Social interactions and market outcome.

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Very preliminary, do not circulate

1 Closed economy

1.1 Demand

Let $\mathcal{N}$ and $\mathcal{H}$ be the sets of varieties and individuals while N and M are the respective masses of varieties and individuals. Individual $h \in \mathcal{H}$ consumes a quantity z_h of homogeneous good and the quantities x_{ih}, $i \in \mathcal{N}$, of differentiated varieties. Its preferences are given by the utility function

$$U_h = z_h + \alpha_0 \int_{\mathcal{N}} \theta_{ih} x_{ih} \mathrm{d}i - \frac{\beta}{2} \int_{\mathcal{N}} x_{ih}^2 \mathrm{d}i - \frac{\gamma}{2N} \int_{\mathcal{N}\times\mathcal{N}} x_{ih} x_{jh} \mathrm{d}i \mathrm{d}j - \frac{\delta_h}{2M} \int_{\mathcal{H}\times\mathcal{N}} (x_{ih} - x_{il})^2 \mathrm{d}l \mathrm{d}i \qquad (1)$$

where α_0 is a common demand shifter while θ_{ih} is a consumer h's taste shifter for variety i, or equivalently, relative willingness to pay for variety i or relative quality of variety i as perceived by consumer h. We further define $\theta_i = \frac{1}{M} \int_{\mathcal{H}} \theta_{ih} \mathrm{d}h$ to be the average of the parameter θ_{ih} over individuals and $\theta = \frac{1}{NM} \int_{\mathcal{N}\times\mathcal{H}} \theta_{ih} \mathrm{d}i \mathrm{d}h$. The last term in (1) captures the *conformism incentives* or *communication effect*. Individual h suffers a discrepancy between her consumption pattern $\mathbf{x}_h$ and the consumption pattern $(\mathbf{x}_h)_{h\in H}$ of the whole population of consumers. The coefficient $\delta_h > 0$ may be viewed as a measure of *conformism* or *communication intensity*. Without loss of generality, we rank individuals in reverse order of their conformism level: $\delta_h < \delta_{h'} \iff h > h'$.

Individual h maximizes its utility under the budget constraint $z_h + \int_{\mathcal{N}} p_i x_{ih} \mathrm{d}i = w_h$ where w_h is its income and the price of the homogeneous good is normalized to one. The first order condition yields the individual inverse demands of consumer h

$$p_i = \alpha_0 \theta_{ih} - (\beta + \delta_h) x_{ih} - \frac{\gamma}{N} \int_{\mathcal{N}} x_{jh} \mathrm{d}j + \frac{\delta_h}{M} \int_{\mathcal{H}} x_{il} \mathrm{d}l. \qquad (2)$$

We show in Appendix A that $\beta + \delta_h > 0$ is necessary and sufficient for the first order condition holds.

We assume that all individuals consume all goods: $x_{ih} > 0 \ \forall i \in N, h \in \mathcal{H}$. Then, we show in Appendix A that the total demand for variety i is $X_i = M x_i$, where $x_i \equiv \frac{1}{M} \int_H x_{ih} dh$ is the average per capita demand for this variety given by

$$x_i = \alpha_0 a_i - \frac{p_i}{\beta} + \frac{\gamma p}{\beta(\beta+\gamma)}$$

where

$$p = \frac{1}{N}\int_{\mathcal{N}} p_j \mathrm{d}j$$

is the average price over varieties. The demand intercept depends on

$$a_i = \frac{1}{\beta}\left[\frac{\beta}{\beta+\gamma}\theta + (\theta_i - \theta) + \mathrm{cov}_h(\theta_{ih},\, \omega_h^0) - \mathrm{cov}_{jh}(\theta_{jh},\, \omega_h^2)\right], \tag{3}$$

where the weights ω_h^0 and ω_h^1 are defined as follows:

