Abstract. We are constructing an imperfect competition in a general equilibrium with non-consumable money and labor market; our toolkit is an equilibrium default model of Shubik-Wilson (1978). Our result has an ‘equilibrium instability’ simultaneously occurring at all three markets: labor, goods, and credit market, where a bank supplies fixed quantity of money. A worker and an entrepreneur strategically trade at these markets. Players are uncertain about each other’s actions, and do not have a convergence of common beliefs of common knowledge. It is impossible to calculate mixed strategies equilibria exactly, but it is possible to identify some Pareto-efficient strategies and study induced prices, wages, interest rates, default, allocations and payoffs, which are all volatile, and their fluctuations are not independent. We present an equilibrium result, when a total default value is bigger than total money supply.
1. Introduction

There is an enormous literature on money, demand for credit and interest rate, see Walsh (2010), or surveys in the volumes of the “Handbook of Monetary theory,” (1990, 2000, 2010). However, micro-foundations\(^1\) for money demand are still not well-transparent.\(^2\)

Samuelson (1958) introduced infinite time to support eternal existence of demand for money. However, this approach avoids resolution of the Hahn paradox (Arrow, Hahn, 1971): why demand for fiat non-consumable money exists in a finite time, when money worth nothing at the last moment of life. This paradox radically contradicts the mainstream infinite-time models of monetary economics, when permanent existence of eternal future justifies holding money earlier. The key puzzle of the paradox is: what could motivate all agents to hold fiat non-consumable money for finite time intervals?

The suggestion of Clower (1967) to resolve the “an embarrassment to economic theory” was to integrate monetary and value theories by involving money in utility function.\(^3\) Shubik\(^4\) and Wilson (1978), further

\(^1\)By micro-foundations we mean individual decision marking of players, given individual resources and individual beliefs about actions of other players. To be precise, these foundations are considered in terms of non-cooperative games.

\(^2\)“Money is something of an embarrassment to economic theory,” Banerjee and Maskin (1996).

\(^3\)The importance of integrating price pricing theory and monetary theory was emphasized by Banerjee and Maskin (1996), Wallace (2010) wrote that “The search for settings in which money is essential is hardly new. Suggestions about absence-of-double-coincidence difficulties go back at least to the first millennium.... However, despite being repeated over and over again ever since, those statements are incomplete. After all, if they were regarded as satisfactory, then the search would long ago have been regarded as over. If it were over, then the problem of integrating price theory and monetary theory would not have been one of the big unsolved problems in economics throughout the twentieth century.”

\(^4\)Shubik wrote in many places that “Money is an institute of trust.”
(SW78), developed his model to resolve the Hahn paradox using fiat money in one period non-cooperative game. They introduced default as a possible equilibrium phenomenon. In contemporary terms, the paper of Shubik and Wilson studied a mechanism design of fiat money holding, not mentioned by Wallace (2010) on mechanism design for monetary economics.

The suggestion of Shubik and Wilson to consider default on credits as an equilibrium phenomenon matches the empirical facts. Although international statistics on defaults is far from being standardized, terminology is not unified, data are rarely public, it is possible to use the US delinquency rates on consumer credits, or on many other types of credits. It is easy to see that default on credits is the sustainable economic phenomenon, see Figure 1.

Our model, based on SW78, unites elements of micro and macro economic analysis - labor market, goods market, credit market and default in monetary terms - into a general equilibrium framework with imperfect competition. This includes: strategic demand for labor, strategic labor supply, strategic trade in a real good, and imperfect competition for credits, as components of an economy. Activity in the economy is organized as one period non-cooperative strategic market game (Shal-ley and Shubik, 1977) between one worker and one entrepreneur. The game has three prices in terms of fiat non-consumable money: nominal wage, monetary price of a consumable good and a nominal interest rate.
Figure 1. Delinquency rate for consumer loans in US
This setup overcomes the ambiguity to choose a means of payment of Walrasian economics.\(^5\)

Barter trade is prohibited in the model, and the role of the credit market is to supply liquidity though an exchange of a credit for liability. Following SH78 we assume that money is non-consumable, have no intrinsic value, but a default in monetary liabilities generates an individual loss in a value of the game\(^6\). The mechanism design operates through an individual punishment for default, what is different\(^7\) from Stiglitz and Weiss (1977). The injection of money into the economy is different from “helicopter money” or a monetary transfer/tax approach of Gu, Wright (2016), and, also, from generations of monetary models classified by Lagos et al. (2017).\(^8\)

The result of our paper is that equilibrium values of all endogenous variables - wage, price, interest rate, default, demand for money, and their interactions - can not be stable. Their values are exposed to unremovable joint volatility. In the general case this volatility can not be expanded into independent volatilities of separated markets. An equilibrium volatility of the game does not supply new information, and creates market distortions without traditional reasons: outside shocks, or/and information asymmetry.

\(^5\)Ambiguity in a choice of a *numeraire* good.

\(^6\)A loss in utility function, what breaks it’s continuity.

\(^7\)The mechanisms are consistent, see further.

\(^8\)Lagos et al. (2017) surveys generations of the models: first-generation models, for example, Kiyotaki and Wright (1989, 1993) with indivisible assets and indivisible goods, “second-generation models make goods divisible and determine prices by bargaining.” “Third-generation models, with goods and assets both divisible, but they all come with some baggage because they must deal with distributions of assets across agents,” cited from Burret, Trejos and Wright (2017)
The equilibrium phenomenon appears from indeterminacies of players about possible actions of each other at all markets simultaneously.\(^9\) We additionally demonstrate Prisoners’ Dilemma at the credit market, when a too small punishment for default can not enough reduce individual demands for credits ("pseudo-wealth"), and a resulting total value of debt exceeds total money supply, injected into the economy.

An economy of the model is a strategic market game of a worker and an entrepreneur. The worker has time, which he shares between a leisure time and working time. He enjoys the leisure and a produced good. The good is purchased at the goods market, and is produced jointly with the entrepreneur.

The entrepreneur has a production factor, which can be transformed into a fixed quantity of consumable good with a labor time of the worker. She consumes part of the produced good, and the rest sells to the worker through the goods market. The entrepreneur enjoys consumption of the good and managing the working time of the worker.

There are two real markets in the economy, goods and labor, but a barter exchange is prohibited. The worker and the entrepreneur compete for non-consumable fiat money credits from a bank, which offers a fixed money supply.\(^10\) The credit is nominated in fiat money, which can not be consumed, but used only as a means of payment for real goods and credits.

The paper has the following structure: Section 2 discusses relation to existing literature, Section 3 presents the formal model, Sections 4

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\(^9\)The idea is the same as in the colonel Blotto game.

\(^10\)We do not study monetary policy in the paper.
and 5 present approximation and numerical results related to microeconomics, Section 6 presents results related to macroeconomics.

2. Relation to literature

The standard understanding of credits assumes that all credits are payed completely and on schedule.\textsuperscript{11} Our paper deviated from this by developing the model from SW78.

Walsh (2010) wrote about existence of the “three alternative modeling strategies” for studying demand for money. This includes representative-agent\textsuperscript{12} models and overlapping-generations models. These models often assume perfect competition, infinite time, (for example, Bigo and Sannikov, 2018) and complete payments of debts. The third approach is an \textit{ad hoc} construction of “equilibrium relationships that are often not derived directly from any decision problem.” Demand for money, in this big strand of literature, is motivated by infinite life of economic agents: they need a means of trade, and share a belief in future of this means of trade. In macroeconomic literature an origin of an interest rate is usually vague or defined from perfect competition considerations.

