On Unifying Local and Global Competition: When Kaldor meets Chamberlin

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Preliminary version

1 Introduction

Two models of product differentiation compete for attention in the economics profession, that is, spatial (or localized) competition and monopolistic competition in which each firm competes symmetrically with all the others.

Consider the following demand system:

$$q_i = a - bp_i + c \sum_{neighbors} p_j + d \sum_{foreign} p_k, \quad b > c \geq d \geq 0, \qquad i = 1, ..., n. \tag{1}$$

To capture the idea that neighboring firms have a stronger impact on firm i than remote, or foreign, firms established in the rest of the world, we find it natural to assume that $b > c > d > 0$. In this way, each firm competes symmetrically with its direct competitors. It also competes symmetrically with its indirect competitors, but less fiercely because they supply poorer substitutes. We first describe the spatial setting that generates the demand functions (1).

The case when $c = d$ has a flavor of monopolistic competition à la Chamberlin (1933), where market competition is a "bellum omnium contra omnes". The case when $d = 0$ corresponds to the Kaldorian approach (Kaldor, 1935), which is based on the idea of local competition. Between these two extremes, there is a myriad of cases in which $c > d > 0$, so that local and global competition are unified.

2 The spatial setting

Consider a region formed two spatially separated districts – S and L – connected by a transport infrastructure. Each district has N spokes of length 1/2 each (like in Chen and Riordan, 2007), but the district S accommodates fewer firms than district L. Let n_r be the number of firms located in district $r \in \{S, L\}$ where $0 < n_S < n_L < N$.

2.1 Shopping behavior

Each district has a population of unit mass which is uniformly distributed over the N spokes with density $2/N$ (see the bold lines in Figure 1). There are two types of consumers: immobile and mobile. The mass $1-\alpha$ of immobile consumers, with $0 \leq \alpha \leq 1$, is the same across neighborhoods. Those consumers have prohibitively high travel costs to go shopping outside their district. Therefore, they focus on the varieties provided within their district. Mobile consumers have a travel cost $t \geq 0$ to go from the center of their district to the center of the other. Therefore, those consumers consider the city as a "global market" in which home and foreign varieties are imperfect substitutes because they have to bear the cost t to visit the remote district. A firm competes *locally* with the firms located in its district for the immobile consumers. However, it also competes *globally* with all firms for the mobile consumers. As a consequence, market demands faced by firms, hence equilibrium prices, depend not only on the total number of firms, but also on the distribution of firms between the two districts.

Each consumer has a most-preferred variety (her first-best) and a second most-preferred variety (her second-best). She dismisses the other varieties when she considers buying. More specifically, a consumer buys one unit of the differentiated product when her first- or second-best is available, but does not buy otherwise. We assume that consumers' first-best are given by the endpoints of the spokes they belongs to. Thus, a consumer has a strictly positive probability to buy her first-best in her district, but a zero probability to buy it from outside. By contrast, her second-best may belong to any district. Whether the second-best is available to the consumer depends on her mobility. If the consumer is immobile, any potential variety within her district can be her second-best with probability $1/(N-1)$. By contrast, a mobile consumer considers the whole range of potential varieties, so that the probability that a potential variety is her second-best is now given by $1/(2N-1)$. To purchase a variety, every consumer must go to the corresponding firm and incur a travel cost which is linear in distance. The full price the consumer pays is the firm price plus the travel cost to this firm.

Consumer's shopping behavior may be described as follows. (i) If a firm is located at the endpoint of a consumer's spoke, this variety is the consumer's first-best and her travel cost is equal to x. As for the second-best variety, three cases may arise. First, the consumer's second-best is available in her district, so that her travel cost equal to $1-x$. Second, the second-best is available in the other district, which means that that her travel cost is now $1-x+t$. In both cases, the consumer buys the variety with the lower full price. Third, the second-best variety is not available and the consumer buys her first-best.

(ii) Assume now that no firm is located at the endpoint of the consumer's spoke. In this case, the consumer buys her second-best. If this one is available in her district, she bears a travel cost equal to $1-x$. Otherwise, her travel cost is equal to $1-x+t$. When the second-best is not available either, the consumer does not buy at all because none of the available varieties corresponds to her first- or second-best.