$$\omega_h^0 = \frac{\frac{1}{\beta+\delta_h}}{\frac{1}{M}\int_{\mathcal{H}}\frac{1}{\beta+\delta_l}\mathrm{d}l}, \quad \text{and} \quad \omega_h^1 = \frac{\frac{1}{\beta+\gamma+\delta_h}}{\frac{1}{M}\int_{\mathcal{H}}\frac{1}{\beta+\gamma+\delta_l}\mathrm{d}l}$$

while

$$\omega_h^2 = \omega_h^0 - \frac{\beta}{\beta+\gamma}\omega_h^1$$

Those weights capture the degree of non-conformism or social independence. Note that mean of the weights $\frac{1}{M}\int_{\mathcal{H}}\omega_h^0 \mathrm{d}h$ and $\frac{1}{M}\int_{\mathcal{H}}\omega_h^1 \mathrm{d}h$ are equal to one and that ω_h^2 is positive.

When all varieties and consumers are symmetric ($\theta_i = 1,\ \forall i \in \mathcal{N},\ \delta_h = \delta_l,\ h,\, l \in \mathcal{H}$), the three last terms in (3) vanish and $a_i = 1$. In this case the demand intercept is constant as in Ottaviano et al. (2002) and Melitz and Ottaviano (2008). When consumers are symmetric ($\theta_{ih} = \theta_{il} = \theta_i$ and $\delta_h = \delta_l$, $h,\, l \in \mathcal{H}$) but have different demands for the varieties ($\theta_{ih} \neq \theta_{jh}$), the two last terms in (3) are zero. A rise in the average taste parameter θ_i of variety i leads to a parallel shift of the demand curve as in Picard and Okubo (2012).

This paper stresses the role of conformism and communication. The demand for a variety i depends on the interaction between the individual's willingness to pay and conformism levels for the purchased variety i

$$\mathrm{cov}_h(\theta_{ih},\, \omega_h^0) = \frac{1}{M}\int_{\mathcal{H}}(\theta_{ih} - \theta_i)\left(\omega_h^0 - 1\right)\mathrm{d}h$$

and for all varieties

$$\mathrm{cov}_{jh}(\theta_{jh},\, \omega_h^2) = \frac{1}{NM}\int_{\mathcal{N}\times\mathcal{H}}(\theta_{jh} - \theta)\left(\omega_h^2 - 1\right)\mathrm{d}h\mathrm{d}j$$

Those co-variances are related to the spread of the taste distributions. Consider a taste distribution θ_{ih} that is an increasing function of h and a mean preserving spread of the taste distribution of variety i in the individual space $\mathcal{H}$ from θ_{ih} to $\theta_{ih} + \varepsilon_{ih}$ where $\varepsilon_{ih} \geq 0$ for $\theta_{ih} \geq \theta_i$ and $\varepsilon_{ih} < 0$ otherwise. One has $\int_{\mathcal{H}}\varepsilon_{ih}\mathrm{d}h = 0$ while we have $\int_0^h \varepsilon_{il}\mathrm{d}l \leq 0$. The mean preserving spread raises $\mathrm{cov}_h(\theta_{ih}, \omega_h^0)$ if and only if

$\text{cov}_h(\varepsilon_{ih},\, \omega_h^0) \geq 0$. Integrating by part, one gets

$$\text{cov}_h(\varepsilon_{ih},\, \omega_h^0) = \frac{1}{M}\int_H \left[\int_0^h \varepsilon_{il}\text{d}l\right] \omega_h^0 \text{d}h \geq 0$$

Therefore, a mean preserving spread of the taste distribution of good i raises $\text{cov}_h(\theta_{ih},\, \omega_h^0)$. That is, if higher valuation consumers have lower conformism. Finally, $\text{cov}_{jh}(\theta_{jh},\, \omega_h^2)$ is simply the average of those effects over all varieties. The opposite property holds when the taste distribution θ_{ih} is a decreasing function of h.