The important contribution to resolve the paradox was done by SW78. Their insight was that default on credit should be incorporated into an equilibrium, while individual demand for money is regulated by a punishment for default.\textsuperscript{13} The model of Shubik and Wilson is a

\textsuperscript{11}Like in IS-LM model.
\textsuperscript{12}In our model agents can differ in many respects.
\textsuperscript{13}Further we use the terms “demand for credit,” “demand for money” and “demand for liquidity” as synonyms, even for practical purposes these terms may have differences.
playable non-cooperative game, where studying off-equilibrium cases is an investigation of feasible opportunities. The innovations of the Shubik-Wilson model is that non-consumable money has value for a borrower only as a means of payment or as a means to pay back a debt. In other words, money has no consumable value, but any debt generates a punishment for an unsuccessful borrower. The intuition is: too little punishment can not restrict demand for liquidity from it’s rise to infinity, but too sever punishment suppresses any demand for liquidity, that will terminate a trade. This logics is repeated in our model too.

The non-converging beliefs, appearing in our model, were described by Stiglitz (2018) as “when two individuals differ in beliefs, they have an incentive to engage in a bet ... Both sides, of course, think that they are going to win, so that the sum of their ‘perceived' wealth is greater than ‘true’ wealth. Pseudo-wealth is being created and destroyed all the time ... Fluctuations in pseudo-wealth help explain one of the paradoxes of macroeconomics: the large fluctuations in the economy in spite of small changes in the physical state variables, the stock of capital, labour, and natural capital.”

The observation of Stiglitz has the direct parallel in our paper. Beliefs of players can not converge, pseudo-wealth is a big size of nominal credit, disappearance of the pseudo-wealth is an excessive debt burden, appearing from credit market externalities. Small changes in physical variables are changes in demands and supplies of the model, which induce volatility of endogenous variables in an equilibrium. In this way
we directly address micro-foundations of macroeconomics to study interac-
tions between leisure/labor trade-off from one side, and interest rate, value of default, price, from another.

A “credit channel” (Bernanke, Gertler, 1990) in our model is in-
dividually motivated demand for credits, that results in liabilities in monetary terms for players. The resulting competition for credits generates an interest rate. We do not address economic growth and policy issues in this paper.

Stiglitz and Weiss (1981) suggest credit rationing as a tool to dis-
 criminate between borrowers. Our model can incorporate this approach in a multi-period development. For this case a size of punishment for default should be related to future credit-rationing loss.

Our model satisfies information efficiency criterion (Fama, 1970), all players know about fundamentals of each other, including structure of payoff functions. But this does not guarantee stability of trade and values of endogenous equilibrium variables. Our result differs from real-business-cycle approach to monetary factors in dynamic stochastic general equilibrium (DSGE, for example, Rotenberg, Woodford, 1997). DSGE is sufficiently based on perfect competition, being an expansion of Walrasian economics with outside shocks, but this paper studies a duopoly without outside influence.

Some DSGE literature studies sticky information, (Mankiw and Reis (2006, 2007) Reis (2009a,b). We also have sticky information, but prices are still volatile. This makes volatility not informative. Mankiew and Reiss (2010) survey results on imperfect information models for
monetary economics, with an emphasis on cases, when “information is dispersed and disseminates slowly across a population of agents who strategically interact in their use of information,” particularly “models with partial information, where agents observe economic conditions with noise, and models with delayed information, where they observe economic conditions with a lag.” In our model agents are equally informed, do not have noise in observation of fundamentals, but the noise exists, and is induced by themselves.

Our model offers one more facet for thinking about incomplete information. Different from existing literature we demonstrate that non-converging conjectures can exist in an equilibrium and induce market volatility.

Our result differs also from the result of the island economy of Lucas (Lucas, 1973), where price-taking agents meet an incomplete information about environment, and prices change as ex post adjustment to a previous unknown outside shock. Volatility in our model is not an adjustment.

Our model is a one-period model. We expect to develop intertemporal part of the model in later papers. The presented model has a simplified vision of default, there are no next period consequences. Many period model will be much more complex. But even in this simplified approach the instability does not need “noisy traders,” Kyle (1977). We can attribute all volatility to “animal spirits” resulting from strategic decisions of market agents.
Importance of default for macroeconomic analysis and applications was emphasized by Goodhart and his co-authors in many recent publications, Bhattacharya, Goodhart, Tsomocos and Vardoulakis, (2015), Goodhart and Tsomocos, (2011), Goodhart, Sunirand and Tsomocos, (2004, 2006), Goodhart, Kashyap, Tsomocos and Vardoulakis (2012, 2013), Goodhart, Peiris and Tsomocos, (2013, 2016), Goodhart, Peiris, Tsomocos and Vardoulakis, (2010) and Goodhart, Tsomocos and Vardoulakis, (2010). However, these literature concentrates around price-taking behavior of borrowers and policy issues. Our model suggests a case when total value of default can exceed injected money, causing a spill-over of default from real to financial sector. But a good share of other strands of academic literature considers default as an empirical accident.

The relation of presented approach to rational expectation benchmark was discussed in our previous paper, Levando and Sakharov, (2018). Relation of rational expectations approach to monetary economics was extensively discussed by Sargent (2008): “the rational expectations equilibrium concept equates all subjective distributions with an objective distribution. By equating subjective distributions for endogenous variables to an equilibrium distribution implied by a model, the rational expectations hypothesis makes agents’ beliefs disappear as extra components of a theory.” Imperfect competition of our model demonstrates how indeterminacy of individual conjectures may appear.

Existing theories do not supply a non-ambiguous answer why people hold non-consumable money for finite time-intervals based based
on individual motivations,\textsuperscript{14} Wallace (2010) wrote about non-adequacy of existing theories as “needless to say, models with cash-in-advance constraints — or, more generally, models with asset-specific transaction costs — and models with real balances as arguments of utility or production functions are not among the candidates for such settings. The former are ruled out because their structure does not permit us to ask about other ways of achieving allocations and the latter are ruled out because they are at best implicit versions of the former.” Our model supplies a way to numerically simulate final allocations, along with credit default values.

A hybrid Phillips curve was offered by Gali and Gertler (1999). Our model combines features of both short and long-run horizons. The short-run horizon is described as a trade-off between non-coordinated decisions of the worker and entrepreneur at the labor and goods markets, what results in wage and price adjustments. The long-run horizon is described by a fixed production.

The model offers a possible answer for the question about price volatility “how should these temporary movements be modeled,” (Klenow and Malin, 2010). The answer is that there are properties of a market, when being adequate to the first order conditions of the expected utility maximization problem means addressing to ill-posed equations and instabilities of their solutions.

Prices at all markets are volatile in our model, and have a “missing middle” (Klenow, Malin, 2010), but this appears not from large

\textsuperscript{14}\textsuperscript{Wallace, (2010): ” So what kinds of settings lend themselves to a mechanism-design analysis of monetary trade and are fruitful?”}
menu cost as in Golosov and Lucas (2007). Midrigan (2009) suggested to address such results in a monetary economy as “departures from monetary neutrality.”

The important property of our model is that even there is no stickiness of price, wage or interest rate and different kinds of menu costs, there is no synchronization volatility of the price, wage, interest rate and default value.

Pricing externalities in our model are similar to those in Gorodnichenko (2008), but here externalities are not necessarily positive. For example, a spill-over effect from competition for a credit to real markets.

Our model exploits the property of simultaneous games, i.e. equal treatment of all markets, however real-life goods, labor and credit markets do have different stickiness.\textsuperscript{15} This requires reconstructing of the model as a sequential one, however it will not change the mathematical core of the problem, ill-posed property, but the formal side will become more complicated.

