Risk and Return in a Behavioural Framework. An application To Russian Financial Markets.

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The expected utility theory (EUT) is the standard descriptive and prescriptive framework behind most risk/return optimisation models in finance. There are a number of motivations for this choice. First, the expected utility hypothesis can be analytically derived from the axiomatic preference theory. Therefore, in so far as the axiomatic theory describes the way individuals choose among alternative bundles, the EUT derivation is able to describe the way they decide, including the case when bundles are represented by monetary payoffs provided by portfolio of securities. Since EUT derives from the assumed individual preference structure, not only does the theory *describes* what investors do, but also it *prescribes* what they should do; because their choice will comply to their natural preferences even in a complex decision making settings.

EUT, in turn, leads to the ubiquitous mean-variance portfolio approach. By definition, expected utility is set as a reference value to identify risk aversion. It can be then proved that return variance can be adopted as the risk measure for risk adverse investors; consequently, minimising it, exploiting portfolio securities correlations, is the common strategy for optimising the risk-return profile of investment portfolios. Portfolio variance is formally a quadratic form, which makes analytically simple to find its minimum, through convex optimisation techniques (and limiting calculations to first to order conditions).

In recent times, alternative measures have arisen which take into account different dimensions of risk. For example, in the case of *semivariance*, one measures only the actually harmful part of the variance, the negative deviations from mean. However, semivariance breaks the convexity of the objective function, therefore requiring second order conditions to identify the minimum. Following the (commercial success of) Value at Risk (VaR) a number of *shortfall measures* have been proposed. Due to its quantile approach VaR proved to be very easy in communicating risk positions of portfolios among financial practitioners. Unfortunately, VaR is an incoherent measure technically, since it does not abide to the coherence rules ordinarily required by formal risk theory; then, to some academics, it is not even a risk measure. Alternatives have been proposed, such as the *Expected Shortfall* measure and the *Conditional Value at Risk*. These measures capitalise on the idea that financial risk is speculative:

there is a negative (losses) and a positive (gains) component in it. There is no need to include the positive component in the risk measure, like the traditional variance approach does. Their success is coupled by the recent improvements of computational power. In fact, they require numerical methods to identify optimal solutions, especially intensive for large portfolios.

The approach that we investigate goes back to the root of decision theory in a descriptive rather than prescriptive way. A number of empirical investigations have proved that investors do not conform their decision making process to what is prescribed by EUT. Deviation from EUT tenets can be traced back to initial experiments in the context of prospect theory.

When we deny the building blocks of EUT, we are left without the "comfort" of the mathematical expectations of random payoffs; indeed, the expected utility is a tranformation of the mathematical expectaion and, under proper assumptions regarding the utility function, we let the exepcted utility behave much like the mathematical expectation. If we give up EUT, we have to work straight on probability distributions.

To this end, we need to define *ad hoc* optimisation procedures, which maximise the probabilities of desired outcomes, rather than minimising variance. Secondly, we need to give a parametric definition of risk. That is, since we optimise the investor preference function, the risk depends on the parameter values characterising this function. Accordingly, we first build the optimisation framework. Once this procedure is given, much like the standard mean-variance approach, we can adjust the functional risk parameters and identify related combination of portfolio weights.