Round-Robin Tournaments with Limited Resources

Dmitry Dagaev* and Andrey Zubanov†

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We study economic foundations behind players' behavior in a round-robin contest environment. This type of contests is beleived to be one of two main contest types used in sports (alongside with elimination contests) and some decision-making processes. Round-robin structure of the contest means that participants are sequentially playing each other, and all prizes are distributed according to the general ranking at the end. The latter assumption distinguishes a tournament from a sequence of independent games and brings out specific participants' behavior patterns. To the best of our knowledge, most of the literature on agents' behavior in such kind of contests (except for [4, 5]) ignores the nature of the round-robin tournament. Making the assumption that each participant plays each other is not enough: it is critical that in a real round-robin, participants are maximizing the expected payoff after the completion of the tournament rather than the sum of payoffs in the separate games.

We consider a general model of a round-robin tournament with three participants and perfect information. Following the literature on contest theory, we take participants' efforts as their resources and thus teams decide on how to spend them. Then, we consider several economic concepts which have the major effect on the distribution of efforts in round-robin tournaments: the type of competition, the cost of resources, the resource constraints, and payoffs structure.

In contest theory, a contest is a strategic interaction between $N \geq 2$ players (teams, employees etc.) wherein each player i chooses the non-negative efforts level e_i, and player i's probability to finish in k-th place $p_k^i = p_k^i(e_1, \ldots, e_N)$ depends on the efforts exerted by all of the competitors, $i, k = 1, \ldots, N$. The efforts are costly, the cost functions $c_i(e_i), i = 1, \ldots, N$, are given. Let v_k be the prize for finishing in k-th place, $k = 1, \ldots, N$. Then, each player i maximizes his expected utility $u_i(e_1, \ldots, e_N) = \sum\limits_{k=1}^{N} p_k^i(e_1, \ldots, e_N)v_k - c_i(e_i)$ with respect to e_i. Basic contest models are single shot games which differ in the functional form of winning probabilities p_k^i and cost functions $c(e_i)$, as well as in presence of random shocks which affect player's actual performance by adding to chosen efforts level. The prize structure is a matter of policy – contest organizers seek to maximize overall efforts level by choosing optimal prize distribution.

One of the first and simplest specifications was proposed by Tullock [9] who considered a 2-player game with $p_1^1(e_1, e_2) = \frac{e_1^r}{e_1^r + e_2^r}$, $r > 0$ (the model is now called a Tullock contest). First-price sealed-bid auction is another

*National Research University Higher School of Economics. E-mail: ddagaev@gmail.com
†Department of Economics, University of Wisconsin–Madison. E-mail: zubanov@wisc.edu

important case, it is defined by winning probability $p_1^1(e_1, e_2)$, where $p_1^1(e_1, e_2) = 1$ if $e_1 > e_2$ and $p_1^1(e_1, e_2) = 0$ if $e_1 < e_2$.

Some other models describe repeated interactions of two players. Konrad and Kovenock [2] propose a (n, m)-contest consisting of repeated battles of two players where first player needs to win n battles to claim the overall victory, and the second player – m battles. There are both prizes for winning the whole contest and winning a single battle. The winner of the battle is determined in an all-pay first-price auction. The authors describe a subgame-perfect equilibrium in mixed strategies. Krumer [3] considers a (2,2)-contest in connection to 'best-of-three' series which are used in various sport tournaments. He separates home and away matches – each team has larger valuation for winning a match at home and pays additional fixed costs for playing away. A stronger team plays 2 matches at home, a weaker team hosts only 1 match (a common practice in sport tournaments is that a team that finishes at a higher place at the previous stage is rewarded by the possibility to play more matches at home at the subsequent stage). The author finds the equilibrium order of games that neither of teams would like to change.

Two special types of sequential contests which are widely exploited in sports are round-robin and knock-out (elimination) tournaments. In round-robin tournaments each participant plays against each other once. Results are aggregated after all matches are finished. Krumer, Megdish, and Sela [4] study round-robin tournaments with 3 and 4 participants. Each game is an all-pay first-price auction. Each player maximizes probability of winning the tournament. The authors show that in case of 3 players a player who plays in the first and in the third round gets the maximal payoffs as well as has the highest probability to win the tournament. In tournament with four participants with sequential games in each of three rounds a player who plays in the first game in each of the first and second round has an advantage. Thus, after proving the first-mover advantage, the authors questioned the fairness of such kind of tournaments.

In standard contest game formulation, efforts are costly but, theoretically, not limited. A special feature that arises in multiple contests is that participants may face the problem of distributing fixed amount of efforts between several games or competitions. This question is closely connected to classical Colonel Blotto game [7], in which each of two opposing colonels must distribute their soldiers between several battlefields. Each of the battlefields is won by the side with more soldiers on this particular battlefield. A player who wins more battlefields, wins the game. Kvasov [6] suggests a contest with limited resources which is similar to Colonel Blotto game due to the budget constraint. There are several objects of equal value for both players. These objects are for sale through the all-pay first-price sealed-bid contest. Each of the players has fixed budget. Both players decide independently on distribution of their bids trying to maximize the expected utility. Contrary to Colonel Blotto game, a player pays his bid irrespectively of the auction outcome. The author shows that there are no equilibria in pure strategies and provides all equilibria in mixed strategies. Erez and Sela [1] investigate a competition with limited and costless efforts. The players play each other; each game is a Tullock contest for the prize valued at 1. The authors find equilibrium in pure strategies as well as efforts-maximizing distribution of prizes between the rounds.

The paper is organized as summarized in Table 1. First, a general model of round-robin tournament with limited effort budgets is defined. Thereafter we suggest two important specifications of the model. Section 3 contains first-price sealed-bid auction specification. For the corresponding game a pure strategy subgame-perfect

Contest type / Efforts	Costless	Costless	Costly	Costly
	Not limited	Limited	Not limited	Limited
Tullock lottery	Trivial case	Chapter 4	Chapter 5	Similar case to
First-price auction	(no constraints)	Chapter 3	Krumer et al (2015)	Costly, Not limited

Table 1: Summary

Nash equilibrium is described. Tullock lottery specification of the model is solved numerically in Sections 4 and 5.

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