3. Model

Let player $i = 1$ be a worker, he owns $Q_1$ units of time, $q_1$ is his labor supply, $q_1 \in [0, Q_1], Q_1 < \infty$, and he enjoys own leisure, $Q_1 - q_1$. The worker also enjoys a consumable good, which can be produced only together with an entrepreneur, $i = 2$.

\textsuperscript{15}For example as heterogeneity in frequencies of information adjustments, as in Carvalho and Schwartzman (2008), maturities of contracts, technological innovations shocks, as in Lorenzoni (2009), etc.
The entrepreneur has $Q_2$ units of non-consumable capital, which can be converted into a consumable good with labor supply $q_1$ from the worker. The entrepreneur can consume the capital only if there is no trade.

The production technology is a mapping of two factors, labor and capital, into the consumable good: $F_2: [0, Q_1] \times [0, \bar{Q}_2] \mapsto Q_2$, where $Q_2 \in \mathbb{R}_+^1$, $Q_2 < \infty$. Labor supply $q_1$ does not effect production outcome $Q_2$, but has a production cost $b_2$ for the entrepreneur. She sells part of the produced good to the worker at the goods market. The entrepreneur values not sold quantity of the good, $Q_2 - q_2$, and the bought worker’s labor time $q_1$.

There are two real markets in the economy: labor, and goods, but a barter exchange is prohibited. Both agents need fiat money for trade. Hence we introduce the third market, a money or market for credits, where a bank sells money and an interest rate is formed.

All three markets are oligopolistic. At the labor market the entrepreneur chooses how much units $b_2$ of fiat non-consumable money to pay for the supplied labor time $q_1$ of the worker. The price of the labor market is a wage, $w = \frac{b_2}{q_1}$, defined as in SMG. After the market closes the entrepreneurs obtains $q_1$ units of labor, the worker obtains $b_1$ units of money.

\footnote{If a game is sequential, then more traditional production function can be used with a costly rise in complexity of the game. We assume that $0 < Q_1, \bar{Q}_2 < \infty$.}

\footnote{The assumption that a manager enjoys power over worker’s time does not seem to contradict observations.}

\footnote{Domains for the monetary payments $b_1$ and $b_2$ are assigned after a description of how money enters into the economy.}
At the goods market the entrepreneur chooses a quantity of the produced good to sell, \( q_2 \), \( 0 < q_2 \leq Q_2 \), and the worker \( i = 1 \) chooses how much units of fiat money \( b_1 \) to pay for the consumable good. The price for consumable good is determined as \( p = \frac{b_1}{q_2} \). After the market closes, the entrepreneur receives \( b_1 \) units of money, and the worker receives \( q_2 \) units of the good to consume. The entrepreneur consumes the residual quantity of good 2, \( Q_2 - q_2 \).

The money market of the model\(^{19}\) has a supply and demand sides. The supply side consists of bank, a dummy player in the game. Its only function to supply a fixed quantity of fiat non-consumable money \( M \). The demand side of the market consists of the worker and the entrepreneur, who submit individual demands for credits \( d_i \). Individual demand for money, \( d_i, i = 1, 2 \), is a promise of \( i \) to pay back the total quantity \( d_i \), \( 0 \leq d_i \leq M, i = 1, 2 \). Fiat money in the model is not consumable, i.e. it has no value, if left after a trade, but it induces a disutility for \( i \), if a debt \( d_i \) is not payed completely. We assume that \( M \) is public information, and every agent \( i = 1, 2 \) can not promise more than \( M \) units of money to return. Competing bids\(^{20}\) for credits form an interest rate as in Shubik and Wilson (1977):

\[
1 + \rho = \frac{d_i + d_{-i}}{M}.
\]

\(^{19}\)The market can be addressed as money market, as it is an institute where money is injected into the economy and reaches agents. The mechanism of injection is a credit, thus the same market can be addressed as a credit market.

\(^{20}\)This non-coordinated competition induces Prisoners' Dilemma if the two demands are too big, but every player desires to obtain as much credit as possible with as little interest rate. These actions are the source of credit market externalities, which have an impact for other markets.
Then $i$ obtains a fiat money credit $\frac{d_i}{d_i + d_{-i}} M$. The credit is the upper bound for a feasible payment of $i$, $b_i \in [0, \frac{d_i}{d_i + d_{-i}} M]$, and depends on an action $d_{-i}$ of another player, $-i$.

In a simultaneous game which we study, $i$’s total cash flow consists of: a credit $\frac{d_i}{d_i + d_{-i}} M$, $d_i, d_{-i} \in [0, M]$; own payment $b_i$ for a good, a payment received $b_{-i}$; and the credit to pay back $d_i$.

Let $\mu$ be a punishment for default, $\mu > 0$, for any negative money holding, $\frac{d_i}{d_i + d_{-i}} M - b_i + b_{-i} - d_i < 0$ after the trade. Following SW78, the punishment has an impact on ex ante motivation to take a credit: if $\mu \to \infty$, then $d_i \to 0$, but if $\mu \to 0$, then $d_i \to \infty$. The payoff from final money holding comprises the role of money as fiat and non-consumable token:

$$\mu \ast \min \left\{ 0; \frac{d_i}{d_i + d_{-i}} M - b_i + b_{-i} - d_i \right\}.$$  

A non-cooperative game requires strategy sets for every player, and let $S_i$ be $i$’s strategy set defined as

$$S_i = \left\{ s_i = (q_i, b_i, d_i) : \begin{cases} \quad q_i \in [0; Q_i] \subset \mathbb{R}_+ \\ d_i, d_{-i} \in [0; M] \subset \mathbb{R}_+ \\ b_i \in [0; \frac{d_i}{d_i + d_{-i}} M] \subset \mathbb{R}_+^2 \end{cases} \right\} \subset \mathbb{R}_+^4,$$

where $s_i$ is a pure strategy of $i$, and it consists of three actions: a supply $q_i$, from the one-dimension compact; a money payment $b_i$, from the two-dimension compact; and a demand for liquidity, $d_i$, from the one-dimension compact. Demand for money $d_{-i}$ from another player $-i$ is not controlled by $i$, but it has an impact on a size of available credit

\footnote{A sequential game adds more realism, but it is more complicated for analysis.}
for $i$, and, imposes a restriction on a range of feasible payments $b_i$. The set $S_i$ is four-dimensional, a boundary of $S_i$ has two one-dimensional facets, and one two-dimensional facet, but $i$ directly controls only three of dimensions. In the general case $S_i$ is neither convex nor concave, see an example below.

Utility of $i$ consists of a payoff from consumption and a payoff from holding final money balance:

$$U_i(q_{-i}, b_{-i}, d_{-i}) (q_i, b_i, d_i) = \phi_i(q_i, q_{-i}) + \mu \min\left\{0; \frac{d_i}{d_i + d_{-i}} M - b_i + b_{-i} - d_i\right\},$$

where $\phi_i(q_i, q_{-i}) = \sqrt{Q_i - q_i} + \sqrt{q_{-i}}$. The utility function captures the ideas that money is required only for trade, to pay the credit back, and it does not have intrinsic value.$^{22}$ The payoffs $U_i(\cdots)(\cdots)$ is written in the operators notation: there are fixed exogenous parameters $(q_{-i}, b_{-i}, d_{-i})$ for $i$, and an operator is defined over the three endogenous arguments $(q_i, b_i, d_i)$. Parameters are interpreted as actions of another player $-i$, arguments are interpreted as own actions of $i$.