2.2 Market demand

Let F_h be the set of active firms in district $h = S, L$. The demand faced by firm $i \in F_h$ is composed by several groups of consumers. First of all, we must distinguish between immobile and mobile consumers whose shopping behaviors differ.

2.2.1 The demand of immobile consumers

We first discuss the demand which stems from immobile consumers.

Group Im-1: There are

$$(1-\alpha) \cdot \frac{N-n_h}{N-1} \cdot \frac{1}{N}$$

immobile consumers residing in district h for whom the first-best is variety i, while their second-best is not available.

Group Im-2: Consider now the immobile consumers for whom i is their second-best, while their first-best is not available. The total mass of such consumers is given by

$$(1-\alpha) \cdot \frac{N-1}{N} \cdot \frac{N-n_h}{N-1} \cdot \frac{n_h}{N-1} \cdot \frac{1}{n_h} = (1-\alpha) \cdot \frac{N-n_h}{N-1} \cdot \frac{1}{N}.$$

In this expression, (i) $\frac{N-1}{N}$ is the probability for a consumer is located on a spoke $j \neq i$, so that i is not her first-best; (ii) $\frac{N-n_h}{N-1}$ is the probability that there is no firm at the endpoint of this spoke; (iii) $\frac{n_h}{N-1}$ is the probability that the consumer's second-best is available in h; and (iv) $\frac{1}{n_h}$ is the probability that i is her second-best.

Group Im-3: There are

$$(1-\alpha) \cdot \frac{1}{N} \cdot \frac{1}{N-1} \cdot \sum_{j \in F_h \setminus \{i\}} 2\widehat{x}_{ij}$$

immobile consumers for whom i is their first-best while their second-best $j \neq i$ is also available in district h.

In this expression, $\frac{1}{N}$ is the total mass of consumers located on spoke i; $\frac{1}{N-1}$ is the probability that $j \neq i$ is their second-best; and (iii) $2\widehat{x}_{ij}$ is the share of consumers who choose their fist-best i where $\widehat{x}_{ij}$ is the marginal consumer given by

$$\widehat{x}_{ij} \equiv \max\left\{0, \, \min\left\{\frac{1+p_j-p_i}{2}, \, \frac{1}{2}\right\}\right\} \tag{2}$$

Group Im-4: There are

$$(1-\alpha)\cdot\frac{1}{N}\cdot\frac{1}{N-1}\cdot\sum_{j\in F_h\backslash\{i\}}2\cdot(\frac{1}{2}-\widehat{x}_{ji})$$

immobile consumers for whom i is their second-best while their first-best is available.

In this expression, (i) $\frac{1}{N}$ is the total mass of consumers located on spoke j, (ii) $\frac{1}{N-1}$ is the probability that i is their second-best; and (iii) $2\cdot(\frac{1}{2}-\widehat{x}_{ji})$ is the fraction of consumers who choose their second-best i.

Therefore, the market demand of immobile consumers for variety i in district h is given by:

$$q_{ih}^{\text{im}}=(1-\alpha)\cdot\frac{1}{N-1}\cdot\frac{2}{N}\left(N-n_h+\sum_{j\in F_h\backslash\{i\}}\max\left\{0,\min\left\{\frac{1+p_j-p_i}{2},1\right\}\right\}\right).$$

Hence, when prices do not differ too much, the immobile consumer's demand is linear:

$$q_{ih}^{\text{im}}=(1-\alpha)\cdot\frac{1}{N-1}\cdot\frac{2}{N}\left(N-n_h+\sum_{j\in F_h\backslash\{i\}}\frac{1+p_j-p_i}{2}\right). \tag{3}$$

2.2.2 The demand of mobile consumers

We now come to decomposition of the demand of mobile consumers for variety i supplied in district h.

Group Mob-1: There are

$$\alpha\cdot\frac{2N-n_h-n_f}{2N-1}\cdot\frac{1}{N}$$

mobile consumers district h for whom i is their first-best while their second-best is not available at all.

