# Dynamical Topology of Highly Aggregated Input-Output Networks

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The dominating approach in economical modeling of systems of interacting economic agents such as, e.g., a multi-sector macroeconomy, is still based on the notions of (single) representative producer and consumer. The validity of this concept is backed by the two main arguments: that of diversification [18] and irrelevance of a firm's (sector's) position in a supply chain (Hulten theorem) [16]. The diversification argument is based on the fact that for a system consisting of a large number of similar independent sectors idiosyncratic shocks are irrelevant due to the central limit theorem. From the Hulten theorem there follows that a response of aggregate total factor productivity (TFP) to the idiosyncratic shock to a sector is completely determined by its relative sales volume and, therefore, its functional role in the supply chain is also irrelevant.

These arguments were however questioned in a number of recent studies. In the seminal paper [13] it was shown that for fat-tailed distributions of firm size the diversification argument does not work. In another important study [1] the role of topology of input-output network in the multi-sector macroeconomic model was analyzed. In the analysis of [1] vertices of the input-output network represent sectors (or firms) and edges correspond to certain buyer-seller relationships between the sectors. It was shown that topological characteristics of the input-output network such as the degree distribution and Katz (or, in some cases, PageRank) centrality determine relative sizes of sales in the competitive equilibrium of the multi-sector macroeconomy under consideration. Therefore, even if Hulten's Theorem does hold, the topology of the network describing relationships between suppliers and customers does play an important role. Some recent studies provide explicit counterexamples to the Hulten theorem. First, it was shown that the theorem does not hold when frictions (e.g. firms entry/exit) are present [3]. Second, in the paper

[4] it was demonstrated that higher-order terms in log-linearization neglected in derivation of the Hulten Theorem do invalidate its results. Thus, investigation of the topological properties of real input-output matrices reflecting existing relationships between industries is of great interest.

The input-output network is a weighted oriented graph characterized by the (weighted) adjacency matrix $Z = \{z_{ij}\}_{i,j=1,...,N,}$ , where $z_{ij}$ is the cost of the products (services) produced by the sector $i$ that are bought by the sector $j$. In other words, the column $j$ of the matrix represents the composition of goods from different sectors used by the sector $j$ to produce its own output. Thus, the orientation of an arrow in the graph corresponds to the direction of the material flow.

The main goal of the paper is to investigate which properties of highly aggregated input-output networks are stable over time and which undergo temporal evolution. The Russian and US input-output networks were analyzed. For the best of the authors' knowledge, the only study of dynamic properties of the Russian input-output network was undertaken in [9], where evolution of its matrix elements was studied by gauging it against a linear regression model. Therefore the analysis of [9] is inherently local and does not address issues related to the nonlocal topological properties of the input-output network. Moreover, for technical reasons the periods of rapid changes (such as the crisis period of 1997-1999 in Russia) had to be excluded from the analysis of [9], whereas the evolution during these periods that network undergoes reflects extremely significant economic transformations that definitely have to be taken into account in the studies of input-output network dynamics. In contrast, the analysis of aggregative network characteristics performed in this paper covers both stable and crisis periods. For the US input-output network there exists considerable literature studying its properties [5, 1, 6][1]. Let us note that in the existing dynamical multi-sector macroeconomic models, see e.g.[17, 14, 8, 15, 5, 11], the input-output matrix remains constant. This property follows from the fact that its matrix elements are determined by the coefficients of production functions (namely, by elasticities of the sectoral output with respect to intermediate goods from other sectors) which are constant in time.

In this study National Input-Output Tables from the World Input-Output Database [21] are used. These tables contain information on 34 sectors[2] and

[1]Other strands of literature related to the current study include the papers analyzing Japanese networks at the firm level [19, 12, 20], sectoral European betworks [2] and the sectoral network of Greece [22]. Some dynamic properties of input-output networks were studied in [7, 10]

[2]This tables provide information on 35 sectors, but, to facilitate comparison between the Russian and US networks, the sector "Private Households with Employed Persons"

cover the period from 1995 to 2011 on an annual basis. In addition to the input-output tables this database provides, within the same classification, other statistical data (e.g. total sectoral output, total final consumption of households, etc.).

The main findings of the paper are closely related to the properties of edge weight distribution of input-output networks, their clustering pattern and identification of the key sectors with respect to different centrality measures and can be formulated as follows:

- The edge weight distributions of the networks are fat-tailed, but the corresponding slopes are evolving in time.
- The clustering properties of the networks are time-dependent. At the same time, we found some approximate stability related to triangle formation reflected in the constancy of the weight fraction of triangles of different types existing in a weighted directed graph.
- Another stable property of the input-output networks is a relatively high rank autocorrelation for centrality vectors. In particular, analysis of PageRank, Hubs and Authorities centralities shows that the Kendall rank correlation, even at relatively large time lags, is about 0.7. However, there exists some evolution in the composition of leading sectors and the network can only be adequately described as a dynamic object.
- The dynamic evolution of input-output networks is also illustrated by studying the evolution of $L^1$ distances between the matrices corresponding to different years.

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