Strategic market games have a continuum of pure strategies equilibria (Peck, Shell, 1978). Hence, one can take any set of pure strategies and construct over them a mixed strategies equilibrium. So the game has many mixed strategies equilibria. We do not address the full generality of the task, but only demonstrate some properties of the game, using pure Pareto-improving strategies. But first, we need to introduce mixed strategies and construct the first order condition.

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$^{22}$This is the direct addressing the Hahn paradox with demand for non-consumable money for a finite time interval.
Let

\[
\Delta_i = \left\{ \sigma_i(s_i) \geq 0, \quad s_i \equiv (q_i, b_i, d_i) : \int_{d_{-i} \in [0:M]} \int_{[0;\frac{d_i}{d_i + \mu_0}M]} \sigma(s_i) ds_i = 1 \right\}
\]

be a set of mixed strategies of \(i\), where normalization condition is the Lebegue integral with a general element for a mixed strategy \(\sigma_i(q_i, b_i, d_i)\). Domain of integration is written explicitly, as it can not be constructed as a cartesian product. This also implies that actions of \(i\) can not be statistically independent.

Expected utility of \(i\) is

\[
EU_i(\sigma_i, \sigma_{-i}) = \int_{S_i \times S_{-i}} \left( \phi_i(q_i, q_{-i}) + \mu \times \min \left\{ 0; \frac{d_i}{d_i + d_{-i}}M - b_i + b_{-i} - d_i \right\} \right) d\sigma_i(q_i, b_i, d_i)d\sigma_{-i}(q_{-i}, b_{-i}, d_{-i}),
\]

subject to \(\int_{S_i} d\sigma_i(q_i, b_i, d_i) = 1\).

Player \(i\) controls only own mixed strategy, \(\sigma_i(q_i, b_i, d_i)\), taking mixed strategy \(\sigma_{-i}(q_{-i}, b_{-i}, d_{-i})\) of \(-i\) as given.

We are interested in a trading equilibrium, which must have incentive compatibility property for both players, \(i = 1, 2\):

\[
EU_i(\mu_i, \mu_{-i}) \geq U_i(Q_i, B_i),
\]
$U_i(Q_i, B_i) \equiv U_i(Q_i, B_i, 0, 0, 0)$ is a payoff from consumption of initial endowment $(Q_i, B_i)$, or a fixed opportunity gain for $i$. Incentive compatibility means that trading mixed strategies equilibrium must guarantee a payoff not less than one from no-trade. Different from traditional game theory problems, opportunity gain here is determined explicitly by fundamentals of the game.

Nash equilibrium is a pair of mixed strategies $(\mu^*_1, \mu^*_2)$ such that for every $\mu_i \neq \mu^*_i$ there is

$$EU_i(\mu^*_i, \mu^*_{-i}) \geq EU_i(\mu_i, \mu^*_{-i}), \quad i = 1, 2.$$ 

Nash equilibrium exists due to generalization of the fixed point theorem for a finite product of infinite-dimensional spaces\footnote{Tikhonov theorem guarantees compactness of the product.} as a mapping of a bounded, closed, convex, continuous set into another, $(\Delta_1, \Delta_2) \mapsto (\Delta_1, \Delta_2)$. Equilibrium incentive compatibility for trade is such that for every $i$ there is

$$EU_i(\mu^*_i, \mu^*_{-i}) \geq U_i(Q_i, B_i).$$
First order condition of $i$ is derived using methods of calculus of variation:

\begin{equation}
\int_{S_{-i}} \left( \phi_i(q_i, q_{-i}) + \mu \cdot \min \left\{ 0, \frac{d_i}{d_i + d_{-i}} M - b_i + b_{-i} - d_i \right\} \right) d\sigma_{-i}(q_{-i}, b_{-i}, d_{-i}) = \lambda_i,
\end{equation}

\begin{equation}
d\sigma_{-i}(q_{-i}, b_{-i}, d_{-i}) \geq 0, \quad \int_{S_{-i}} d\sigma_{-i} = 1
\end{equation}

for $\forall s_i = (q_i, b_i, d_i) \in S_i$,

where $\lambda_i \neq 0$ is any non-zero Lagrangian multiplier, $\sigma_{-i}(q_{-i}, b_{-i}, d_{-i})$ is unknown probability distribution or unknown mixed strategy, $\sigma_{-i} \in \Delta_{-i}$. Informally, the first order condition means that for every pure strategy $s_i$, $s_i \in S_i$, the payoff of one player should be smoothed by mixed strategies of another player, and it must be true for every $s_i$.

Equation (2) is first kind integral equation of Fredholm, notoriously known for its bad numerical properties, (Kabanikhin, 2005, Petrov and Sizikov, 2005), and usually, can not be precisely calculated. The complications with this equation type are similar to those appearing with systems of linear equations, when a matrix of coefficients is severely ill-conditioned or singular. The reason is that properties of integral operators in infinite-dimensional spaces are richer than properties of matrix operators in finite spaces, and methods developed for analysis of finite-dimension spaces can be inadequate.

Ill-posed problem is often approximated by Tikhonov regularization procedure. However, this method does not guarantee that an obtained solution is non-negative and can be appropriately normalized. At the
moment it is unclear, how to approximate mixed strategies for this kind of strategic market games. Our approach is explained in details further.

4. Construction of an approximation

The technical difference with our previous paper (Levando and Sakharov, 2018) is that the game is the generalized Nash game (Debreu, 1952?), where a strategy of one player has an impact on a feasibility of strategies for other players.

4.1. Discrete approximation of pure strategies. Let for one dimension domains \([0, Q_i]\) and \([0, M]\) an integer \(dim\) be an number of approximating points:

\[
0 < q_{i,1} < \cdots < q_{i,dim} < Q_i,
\]

and

\[
0 < d_{i,1} < \cdots < d_{i,dim} < M.
\]

Nodes of Tchebyshev polynomials of the 1-st kind, or collocation points, are locations for these points. These polynomials provide the best approximation in a class of polynomials of the order no bigger than \(dim\). Collocation points are more sparse in the middle and more concentrated closer to intervals bounds.

The approximating set of \(i\)'s strategies has 4 dimensions, each dimension has index in the range 1, \ldots, \(dim\). Let index \(t_1\) (or \(t_{i1}\) to emphasize that it is a decision variable of the player to move \(i\)) be reserved for a supply \(q_i\) of player \(i\), the index \(r_1\) (or \(r_i\) similarly) is reserved for
demand for credit $d_i$, (or $d_i$ similarly). The index $t_2$ (or $t_{2,i}$ similarly) is reserved for a payment $b_i$, which is constructed below. Demand for credit $d_{-i}$ has the index $r_2$ (or $r_{-i}$). Indices $l_1$ and $l_2$ (or $l_{-i,1}$, $l_{-i,2}$) are reserved for actions of $q_{-i}$ and $b_{-i}$ respectively.