Group Mob-2: There are

$$2\cdot\alpha\cdot\frac{2N-1}{2N}\cdot\frac{2N-n_h-n_f}{2N-1}\cdot\frac{n_h+n_f}{2N-1}\cdot\frac{1}{n_h+n_f}=\alpha\cdot\frac{2N-n_h-n_f}{2N-1}\cdot\frac{1}{N}$$

mobile consumers (no matter where they are located) for whom i is their second-best while their first-best is not available. Note that the total mass of consumers in the city is 2.

Group Mob-3: The mass of mobile consumers for whom i is their first-best while their second-best is available in the city is given by

$$\alpha\cdot\frac{1}{N}\cdot\frac{n_h+n_f-1}{2N-1}\cdot\left(\frac{n_h-1}{n_h+n_f-1}\cdot\frac{1}{n_h-1}\cdot\sum_{j\in F_h\backslash\{i\}}2\widehat{x}_{ij}+\frac{n_f}{n_h+n_f-1}\cdot\frac{1}{n_f}\cdot\sum_{k\in F_f}2\widetilde{x}_{ik}\right)$$

$$= \alpha \cdot \frac{2}{N} \cdot \frac{1}{2N-1} \cdot \left(\sum_{j \in F_h \setminus \{i\}} \widehat{x}_{ij} + \sum_{k \in F_f} \widetilde{x}_{ik} \right),$$

where $\widehat{x}_{ij}$ is the "within-district" marginal consumer given by (2), while $\widetilde{x}_{ik}$ is the "between-district" marginal consumer defined as follows:

$$\widetilde{x}_{ik} \equiv \max \left\{ 0, \, \min \left\{ \frac{1 + p_k + t - p_i}{2}, \, \frac{1}{2} \right\} \right\}.$$

In the above expression, (i) $\frac{n_h+n_f-1}{2N-1}$ is the probability that the second-best is available in the city; (ii) $\frac{n_h-1}{n_h+n_f-1}$ is the probability that the second-best is supplied in the same district as i; (iii) $\frac{n_f}{n_h+n_f-1}$ is the probability that the second-best is available in district $f \neq h$; (iv) $\frac{1}{n_h-1}$ is the probability that $j \in F_h \setminus \{i\}$ is the second-best; and (v) $\frac{1}{n_f}$ is the probability that $k \in F_f$ is the second-best.

Group Mob-4: The mass of mobile consumers for whom the first-best is available in the city while i is their second-best:

$$\alpha \cdot \frac{1}{N} \cdot \frac{1}{2N-1} \cdot \left[\sum_{j \in F_h \setminus \{i\}} 2 \cdot \left(\frac{1}{2} - \widehat{x}_{ji} \right) + \sum_{k \in F_f} 2 \cdot \left(\frac{1}{2} - \widetilde{x}_{ki} \right) \right]$$

$$= \alpha \cdot \frac{2}{N} \cdot \frac{1}{2N-1} \cdot \left[\sum_{j \in F_h \setminus \{i\}} \left(\frac{1}{2} - \widehat{x}_{ji} \right) + \sum_{k \in F_f} \left(\frac{1}{2} - \widetilde{x}_{ki} \right) \right].$$

Here, (i) $\frac{1}{N}$ is the mass of consumers on a given spoke ($j \in F_h \setminus \{i\}$ or $k \in F_f$); (ii) $\frac{1}{2N-1}$ is the probability that i is the second-best; (iii) $2 \cdot (\frac{1}{2} - \widehat{x}_{ji})$ is a fraction of consumers who choose i when j is available in the same district as i.