1.2 Supply

For the sake of clarity, we put more structure between taste and conformism. In particular, we assume that $\theta_{ih} \equiv \mu_h \sigma_i + \vartheta_h$ where σ_i is the "quality" of good i ,μ_h is the consumer h's (idiosyncratic) taste for quality and ϑ_h is consumer h's independent (idiosyncratic) taste for goods (fixed effect). Therefore $\theta_i \equiv \mu\sigma_i + \vartheta$, where $\mu = \frac{1}{M}\int_{\mathcal{H}} \mu_h \text{d}h > 0$, and $\vartheta = \frac{1}{M}\int_{\mathcal{H}} \vartheta_h \text{d}h$. Then $\text{cov}_h(\theta_{ih}, \omega_h^0) = \sigma_i \text{cov}_h(\mu_h, \omega_h^0) + \text{cov}_h(\vartheta_h, \omega_h^0)$ is a linear function of σ_i and $\text{cov}_{jh}(\mu_h\sigma_j + \vartheta_h,\, \omega_h^0) = \sigma\text{cov}_h(\mu_h,\, \omega_h^0) + \text{cov}_h(\vartheta_h,\, \omega_h^0)$, where $\sigma = \frac{1}{N}\int_{\mathcal{N}} \sigma_j \text{d}j$ and $N = \int_{\mathcal{N}} \text{d}j$. So, $\theta \equiv \mu\sigma + \vartheta$.

After entry, each firm i draws a pair of marginal cost c_i and a taste distribution $\sigma_i : \mathcal{N} \to R_0^+$ where cost and average quality are related. Each firm can improve the average willingness to pay for its variety by paying a higher marginal cost. For simplicity, we assume the linear production function, $\eta\sigma_i = c_i$, where η is a quality production parameter. Firms sell their varieties under monopolistic competition. Because it is infinitely small, each firm i chooses the prices p_i that maximizes its profit $\pi_i = x_i(p_i - c_i) - f$ taking the average price index p as a given. Its optimal price is equal to

$$p_i = \frac{1}{2}\eta\sigma_i + \frac{\alpha_0 \beta a_i}{2} + \frac{\gamma p}{2(\beta + \gamma)}$$

When all product markets clear, all prices p_i must be consistent with the price index p. Solving for the fixed point of $\frac{1}{N}\int_{\mathcal{N}} p_i \text{d}i = p$, we get the average price

$$p = \frac{(\beta + \gamma)\eta\sigma + \alpha_0\beta(\vartheta + \mu\sigma)}{2\beta + \gamma} + \alpha_0 \cdot \frac{\beta}{2\beta + \gamma}\text{cov}_{jh}(\theta_{jh},\, \omega_h^1)$$

which gives the following equilibrium markup:

$$p_i^* = m_i^* + \eta\sigma_i,$$

where

$$m_i^* = \beta\frac{\alpha_0(\vartheta+\mu\sigma)-\eta\sigma}{2\beta+\gamma} + (\alpha_0\mu-\eta)\frac{(\sigma_i-\sigma)}{2}$$
$$+\frac{\alpha_0}{2}\mathrm{cov}_h(\theta_{ih},\,\omega_h^0) - \frac{\alpha_0}{2}\mathrm{cov}_{jh}(\theta_{jh},\,\omega_h^0) + \frac{\alpha_0\beta}{2\beta+\gamma}\mathrm{cov}_{jh}(\theta_{jh},\,\omega_h^1).$$

We also simplify covariance terms and observe that

$$m_i = \beta\frac{\alpha_0\left(\vartheta+\sigma\mu\right)-\sigma\eta}{2\beta+\gamma}$$
$$+\frac{1}{2}\left[\alpha_0\mu-\eta+\alpha_0\mathrm{cov}_h(\mu_h,\,\omega_h^0)\right]\left(\sigma_i-\sigma\right)$$
$$+\frac{\alpha_0\beta}{2\beta+\gamma}\left[\mathrm{cov}_h(\vartheta_h,\,\omega_h^1)+\sigma\mathrm{cov}_h\left(\mu_h,\,\omega_h^1\right)\right]$$