Let $d_{i,r_i}$ be $i$’s demand for money, and demand for money from $-i$ is $d_{i,r_{-i}}$. Then, let $\frac{d_{i,r_i}}{d_{i,r_i} + d_{-i,r_{-i}}} M$ be a credit, and a total set of possible credits for $i$ make a conditional feasible payment set

$$B_{i,r_i,r_{-i}} = \left\{ \frac{b_{i,t_2,r_i,r_{-i}}}{d_{i,r_i} + d_{-i,r_{-i}}} : \frac{b_{i,t_2,r_i,r_{-i}}}{d_{i,r_i} + d_{-i,r_{-i}}} \in [0; \frac{d_{i,r_i}}{d_{i,r_i} + d_{-i,r_{-i}}}] \right\},$$

or a quantity of fiat and non-consumable money available for $i$ with a general element $b_{i,t_2,r_i,r_{-i}}$. The set $B_{i,r_i,r_{-i}}$ is a two dimension compact. Every $B_{i,r_i,r_{-i}}$ is discretized by collocation points with index $t_2$ (or $t_{2,i}$). i.e.

$$0 < b_{i,1,r_i,r_{-i}} < \ldots < b_{i,t_2,r_i,r_{-i}} < \ldots < b_{i,dim,r_i,r_{-i}} < \frac{d_{i,r_i}}{d_{i,r_i} + d_{-i,r_{-i}}},$$

such that for fixed $r_i$ and $r_{-i}$ player $i$ has a feasible payment set, which we label the same as above. Due to discretization player $i$ has $dim$ payment actions for every $B_{i,r_i,r_{-i}}$.

The total set of payment actions of $i$ is

$$B_i = \left\{ B_{i,r_i,r_{-i}} : r_i, r_{-i} \in \{1, \ldots, n\} \right\}.$$

For numerical simulation we take values of endowments $Q_1 = Q_2 = 10$, and money supply $M = 3$. Table ?? demonstrates different feasible payment sets $B_{i,r_i,r_{-i}}$ set for $i = 1$ from different demands for credits $d_{-i,r_{-i}}$ from $-i$. The variable $r_{-i}$ is an exogenous parameter for every
layer of possible actions of \( i \). The 4D set appears as we need to enumerate internal points of the set of strategies, not only discrete number of points on the boundary of \( S_i \). It is easy to see on Figure ?? that the surface of \( S_i \) is neither convex, nor concave.

**Table 1.** Form of a strategy set

<table>
<thead>
<tr>
<th>Demand for credit ( d_{i,r_i} ) vs payment ( b_i(i,r_i,r_{-i}) )</th>
<th>Supply ( q_{t_1, t_2, r_i, r_{-i}}, d_{i,r_i} ), payment ( b_i(i,r_i,r_{-i}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{dim}=5 )</td>
<td>( \text{dim}=6 )</td>
</tr>
</tbody>
</table>

Restrictions for demand for money: \( 0 < d_1, d_2 \leq M, M = 3 \).

4.2. **Approximation of the payoff function.** Payoff function \( U_i(q_i, b_i, d_i, q_{-i}, b_{-i}, d_{-i}, \) \) is defined over \( S_i \times S_{-i} \subset \mathbb{R}_+^6 \). Every player controls only three parameters, \( q_i, b_i, d_i \), but \( d_{-i} \) imposes an upper bound for \( i \)'s strategy set. Let \( \sigma_i(q_{t_1, t_2, r_i, r_{-i}}, d_{i,r_i}) \) be a probability for a pure strategy \( s_i = (q_{t_1, t_2, r_i, r_{-i}}, d_{i,r_i}) \in S_i \subset \mathbb{R}_+^4 \) with indices \( t_1, t_2, r_i \) chosen by \( i \), while \(-i\) chooses \( d_{-i,r_{-i}} \) as a demand for money with an index \( r_{-i} \).
The approximating set of mixed strategies $\sigma_i(q_{i,t_1}, b_{i,t_2,r_i}, r_{-i}, d_{i,r_1})$ of $i$ is a discreet set defined over $\text{dim}^4$ points; a domain for $\Delta_i(S_i)$, is approximated by $\text{dim}^4$ points.

Utility from consumption is borrowed from SW78:

$$
\phi_i(q_i, q_{-i}) = \sqrt{Q_i - q_i} + \sqrt{q_{-i}}.
$$

For player $i = 1$ the first term $\sqrt{Q_i - q_i}$ is the leisure, for $i = 2$ is consumption of the good. The second term $q_2$ for $i = 1$ is consumption, for $i = 2$ is working time.

We approximate $U_i$ and construct a block-diagonal matrix with a trapezoid rule, and do this in two steps. First, to construct a block we fix indices $r_i$ and $r_{-i}$ and vary indices $t_{i,1}, t_{i,2}, l_{i,1}, l_{i,2}$ for $q_1, b_1, q_2, b_2$. Then, we move across blocks, changing $r_i$ and $r_{-i}$, what changes the interest rate.

In every row of every block an interest rate is fixed, and supply and demand of $-i$ for real good markets vary. In a raw we change $q_{-i}$ and $b_{-i}$ by varying $l_{i,1}, l_{i,2}$, in columns we change $q_i$ and $b_i$ by varying $t_{i,1}, t_{i,2}$. 
FIAT MONEY OLIGOPOLISTIC ECONOMY WITH LABOR/LEISURE TRADE-OFF AND EQUILIBRIUM DEFAULT

Let \( q_{i,t} = \text{const}_1 \), \( d_{i,r_i} = \text{const}_2 \), \( d_{-i,r_{-i}} = \text{const}_3 \) and \( b_{i,t_2,r_i,r_{-i}} = \text{const}_4 \), where \( \text{const}_1, \ldots, \text{const}_4 \) are some relevant constants. A payoff for a fixed interest rate and fixed \((q_i, b_i)\) can be approximated as:

\[
\int_{q_{-i} \in [0,Q_{-i}]}^{b_{-i,r_{-i}} \in [0,M]} U_i(s_i, s_{-i}) \sigma_{-i}(s_{-i}) d(q_{-i}, b_{-i}, d_{-i})
\]

\[
\approx \sum_{l_1, l_2 \in \{1, \ldots, \text{dim}\}} \alpha_{t_1,t_2,l_1,l_2,r_i,r_{-i}} U_i(q_{i,t_1}, b_{i,t_2,r_i,r_{-i}}, d_{i,r_i}, q_{-i,l_1}, b_{-i,l_2,r_{-i},r_{-i}}, d_{-i,r_{-i}})
\]

where \( \alpha_{t_1,t_2,l_1,l_2,r_i,r_{-i}} = (1/2)^n \), \( n \) - is a number of jointly non-equal to 1 or \( \text{dim} \) indices. The coefficients \( \alpha_{t_1,t_2,l_1,l_2,r_i,r_{-i}} \) appear from an approximation of 2-dimensional plane in a 6-dimensional space \( S_i \times S_{-i} \). They can be independently calculated from orthogonal properties of Tchebyshev polynomials. For notational simplicity let

\[
x_i(t_1, t_2, l_1, l_2, r_i, r_{-i}) := \alpha_{t_1,t_2,l_1,l_2,r_i,r_{-i}} U_i(q_{i,t_1}, b_{i,t_2,r_i,r_{-i}}, d_{i,r_i}, q_{-i,l_1}, b_{-i,l_2,r_{-i},r_{-i}}, d_{-i,r_{-i}}).
\]

The indices pair \( r_i, r_{-i} \) fixes an interest rate, and for these indices we make a matrix

\[
X(r_i, r_{-i}) = \begin{pmatrix}
  x_i(1, 1, 1, r_i, r_{-i}) & \cdots & x_i(1, 1, \text{dim}, r_i, r_{-i}) \\
  \vdots & \ddots & \vdots \\
  x_i(\text{dim}, \text{dim}, 1, r_i, r_{-i}) & \cdots & x_i(\text{dim}, \text{dim}, \text{dim}, r_i, r_{-i})
\end{pmatrix}
\]

The indices \( l_1 \) and \( l_2 \) increase along rows of the matrix \( X(r_i, r_{-i}) \), from 1 to \( \text{dim} \); indices \( t_1 \) and \( t_2 \) increase along columns, from 1 to \( \text{dim} \).

\[\text{Every where in the approximation section ‘fixing a strategy’ means ‘fixing an index for the strategy’}.\]
Thus, there $\dim^2$ rows and $\dim^2$ columns in every block. Every $X_i(r_i, r_{-i})$ is constructed for two fixed demands for liquidity from both agents, and there are $\dim^2$ of such matrices.