Therefore, the mobile consumers' market demand for variety i is given by:

$$q_{ih}^{\text{mob}} = \alpha \cdot \frac{2}{N} \cdot \frac{1}{2N-1} \cdot \left[2N - n_h - n_f + \sum_{j \in F_h \setminus \{i\}} \widehat{x}_{ij} + \sum_{j \in F_h \setminus \{i\}} \left(\frac{1}{2} - \widehat{x}_{ji} \right) + \sum_{k \in F_f} \widetilde{x}_{ik} + \sum_{k \in F_f} \left(\frac{1}{2} - \widetilde{x}_{ki} \right) \right]$$

$$\begin{aligned} = \;& \alpha \cdot \frac{2}{N} \cdot \frac{1}{2N-1} \cdot \left[2N - n_h - n_f + \sum_{j \in F_h \setminus \{i\}} \max \left\{ 0, \min \left\{ \frac{1 + p_j - p_i}{2}, 1 \right\} \right\} \right. \\ & \left. + \max \left\{ 0, \, \min \left\{ \frac{1 + p_k + t - p_i}{2}, \, \frac{1}{2} \right\} \right\} + \min \left\{ \frac{1}{2}, \, \max \left\{ \frac{p_k - t - p_i}{2}, \, 0 \right\} \right\} \right] \end{aligned}$$

2.2.3 Market demand

In equilibrium, we expect all firms located in the same district to charge the same price: $p_i = p_L$ for all $i \in F_L$ and $p_k = p_S$ for all $k \in F_S$. Furthermore, we also expect that $p_S > p_L$ to hold in equilibrium because the district L involves more firms than the district S. Given that these conjectures are correct, three cases may arise.

Case 1: Assume that prices are very close to each other, such that

$$0 < p_k - p_i < t \quad \text{for all } i \in F_L \text{ and } k \in F_S.$$

In this case mobile consumers who really choose between a purchase in two districts prefer to buy in their home district since the possible gain from price difference is smaller than transport costs. The only flow of consumers who actually go is formed by those mobile consumers with the only second-best variaty available in other district. There are no between-district price competition. The market demand for variety i in each discrict is given by

$$q_{ih} = a_h - b_h p_i + c \sum_{j \in F_h \setminus \{i\}} p_j,$$

where $h,\ f \in \{L, S\}$, and $h \neq f$, while the coefficients $a_h(t)$, b_h, c and d are defined as follows:

$$a_L \equiv \frac{1-\alpha}{N} \cdot \frac{2N - n_L - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{4N - n_L - n_S - 1}{2N-1}, \tag{4}$$

$$a_S \equiv \frac{1-\alpha}{N} \cdot \frac{2N - n_S - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{4N - n_L - n_S - 1}{2N-1}, \tag{5}$$

$$b_L \equiv \left(\frac{1-\alpha}{N} \cdot \frac{1}{N-1} + \frac{\alpha}{N} \cdot \frac{1}{2N-1}\right)(n_L - 1), \qquad b_S \equiv \left(\frac{1-\alpha}{N} \cdot \frac{1}{N-1} + \frac{\alpha}{N} \cdot \frac{1}{2N-1}\right)(n_S - 1), \tag{6}$$

$$c \equiv \frac{1-\alpha}{N} \cdot \frac{1}{N-1} + \frac{\alpha}{N} \cdot \frac{1}{2N-1}, \qquad d \equiv 0. \tag{7}$$

Not surprisingly, the demand does not depend on the value of t. We observe a'la pure Kaldorian market.

Case 2: Assume now that prices only slightly differ within each district, while t is small enough for the prices p_k in district S to slightly exceed the prices p_i in district L. In this case, the following inequality holds:

$$0 < p_k - p_i - t < 1 \quad \text{for all } i \in F_L \text{ and } k \in F_S. \tag{8}$$

Proposition 1. *Assume that* (8) *holds. The market demand for variety $i \in F_L$ in a neighborhood of the equilibrium is linear and given by*

$$q_{iL} = \frac{1-\alpha}{N} \cdot \frac{2N - n_L - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{4N - n_L - n_S - 1}{2N-1} - \frac{\alpha}{N} \cdot \frac{n_S}{2N-1} \cdot t$$

$$- \left(\frac{1-\alpha}{N} \cdot \frac{n_L - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{n_L + n_S - 1}{2N-1} \right) p_i$$