This expression makes it clear that what determines the behavior of the price pattern is actually the *joint* distribution of willingness-to-pay and intensity of communication. To get more insight about the impact of covariances on prices, markups and (in what follows) selection, we make a linear approximation of the weights ω_h^0 and ω_h^1:

$$\omega_h^0 \simeq 1-\frac{1}{\beta+\delta}\left(\delta_h-\delta\right), \qquad \omega_h^1 \simeq 1-\frac{1}{\beta+\gamma+\delta}\left(\delta_h-\delta\right). \tag{4}$$

Using (4), we get

$$\mathrm{var}\left(\omega_h^0\right) \simeq \frac{1}{\left(\beta+\delta\right)^2}\mathrm{var}\left(\delta_h\right), \qquad \mathrm{var}\left(\omega_h^1\right) \simeq \frac{1}{\left(\beta+\gamma+\delta\right)^2}\mathrm{var}\left(\delta_h\right), \tag{5}$$

$$\mathrm{cov}\left(\vartheta_h,\omega_h^0\right) = -\frac{1}{\beta+\delta}\mathrm{cov}\left(\vartheta_h,\delta_h\right), \qquad \mathrm{cov}\left(\vartheta_h,\omega_h^1\right) = -\frac{1}{\beta+\gamma+\delta}\mathrm{cov}\left(\vartheta_h,\delta_h\right), \tag{6}$$

$$\mathrm{cov}\left(\mu_h,\omega_h^0\right) = -\frac{1}{\beta+\delta}\mathrm{cov}\left(\mu_h,\delta_h\right), \qquad \mathrm{cov}\left(\mu_h,\omega_h^1\right) = -\frac{1}{\beta+\gamma+\delta}\mathrm{cov}\left(\mu_h,\delta_h\right). \tag{7}$$

The precision of these approximations increases when $|\delta_h-\delta|$ becomes smaller relative to $\beta+\delta$. In particular, if the distribution of the communication intensities δ_h has compact support $[\underline{\delta},\overline{\delta}]$, the approximation becomes arbitrarily precise when $|\overline{\delta}-\underline{\delta}|$ decreases.

1.3 Firm selection

Firm i operates in the market if it breaks even. That is,

$$\pi_i^* = \frac{M}{\beta}\left(m_i^*\right)^2 - f \geq 0.$$

where f is a fixed production cost.

The set of active firms is given by $\mathcal{N} = \{i : \pi_i^* \geq 0\}$. Prices, markups and profits are determined by the sufficient statistics on the firm i (σ_i) and sufficient statistic of the markets (some covariances between μ_h, σ_j, ω_h^0, ω_h^1 and ω_h^2).

Without loss of generality, we can rank firms i with increasing markups m_i^*. This occurs if

$$i > j \iff m_i^* > m_j^*.$$

Then, the firms can be ranked as an increasing function of σ_i if $\alpha_0 \left(\mu + \mathrm{cov}_h(\mu_h, \omega_h^0)\right) > \eta$, which we assume now (average demands are large enough and covariance $\mathrm{cov}_h(\mu_h, \omega_h^0)$ is not too negative). Given this ranking, the set of active firms is given by $\mathcal{N} = \{i : m_i^* \geq 0\}$ where $m_D^* = 0$. Firms do not exit their markets if they offer high enough quality σ_i. Finally, the profit is equal to

$$\pi_i^* = \frac{M}{\beta} \cdot \max\left[(m_i^*)^2 - f,\, 0\right]$$

In this case, the markup is given by

$$m(\sigma_i, \sigma) = m_0 + m_1\sigma_i - m_2\sigma, \tag{8}$$

where

$$m_0 = \frac{\alpha_0\beta}{2\beta + \gamma}\left(\vartheta + \mathrm{cov}_h(\vartheta_h, \omega_h^1)\right) > 0,$$