A matrix of unknown probabilities for a pair $(r_i, r_{-i})$ is a $\dim^2$ column vector $\sigma_{-i}(r_i, r_{-i})$:

$$\sigma_{-i}(r_i, r_{-i}) = \begin{pmatrix} \sigma_{-i}(q_{-i,l_1}, b_{-i,l_2}, r_i, d_{-i,r_{-i}}) \\ \vdots \\ \sigma_{-i}(q_{-i,l_1}, b_{-i,l_2}, r_i, d_{-i,r_{-i}}) \\ \vdots \\ \sigma_{-i}(q_{-i,\dim}, b_{-i,\dim}, r_i, d_{-i,r_{-i}}) \end{pmatrix},$$

where $(l_1, l_2)$ is a number of (approximating) mixed strategies $\sigma_{-i}(q_{-i,l_1}, b_{-i,l_2}, r_i, d_{-i,r_{-i}})$ in a column, $1 \leq l_1, l_2 \leq \dim$. Every row is an unknown mixed strategy of $-i$, i.e. a probability $\sigma_{-i}(q_{-i,l_1}, b_{-i,l_2}, r_i, d_{-i,r_{-i}})$, that player $-i$ chooses a strategy $(q_{-i,l_1}, b_{-i,l_2}, r_i, d_{-i,r_{-i}})$. The two indices $l_1, l_2$ change from 1 to $\dim$ for every $\sigma_{-i}(r_i, r_{-i})$.

Approximating the first order condition for a fixed $(r_i, r_{-i})$ we have:

$$X_i(r_i, r_{-i})\sigma_{-i}(r_i, r_{-i}) = \lambda_i(r_i, r_{-i}),$$

where $\lambda_i(r_i, r_{-i})$ is a $\dim^2$ column vector.

$$\lambda_i(r_i, r_{-i}) = \begin{pmatrix} \lambda_{1,1,1} \\ \vdots \\ \lambda_{1,t_1,t_2} \\ \vdots \\ \lambda_{1,\dim,\dim} \end{pmatrix},$$
indices \( t_1, t_2 = 1, \ldots, \text{dim} \) correspond to strategies of \( i \), \( \lambda_{t_1, t_2} \) corresponds to fixed \( q_{i, t_1} \) and \( b_{i, t_2} \) actions of \( i \). The same in matrix notation

\[
X_i(r_i, r_{-i}).\sigma_{-i}(r_i, r_{-i}) = \lambda_i(r_i, r_{-i})
\]

meaning that given \( r_i, r_{-i} \) and respective money demands \( (d_{i, r_i}, d_{-i, r_{-i}}) \) equilibrium condition smoothes all fluctuations for all indices \( t_1, t_2 = 1, \text{dim} \).

Approximation of the first order condition (2) for all \( (r_i, r_{-i}) \in \{1, \text{dim}\} \) generates a block-diagonal matrix, size \( \text{dim}^2 \times \text{dim}^2 \):

\[
\begin{pmatrix}
X_i(1,1) & 0 & \vdots & 0 \\
0 & X_i(1,2) & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & X_i(\text{dim}, \text{dim} - 1) & 0 \\
0 & \vdots & 0 & X_i(\text{dim}, \text{dim})
\end{pmatrix}
\begin{pmatrix}
\sigma_{-i}(1,1) \\
\sigma_{-i}(1,2) \\
\vdots \\
\sigma_{-i}(\text{dim}, \text{dim} - 1) \\
\sigma_{-i}(\text{dim}, \text{dim})
\end{pmatrix}
= 
\begin{pmatrix}
\lambda(1,1) \\
\lambda(1,2) \\
\vdots \\
\lambda(\text{dim}, \text{dim} - 1) \\
\lambda(\text{dim}, \text{dim})
\end{pmatrix}
\]

Let \( X_i(\text{dim}_i, \text{dim}_{-i}) \) be

\[
X_i(\text{dim}_i, \text{dim}_{-i}) = 
\begin{pmatrix}
X_i(1,1) & 0 & \vdots & 0 \\
0 & X_i(1,2) & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & X_i(\text{dim}, \text{dim} - 1) & 0 \\
0 & \vdots & 0 & X_i(\text{dim}, \text{dim})
\end{pmatrix}
\]
\[ M_{-i} \] is a column vector, size \( 1 \times \text{dim}^4 \), \( M_{-i} = \begin{pmatrix} \sigma_{-i}(1,1) \\ \sigma_{-i}(1,2) \\ \vdots \\ \sigma_{-i}(\text{dim},\text{dim}-1) \\ \sigma_{-i}(\text{dim},\text{dim}) \end{pmatrix} \), and

\[ \Lambda_i \] is a column vector, size \( \text{dim}^4 \), \( \Lambda_i = \begin{pmatrix} \lambda(1,1) \\ \lambda(1,2) \\ \vdots \\ \lambda(\text{dim},\text{dim}-1) \\ \lambda(\text{dim},\text{dim}) \end{pmatrix} \).

Then the approximating system of equations is

\[ X_i M_{-i} = \Lambda_i. \]

Instability of inverting \( X_i \) to an increase in approximation parameter \( \text{dim} \) can be demonstrated by conditional number (Fadeev, 1959):

\[
\text{cond} (X_i'X_i + \epsilon I(\text{dim})) = \sqrt{\frac{\lambda_i(X_i'X_i + \epsilon I(\text{dim}))_{max}}{\lambda_i(X_i'X_i + \epsilon I(\text{dim}))_{max}}}.
\]

\( X_i'X_i + \epsilon I(\text{dim}) \) is a smoothed matrix for some \( \epsilon \geq 0 \). The sign ‘ is a transposition operation, and \( I(\text{dim}) \) is a unit matrix of an appropriate order. Some conditional numbers for \( \text{dim} = 7 \) and players with \( Q_1 = Q_2 = 10 \) are presented in the Table 2.

<table>
<thead>
<tr>
<th>( \epsilon )</th>
<th>0</th>
<th>1/500</th>
<th>1/100</th>
<th>1/10</th>
<th>1/5</th>
</tr>
</thead>
<tbody>
<tr>
<td>player 1</td>
<td>2.7E9</td>
<td>6.24E6</td>
<td>1.25E6</td>
<td>1.25E5</td>
<td>6.24E4</td>
</tr>
<tr>
<td>player 2</td>
<td>7.46E10</td>
<td>6.25E6</td>
<td>1.25E6</td>
<td>1.25E5</td>
<td>6.25E4</td>
</tr>
</tbody>
</table>
Conditional numbers in Table ?? demonstrate that a decrease in regularization parameter $\epsilon$ explodes the condition number of an approximating matrix $X_i$. The observed instability of $X_i(dim)$ immediately implies the ill-posed property of the original integral equation (2).

5. Construction of Pareto-improving strategies

We study only Pareto-improving strategies, as in Levando and Sakharov, (2018). Their approach guarantees that *ex ante* every player will be better off in comparison to no-trade equilibrium, a trade will occur, but it does not guarantee absence of default. We will numerically demonstrate this effect below.

A set of all Pareto-improving strategies will have positive probability in some mixed strategies equilibrium by the following reasoning. Assume an opposite, and they do not have an equilibrium at these points. Then it is possible to find another pure strategies profile, which will provide a better result for a player, but this does not guarantee that another player will be not be worse off. Construction of the set of PE strategies is based on incentive compatibility, thus another player may not trade in this case, and the deviation will result in no-trade equilibrium. In LS we demonstrated numerically a case, why there is no strict dominance in the set.