$$+ \left(\frac{1-\alpha}{N} \cdot \frac{1}{N-1} + \frac{\alpha}{N} \cdot \frac{1}{2N-1} \right) \sum_{j \in F_L \setminus \{i\}} p_j$$

$$+ \frac{\alpha}{N} \cdot \frac{1}{2N-1} \cdot \sum_{k \in F_S} p_k. \tag{9}$$

Furthermore, the market demand for variety $i \in F_S$ is given by

$$q_{iS} = \frac{1-\alpha}{N} \cdot \frac{2N - n_S - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{4N - n_L - n_S - 1}{2N-1} + \frac{\alpha}{N} \cdot \frac{n_L}{2N-1} \cdot t$$

$$- \left(\frac{1-\alpha}{N} \cdot \frac{n_S - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{n_S + n_L - 1}{2N-1} \right) p_i$$

$$+ \left(\frac{1-\alpha}{N} \cdot \frac{1}{N-1} + \frac{\alpha}{N} \cdot \frac{1}{2N-1} \right) \cdot \sum_{j \in F_S \setminus \{i\}} p_j$$

$$+ \frac{\alpha}{N} \cdot \frac{1}{2N-1} \cdot \sum_{k \in F_L} p_k. \tag{10}$$

Proof. The total market demand $q_{ih} \equiv q_{ih}^{\text{im}} + q_{ih}^{\text{mob}}$ for variety i supplied in district h is given by

$$q_{ih} = (1-\alpha) \cdot \frac{1}{N-1} \cdot \frac{2}{N} \left(N - n_h + \sum_{j \in F_h \setminus \{i\}} \max \left\{ 0, \min \left\{ \frac{1 + p_j - p_i}{2}, 1 \right\} \right\} \right) + \tag{11}$$

$$\alpha \cdot \frac{2}{N} \cdot \frac{1}{2N-1} \cdot \left(2N - n_h - n_f + \sum_{j \in F_h \setminus \{i\}} \max \left\{ 0, \min \left\{ \frac{1 + p_j - p_i}{2}, 1 \right\} \right\} \right) +$$

$$+ \alpha \cdot \frac{2}{N} \cdot \frac{1}{2N-1} \cdot \sum_{k \in F_f} \max \left\{ 0,\, \min \left\{ \frac{1 + p_k + t - p_i}{2},\, \frac{1}{2} \right\} \right\} +$$

$$+\alpha \cdot \frac{2}{N} \cdot \frac{1}{2N-1} \cdot \sum_{k \in F_f} \min \left\{ \frac{1}{2}, \max \left\{ \frac{p_k - t - p_i}{2}, 0 \right\} \right\}$$

Here h, $f \in \{L, S\}$, and $h \neq f$. Combining (11) with assumptions (i) and (ii) yields (9) – (10). See Appendix for details. □

Two properties of the demand system (9) – (10) are worth discussing. First, this demand system is linear and has the same structure as (1). Hence, our setup rationalizes the market demand which combines Chamberlinian and Kaldorian features. Indeed, the demand system may be represented as follows:

$$q_{ih} = a_h(t) - b_h p_i + c \sum_{j \in F_h \setminus \{i\}} p_j + d \sum_{k \in F_f} p_j, \tag{12}$$

where h, $f \in \{L, S\}$, and $h \neq f$, while the coefficients $a_h(t)$, b_h, c and d are defined as follows:

$$a_L(t) \equiv \frac{1-\alpha}{N} \cdot \frac{2N - n_L - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{4N - n_L - n_S - 1}{2N-1} - \frac{\alpha}{N} \cdot \frac{n_S}{2N-1} \cdot t, \tag{13}$$

$$a_S(t) \equiv \frac{1-\alpha}{N} \cdot \frac{2N - n_S - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{4N - n_L - n_S - 1}{2N-1} + \frac{\alpha}{N} \cdot \frac{n_L}{2N-1} \cdot t, \tag{14}$$

$$b_L \equiv \frac{1-\alpha}{N} \cdot \frac{n_L - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{n_L + n_S - 1}{2N-1}, \qquad b_S \equiv \frac{1-\alpha}{N} \cdot \frac{n_S - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{n_S + n_L - 1}{2N-1}, \tag{15}$$

$$c \equiv \frac{1-\alpha}{N} \cdot \frac{1}{N-1} + \frac{\alpha}{N} \cdot \frac{1}{2N-1}, \qquad d \equiv \frac{\alpha}{N} \cdot \frac{1}{2N-1}. \tag{16}$$

