team I given the solution of the previous step (optimal effort level of the whole team x_s^*) individually and simultaneously decide on the personal effort level $x_{k,s}, k = 1 \dots m$ on the current tournament stage. They maximize their personal utility with s stage remains $U_{k,s}$, which consists of 2 parts. First part is member's revenue as his part of team's expected revenue, we suppose that there is an equal sharing between members of the team. The second part is his personal costs in the stage. For simplicity costs are calculated as $c(x_{k,s}) = x_{k,s}^2$. The equation of this step of the game is

$$U_{k,s} = \max_{x_{k,s}}\{\frac{P_s EV_{s-1} + [1 - P]W_{s+1}}{m} - x_{k,s}^2\}. \tag{3}$$

Equilibrium analysis

Pareto optimal solution of the game is an outcome, when all teammates choose effort level while s stages remain as $x_{k,s}^* = \frac{x_s}{m}$. To chech the stability of the outcome assume that the 1st player will deviate from Pareto optimal solution as $x_{1,s} = \frac{x_s}{m} - \delta$, where δ is small and positive [1], $\delta \in (0; \frac{x_s}{m})$. Substitute $x_{1,s}$ and [1] in [3] and deduce utility function of the 1st player at each stage given Pareto optimal strategies of other team members:

$$\widetilde{U_{1,s}} = \max_{x_{k,s}-\delta}\{\frac{\gamma_i(x_s - \delta)}{m(\gamma_i(x_s - \delta) + \gamma_j y_s)}EV_{s-1} + \frac{\gamma_j x_j}{m(\gamma_i(x_s - \delta) + \gamma_j y_s)}W_{s+1} - (x_{k,s} - \delta)^2\}. \tag{4}$$

[1] δ can be only positive, since utility includes quadratic cost function. Consequently, increase in effort leads to utility decline.

In this case the players expected revenue will decline (as probability of winning at each stage declines), and costs will decline, too.

Consequently, if the utility, when she deviates, is less or equal to the utility with Pareto optimal strategy $U_{1,s} \geq \widetilde{U}_{1,s}$ for each tournament stage, then she doesn't have incentives to deviate. In simple case all teammates are symmetric and have similar conditions to plat Pareto optimally. The solution of the system of conditions for all players is a restriction on award structure, that prevents free riders problem.

Conclusion

In this paper we adress an issue of team co-production in elimination tournaments. We develop a model to analyze the incentives of team member to shirk in team competitions. Taking into account the motivation of tournament organizer's to maximize the spectacular value and, therefore, the effort of each team member, we analyze the conditions, under which the optimal strategy of each player is to maximize the effort. Those set of conditions is compared to the indivirual elimantion tournament theory of Rosen (1986).