$$m_1 = \frac{1}{2}\left[\alpha_0\mu - \eta + \alpha_0 \mathrm{cov}_h(\mu_h, \omega_h^0)\right],$$

$$m_2 = \frac{\gamma}{2(2\beta + \gamma)}(\alpha_0\mu - \eta) - \frac{\alpha_0\beta}{2\beta + \gamma}\mathrm{cov}_h\left(\mu_h, \omega_h^1\right) + \frac{\alpha_0}{2}\mathrm{cov}_h(\mu_h, \omega_h^0)$$

where m_0, m_1, and m_2 are constant.[1]

Also, $m_1 > m_2$ if and only if

$$\mu - \eta/\alpha_0 > -\mathrm{cov}_h\left(\mu_h, \omega_h^1\right). \tag{9}$$

We can write the profit as a function of the average taste parameter σ_i, and σ is the average of the σ_i's over the set of active firms $\mathcal{N}$:

$$\pi(\sigma_i, \sigma) = \frac{M}{\beta} \cdot \left(m(\sigma_i, \sigma)\right)^2 - f.$$

We assume that $m_1 > 0$, so that each firm i with $\sigma_i > \sigma_D$ survives where σ_D is the zero-profit cutoff

[1] Note that $m_0 > 0$ because $\vartheta + \mathrm{cov}_h(\vartheta_h, \omega_h^1) = \frac{1}{M}\int_H \left[\vartheta_h + (\vartheta_h - \vartheta)\left(\omega_h^1 - 1\right)\right] \mathrm{d}h = \frac{1}{M}\int_H \vartheta_h\omega_h^1 \mathrm{d}h > 0$.

of σ_i given by

$$\sigma_D = \frac{1}{m_1}\left(m_2\sigma - m_0 + \sqrt{\frac{\beta}{M}f}\right) \tag{10}$$

The assumption that $m_1 > 0$ is satisfied for sufficiently high willingness to pay compared to margianl costs and sufficiently low covariance effects between conformism and willingness to pay. Note that σ depends on the set of surviving firms and cutoff level σ_D. Higher fixed costs f reduce the number of varieties that survive in the market. A sufficiently high fixed cost f guarantees that everybody buys.

1.4 Entry

We define the set of entrants as $\overline{\mathcal{N}} = [0, \overline{N}] \times [0, \infty)$. Each entrant draws a firm specific σ_i from a cumulative distribution $G(\sigma_i) : [0, \infty) \to [0, 1]$. Ordering varieties so that σ_i is increasing, we can suppress the index i. The number of active firms equal to $N = \overline{N}\,(1 - G(\sigma_D))$, so, $\mathcal{N} = [0, \overline{N}] \times [\sigma_D, \infty)$. Under that condition σ is a function of σ_D solely since

$$\sigma = \frac{\int_{\sigma_D}^{\infty} \xi \mathrm{d}G(\xi)}{\int_{\sigma_D}^{\infty} \mathrm{d}G(\xi)}. \tag{11}$$

Plugging this into (10) yields the equation that determines σ_D and σ independently of $\overline{N}$. The entry condition is

$$\int_{\sigma_D}^{\infty} \pi(\xi,\, \sigma)\mathrm{d}G(\xi) = f_e\left(\frac{\overline{N}}{M}\right), \tag{12}$$

where f_e is the entry cost which is an increasing function. This expresses the possible congestion in the design of new variety. Plugging (8) we obtain

$$\int_{\sigma_D}^{\infty} \pi(\xi,\, \sigma)\mathrm{d}G(\xi) = \frac{M}{\beta}\int_{\sigma_D}^{\infty} \left[(m(\xi,\, \sigma))^2 - (m(\sigma_D,\, \sigma))^2\right] \mathrm{d}G(\xi)$$