Observe that $a_L(t) < a_S(t)$ while $b_L > b_S > c > d > 0$, which makes (12) fully consistent with (1). Note that α must be smaller than 1 for $c > d$ to hold, that is, for the demand system (12) to feature both Chamberlinian and Kaldorian characteristics. When $\alpha = 1$, consumers weight equally domestic and foreign varieties.

Second, a drop in transport cost t between the two districts shifts firms' demands upward in the larger district and downward in the small one because S provides less diversity than L.

Case 3: The last case corresponds to the possible market configuration, when the price gap between two districts is extremely large, such that

$$p_k - p_i > 1 + t \quad \text{for all } i \in F_L \text{ and } k \in F_S.$$

In this case again there is again no competition for mobile consumers: everyone who has a choice goes to the cheapest district, presumably, it is L. What is observed is again "pure"

Kaldorian competition within firm in the same district, but the demand is reorganized in favor of the large district firms. It looks as follows

$$q_{ih} = a_h - b_h p_i + c \sum_{j \in F_h \setminus \{i\}} p_j,$$

where h, $f \in \{L, S\}$, and $h \neq f$, while the coefficients $a_h(t)$, b_h, c and d are defined as follows:

$$a_L \equiv \frac{1-\alpha}{N} \cdot \frac{2N - n_L - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{4N - n_L - 1}{2N-1}, \tag{17}$$

$$a_S \equiv \frac{1-\alpha}{N} \cdot \frac{2N - n_S - 1}{N-1} + \frac{\alpha}{N} \cdot \frac{4N - 2n_L - n_S - 1}{2N-1}, \tag{18}$$

$$b_L \equiv \left(\frac{1-\alpha}{N} \cdot \frac{1}{N-1} + \frac{\alpha}{N} \cdot \frac{1}{2N-1} \right)(n_L - 1), \qquad b_S \equiv \left(\frac{1-\alpha}{N} \cdot \frac{1}{N-1} + \frac{\alpha}{N} \cdot \frac{1}{2N-1} \right)(n_S - 1), \tag{19}$$

$$c \equiv \frac{1-\alpha}{N} \cdot \frac{1}{N-1} + \frac{\alpha}{N} \cdot \frac{1}{2N-1}, \qquad d \equiv 0. \tag{20}$$

3 The price equilibrium

Deriving from FOC, it is easy to obtain in term of demand coefficients a_h, b_h, c, and d that equilibrium prices in large and small districts are

$$p_S^* = \frac{a_S(t)b_L + d(a_S(t)n_S + a_L(t)n_L)}{b_S b_L + d(b_S n_S + b_L n_L)},$$

$$p_L^* = \frac{a_L(t)b_S + d(a_S(t)n_S + a_L(t)n_L)}{b_S b_L + d(b_S n_S + b_L n_L)}.$$

Since $a_S > a_L$, $b_L > b_S$, we have $p_S^* > p_L^*$. This result reflects the fact that there is more competition in L than in S.

Furthermore,

$$p_S^* - p_L^* = \frac{1}{1 + d\left(\frac{n_S}{b_L} + \frac{n_L}{b_s}\right)} \left(\frac{a_S}{b_S} - \frac{a_L}{b_L} \right) > 0$$

so that the choke price a_S/b_S of a S-firm is higher than that of a L-firm, given by a_L/b_L.

Note that even when all cosumers are mobile ($\alpha = 1$), prices do not converge to the same value:

$$p_S^* - p_L^*|_{\alpha=1} = \frac{n_L - n_S}{2n_L + 2n_S + 1} \cdot t > 0.$$

Note that when $\alpha = 1$, prices do not converge to the same value:

$$p_S^* - p_L^*|_{\alpha=1} = \frac{n_L - n_S}{2n_L + 2n_S + 1} \cdot t > 0.$$

In cases 1 and 3 equilibrium prices have a more simple view:

$$p_h^* = \frac{a_h}{b_h},$$

where $h = L, H$, and coefficients a_h, b_h are determined above.