Let $\left(q_i^P, b_i^P\right)_{i=1,2}$ be a pair of such Pareto-improving strategies. Further examples present simulations for endogenous variables constructed only for this type of strategies.
5.1. Approximation of prices, interest rate and default. Table ?? demonstrates an effect of a punishment increase to a number of Pareto-improving strategies, a number of PE points decreases. We can also see that an increase in a number of discretization points \( \text{dim} \) increases number of points under investigation.

In further calculations we use values \( M = 3 \) for money supply and \( \mu = 1 \) for punishment. Table ?? presents frequencies of Pareto-improving supply of good \( q_1 \); payment \( b_1 \); and demand for money \( d_1 \) only for the first player. We use frequency distributions from occurrences of values of PE pure strategies, but not probabilities, as we do not calculate probabilities. Frequency distributions supply information on what can be realized, and this satisfies a goal of our paper.

We presents results only for player \( i = 1 \). Every graph has values of a pure strategy along X-axes, and frequencies along Y-axes. Values for payments \( b_1 \) depend also on demand for credit \( d_{-i} \) of another player. We can easily observed that supply \( q_1 \) of consumable good, payment \( b_1 \) from player, and demand for credit \( d_1 \) are not stable.

Comparison of effects of discretization and punishment deserves a special consideration, what is reserved for the next papers. The main message here is that all endogenous variables are all unstable due to indeterminacy of beliefs, and these indeterminacies are interdependent.

Terms of trade are constructed as ratios of demand over supply: for the goods market a price is \( p_i = \frac{b_i}{q_i} \), for credit market interest rate is \( 1 + r = \frac{M}{d_i + d_{-i}} \). Total value of default in the economy as a sum of non
full-filed credit liabilities of both agents is:

\[
\min \left\{ 0; \frac{d_{i,r_i}}{d_{i,r_i} + d_{-i,r_i}} M - b_{i,t_i,r_{-i}} + b_{-i,t_{-i},r_{-i},r_i} - d_{i,r_i} \right\} + \\
\min \left\{ 0; \frac{d_{-i,r_{-i}}}{d_{i,r_i} + d_{-i,r_{-i}}} M + b_{-i,t_{-i},r_{-i},r_i} - b_{i,t_i,r_i,r_{-i}} - d_{-i,r_{-i}} \right\}.
\]

The numerical example is presented in Table ??, and as earlier, all variables are non-stable. Table ?? presents a numerical example, when both agents have default for a fixed \( dim \) and \( \mu \). Easy to see that an increase in punishment decreases a share of default cases.

6. Macroeconomic features of the model

Above we have demonstrated the approximation machinery of our approach. In this section we address to simulation of interactions between variables. Volatilities at different markets are not independent, thus, we need 3D diagrams to demonstrate frequency distributions of possible outcomes. We present only frequency distributions of domains for points, which appear with positive probabilities.

Table ?? presents 3D frequency distributions of leisure versus interest rate, price, and default. By the same reasons as above, that these relations should be unstable too. This diagrams can be considered as an analogue for the Phillips curve, when players have indeterminacy about decisions of each other. Rational expectations approach fails to describe such situation.

The results are similar to those, discussed above. There are many Pareto-improving outcomes, but all endogenous variables fluctuate: leisure \( Q_1 - q_1 \), supply of good, \( q_2 \), wage \( w \), etc. The situation replicates
the case of non-informative price instability presented in LS18, but for multiple markets. We leave research into the structure of instabilities for the future.

However, there are cases, when an outcome is Pareto-efficient, but there is a default for players. Existence of default among Pareto-improving strategies (without a domination of one) is a possible source for strategic default caused by a moral hazard or adverse selection. A default with Pareto-efficient outcome does not allow to discriminate between strategic default and a default, caused by a bad luck.

This conclusion matches the sustainable empirical existence of positive delinquency rates on consumer credits, mentioned in the Introduction. Every consumer demands a credits being not aware of demands from others, and her/his decision is individually motivated. Every consumer chooses demand for credit individually as expects to be better off, being imperceptibly in competition with others for the credits.

If credit standards are loose (not enough credit rationing in terminology of Stiglitz and Weiss) this immediately leads to over-crediting, and to the Prisoner’s dilemma in terms of bigger debts and defaults from externalities.\(^{25}\)

Actually, the situation can be much worse, value of defaults from borrowers can exceed total lended money. We assumed that a demand for credit is limited from above by total money supply. If a punishment for default is not big, total debts can wipe out value of lended money, see Figure ??.

\(^{25}\)The discussion of a role of credit standards can be found in “Managing the leverage cycle,” of Geanakoplos (2011).
vertical line in the column $\mu = 1$. An increase the punishment from $\mu = 1$ to $\mu = 3$ diminishes demand for credits, and total value of debts does not exceed $M = 3$.

Our result differs from approach of Lucas and financial literature on the signaling role of price fluctuations. Both agents, perfectly informed about fundamentals of the economy, can not discriminate between reasons for non-stable price (or interest rate, wage, labor supply). Non-stability is caused either by asymmetric information, outside shocks, or/and oligopolistic trade.

Another side effect of the same question is: simulated market instabilities are market specific or not? The outcome of our model is that the reasons are not market specific. They come from indeterminacies of conjectures between players, operating at few markets simultaneously,\textsuperscript{26} that can not be expanded into market specific components.\textsuperscript{27} This was impossible to demonstrate with only two strategies and one market in LS18.

The important feature of the suggested interpretation of SW78 is a consistent general equilibrium model, which assembles production, unemployed resource, non-consumable money and default in fiat non-consumable money. These are features specific for static macroeconomic analysis, not microeconomic. In other words, the suggested interpretation is the oligopolistic general equilibrium model, which includes three markets: labor, good and credit.

\textsuperscript{26}Like in the colonel Blotto game.
\textsuperscript{27}The reason is that in we can expand a multidimension random variable into a product of probabilities of components only when there is independence between components: $\sigma_i(q_i, b_i, d_i) \neq \sigma_i^q(q_i)\sigma_i^b(b_i)\sigma_i^d(d_i)$.
Shubik and Wilson (1978) suggested the perfect competition to fill in the gap between micro and macroeconomics. Our paper expands their approach to the non-perfect competition in a simultaneous game.

7. Discussion

We construct a model with imperfect competition at three markets: labor, goods and credits. Following SW78 the model has fiat non-consumable money and default. We numerically demonstrate some special properties of the model, impossibility to construct an exact solution, and multiplicity of Pareto-efficient outcomes. Labor solution of a worker is described as a labor/leisure trade-off. Competition at goods market is similar to Bertrand competition. Money is injected into the economy through a competition mechanism too.

For numerically Pareto-improving cases, we demonstrate instability of prices, interest rate, volume of default, labor/leisure trade-off with these parameters, etc. Our result is that strategic actions of rational players are able to induce macroeconomic non-removable instabilities, without any information inflows or outside shocks. It happens only from non-coordinated indeterminacy and non-converging beliefs of agents about strategies of each other. Stiglitz (2018) recently wrote about non-converging beliefs as about an empirical fact: “There is ample evidence that individuals differ in their beliefs. ... they are not consistent with common knowledge, where everyone has the same beliefs.” Our paper demonstrated the same effect, but offers another mechanism in comparison to Guzman and Stiglitz (2016a).
The implication is that agents can not consider observed changes of price, interest rate, default value, unemployment etc. as a signal for new information, or as a market coordination device.