It is easy to derive that for the case 1 equilibrium prices are given by

$$p_S^* = \frac{2}{Nc(n_S - 1)} - \frac{\alpha}{Nc(2N-1)} \cdot \frac{n_L}{n_S - 1} - 1,$$

$$p_L^* = \frac{2}{Nc(n_L - 1)} - \frac{\alpha}{Nc(2N-1)} \cdot \frac{n_S}{n_L - 1} - 1.$$

Since this case corresponds to the inequality $0 < p_S^* - p_L^* < t$, this happens if t is large enough, such that

$$t > t_{\text{right}} = \frac{2(n_L - n_S)}{Nc(n_S - 1)(n_L - 1)} - \frac{\alpha}{Nc(2N-1)} \cdot \frac{n_L}{n_S - 1} + \frac{\alpha}{Nc(2N-1)} \cdot \frac{n_S}{n_L - 1}.$$

In the case 3 equilibrium prices are more distant from each other

$$p_S^* = \frac{2}{Nc(n_S - 1)} - \frac{\alpha}{Nc(2N-1)} \cdot \frac{2n_L}{n_S - 1} - 1,$$

$$p_L^* = \frac{2}{Nc(n_L - 1)} - 1.$$

This holds for

$$t < t_{\text{left}} = \frac{2(n_L - n_S)}{Nc(n_S - 1)(n_L - 1)} - \frac{\alpha}{Nc(2N-1)} \cdot \frac{2n_L}{n_S - 1} - 1.$$

Note that $t_{\text{left}} < t_{\text{right}}$ for all values of α, n_S, n_L, so that the whole range of possible values of transport costs splits into several parts corresponding to cases 1, 2, and 3.

4 Comparative statics

Consider first the impact of an infrastructural improvement (i.e. a reduction in t) on equilibrium prices.

Proposition 2. *Decreasing transport costs between the two districts lead to lower prices in the smaller district and higher prices in the larger district.*

Proof. Differentiating the equilibrium prices p_S^* and p_L^* in t, we obtain:

$$\frac{\partial p_S^*}{\partial t} = \frac{(b_L + dn_S)\,(a_S)'\,(t) + dn_L\,(a_L)'\,(t)}{b_S b_L + d(b_S n_S + b_L n_L)},$$

$$\frac{\partial p_L^*}{\partial t} = \frac{(b_S + dn_L)\,(a_L)'\,(t) + dn_S\,(a_S)'\,(t)}{b_S b_L + d(b_S n_S + b_L n_L)}.$$

Using (13) – (14), we get

$$(a_L)'\,(t) = -\frac{\alpha}{N} \cdot \frac{n_S}{2N-1}, \qquad (a_S)'\,(t) = \frac{\alpha}{N} \cdot \frac{n_L}{2N-1}.$$

Plugging these expressions into $\frac{\partial p_S^*}{\partial t}$ and $\frac{\partial p_L^*}{\partial t}$ yields after simplifications:

$$\text{sign}\left(\frac{\partial p_S^*}{\partial t}\right) = \text{sign}(b_L n_L) > 0,$$

$$\text{sign}\left(\frac{\partial p_L^*}{\partial t}\right) = \text{sign}\left(-b_S n_S\right) < 0.$$

This completes the proof.□

Since more consumers visit their foreign district, decreasing t leads to a market expansion effect. Likewise, lowering t intensifies competition in the two districts. What the above proposition says is that the former effect dominates the latter in the larger district whereas the opposite holds in the small district. Moreover, when t falls the price gap shrinks but never vanishes when t goes to 0 so long $\alpha > 0$. This is because each firm has a captive market formed by the loyal immobile consumers located on its spoke. Evidently, when $\alpha = 1$ the price gap disappears when $t = 0$ but still exists when $t > 0$.

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