Some Pareto-efficient outcomes have default, what does not allow to discriminate strategic default and a default from bad luck.

Being different from DSGE model, our model satisfies the "four requirements for a new core model", suggested by Vives and Wills (2018) to improve DSGE. First, there are financial friction appearing from imperfect information at the credit market. Frictions in our model are different from search costs. Second, rational expectations approach can not be used here. Third, agents can be heterogenous in their endowments and payoffs. And finally, the model has rational individual micro-foundations.

Vives and Wills (2018) emphasize, that “the treatment of consumption needs to recognize finite horizons, liquidity constraints, ... a distribution of consumers, ...,” and further that “Keynes learned in the 30s, markets interact,”. This property is satisfied by the general equilibrium framework of the model. Our version of the result of Shubik and Wilson satisfies all these requirements.

To be able to predict a crisis from over-borrowing the model need to be multi-period. But already it predicts that Pareto-improving trade and production do not guarantee absence of default. Spill-over of default to financial sector is also a prediction of the model.

Hicks (1935) wrote that “we have to look frictions in the face, and see if they are really so refractory after all.” Recently, Blanchard (2017)
rephrased the question: “What are the distortions that are central to understanding short-run macroeconomic evolutions?” including distortions for finite horizons. Our partial response is that information frictions co-existing with that imperfect competition do matter for monetary economics, even in one period economy with rational agents and complete information. And these frictions do not seem to be removable. The origin of distortion is different from imperfect information of Lucas (1973), and staggering decisions of Fisher (1977) and Taylor (1980).

Another important result is that fluctuations at different markets, good, labor and money, are connected with each other, and can be separated only with a loss of information. Further research to follow.

REFERENCES


also Blanchard (2017) was cited as "suggests incorporating finite horizons, not necessarily coming from finite lives and incomplete bequests, but instead from bounded rationality or from myopia."


[40] LEVANDO, Dmitry, AND Maxim Sakharov, ”Impossibility of stable equilibrium price”, SSRN.


FIAT MONEY OLIGOPOLISTIC ECONOMY WITH LABOR/LEISURE TRADE-OFF AND EQUILIBRIUM DEFRAUDING


Figure 2. Non-convex and non-concave facet of $S_i$

Table 3. Comparison of total number of approximating number of strategies and Pareto-improving strategies for different discretization parameter $dim$ and punishment $\mu$

<table>
<thead>
<tr>
<th>$dim$</th>
<th>$\mu$</th>
<th>total points</th>
<th>Pareto improving</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1 (punishment)</td>
<td>3125</td>
<td>1936</td>
</tr>
<tr>
<td>5</td>
<td>3 (punishment)</td>
<td>3125</td>
<td>1495</td>
</tr>
<tr>
<td>6</td>
<td>1 (punishment)</td>
<td>7776</td>
<td>4677</td>
</tr>
<tr>
<td>6</td>
<td>3 (punishment)</td>
<td>7776</td>
<td>3525</td>
</tr>
</tbody>
</table>
Table 4. Frequency distributions of Pareto-improving sell $q_1$, buy $b_1$ and demand for money $d_1$ of player 1, $Q_1 = 10$, $Q_2 = 10$

<table>
<thead>
<tr>
<th></th>
<th>$q_1$</th>
<th>$b_1$</th>
<th>$d_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>dim = 5, $\mu = 1$</td>
<td><img src="chart1.png" alt="Histogram" /></td>
<td><img src="chart2.png" alt="Histogram" /></td>
<td><img src="chart3.png" alt="Histogram" /></td>
</tr>
<tr>
<td></td>
<td><img src="chart1.png" alt="Histogram" /></td>
<td><img src="chart2.png" alt="Histogram" /></td>
<td><img src="chart3.png" alt="Histogram" /></td>
</tr>
<tr>
<td>dim = 5, $\mu = 3$</td>
<td><img src="chart1.png" alt="Histogram" /></td>
<td><img src="chart2.png" alt="Histogram" /></td>
<td><img src="chart3.png" alt="Histogram" /></td>
</tr>
<tr>
<td></td>
<td><img src="chart1.png" alt="Histogram" /></td>
<td><img src="chart2.png" alt="Histogram" /></td>
<td><img src="chart3.png" alt="Histogram" /></td>
</tr>
<tr>
<td>dim = 6, $\mu = 1$</td>
<td><img src="chart1.png" alt="Histogram" /></td>
<td><img src="chart2.png" alt="Histogram" /></td>
<td><img src="chart3.png" alt="Histogram" /></td>
</tr>
<tr>
<td></td>
<td><img src="chart1.png" alt="Histogram" /></td>
<td><img src="chart2.png" alt="Histogram" /></td>
<td><img src="chart3.png" alt="Histogram" /></td>
</tr>
<tr>
<td>dim = 6, $\mu = 3$</td>
<td><img src="chart1.png" alt="Histogram" /></td>
<td><img src="chart2.png" alt="Histogram" /></td>
<td><img src="chart3.png" alt="Histogram" /></td>
</tr>
</tbody>
</table>


Table 5. Frequency distributions of prices, interest rate and default

<table>
<thead>
<tr>
<th>price 1</th>
<th>dim=5</th>
<th>( \mu = 1 )</th>
<th>dim=6</th>
<th>( \mu = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>price 2</th>
<th>dim=5</th>
<th>( \mu = 1 )</th>
<th>dim=6</th>
<th>( \mu = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
<td><img src="image7.png" alt="Graph" /></td>
<td><img src="image8.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>interest rate</th>
<th>( 1 + \rho = M / (d_1 + d_2) )</th>
<th>dim=5</th>
<th>( \mu = 1 )</th>
<th>dim=6</th>
<th>( \mu = 1 )</th>
<th>dim=5</th>
<th>( \mu = 3 )</th>
<th>dim=6</th>
<th>( \mu = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image9.png" alt="Graph" /></td>
<td><img src="image10.png" alt="Graph" /></td>
<td><img src="image11.png" alt="Graph" /></td>
<td><img src="image12.png" alt="Graph" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>total value of default</th>
<th>dim=5</th>
<th>( \mu = 1 )</th>
<th>dim=6</th>
<th>( \mu = 1 )</th>
<th>dim=5</th>
<th>( \mu = 3 )</th>
<th>dim=6</th>
<th>( \mu = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image13.png" alt="Graph" /></td>
<td><img src="image14.png" alt="Graph" /></td>
<td><img src="image15.png" alt="Graph" /></td>
<td><img src="image16.png" alt="Graph" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Comparison of a share of events of defaults among Pareto-efficient strategies

<table>
<thead>
<tr>
<th>Share of default in the set of Pareto-improving strategies</th>
<th>( dim = 5, \mu = 1 )</th>
<th>( dim = 6, \mu = 1 )</th>
<th>( dim = 5, \mu = 3 )</th>
<th>( dim = 6, \mu = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>59%</td>
<td>66%</td>
<td>32%</td>
<td>42%</td>
</tr>
</tbody>
</table>
FIAT MONEY OLIGOPOLISTIC ECONOMY WITH LABOR/LEISURE TRADE-OFF AND EQUILIBRIUM DEFAULT

Table 7. Unstable leisure vs price, interest rate ad default, dim = 6, μ = 3

<table>
<thead>
<tr>
<th>leisure $Q_1 - q_1$ vs interest rate</th>
<th>leisure $Q_1 - q_1$ vs consumable good price</th>
<th>leisure $Q_1 - q_1$ vs total default</th>
</tr>
</thead>
</table>

Table 8. Every player a continuum responses, for every fixed parameter, the set of responses is linear.