Assume now Pareto cumulative distribution $G(\xi) = 1 - \left(\frac{\sigma_M}{\xi}\right)^k$, $\xi > \sigma_M$ and $k > 2$. A larger k gives a more concentrated distribution. The density of distribution is $g(\xi) = \frac{k}{\sigma_M}\left(\frac{\sigma_M}{\xi}\right)^{k+1}$. Then (11) takes the form

$$\sigma = \frac{\int_{\sigma_D}^{\infty} \xi \mathrm{d}G(\xi)}{\int_{\sigma_D}^{\infty} \mathrm{d}G(\xi)} = \frac{k}{k-1}\sigma_D.$$

Plugging it into equation (11) we obtain a unique interior cutoff level $\sigma_D = \hat{\sigma}_D$ if $\sigma_M < \hat{\sigma}_D$, where

$$\hat{\sigma}_D = \frac{\sqrt{\frac{\beta}{M}f} - m_0}{m_1 - \frac{k}{k-1} \cdot m_2}. \tag{13}$$

Otherwise, when the $\hat{\sigma}_D < \sigma_M$, all type of firms enter and there is no selection while $\sigma_D = \sigma_M$. In the sequel we assume that $\sigma_M < \hat{\sigma}_D$ so that there is always some firms that exit the market. This is satisfied if

$$\sqrt{\frac{\beta}{M} f} > m_0$$

and

$$k > \frac{m_1}{m_1 - m_2} = \frac{2\beta + \gamma}{2\beta} \cdot \frac{\mu - \eta/\alpha_0 + \text{cov}_h(\mu_h, \omega_h^0)}{\mu - \eta/\alpha_0 + \text{cov}_h(\mu_h, \omega_h^1)}.$$

The firm type distribution should be sufficiently concentrated (high k). This includes the usual set-up of homogenous taste. If the taste distribution G is concentrated on a mass point at the value σ_M ($k = \infty$), then all firms get the same strictly positive markups $m_i = m_0 + m_1\sigma_M - m_2\sigma_M > 0$ and survive iff $\hat{\sigma}_D = \left(\sqrt{\frac{\beta}{M} f} - m_0\right) / (m_1 - m_2) < \sigma_M$ Then, the fixed cost should be low enough. In the absence of taste heterogeneity the condition implies that

$$\gamma < 2\beta \tag{14}$$

so that varieties are sufficiently differentiated. The assumption of high enough fixed costs shall also guarantee positive individual demands, $x_{ih} > 0$.

The entry condition (12) becomes

$$2\frac{M}{\beta} m_1 \frac{\sigma_M^k}{\sigma_D^{k-2}} \frac{m_1 + k(k-2)(m_1 - m_2) + (k-2)(k-1)m_0\sigma_D^{-1}}{(k-2)(k-1)^2} = f_e\left(\frac{\overline{N}}{M}\right)$$

which simplifies to

$$\frac{1}{\beta(k-2)(k-1)} \frac{\sigma_M^k}{\widehat{\sigma}_D^{k-2}} m_1 \left(m_1 + \frac{k-2}{\widehat{\sigma}_D}\sqrt{\frac{\beta}{M} f}\right) = \frac{1}{2M} f_e\left(\frac{\overline{N}}{M}\right)$$

Hence, the number of entrants $\overline{N}$ rises with higher m_1 and smaller $\widehat{\sigma}_D$. However the number of active firms equal to $N = \overline{N}(1 - G(\sigma_D)) = \left(\frac{\sigma_M}{\widehat{\sigma}_D}\right)^k$ rises with smaller $\widehat{\sigma}_D$ only. Since $\hat{\sigma}_D$ falls with higher m_1, it comes that both the numbers of entrants $\overline{N}$ and of active firms N are decreasing functions of the threshold $\widehat{\sigma}_D$. Also, if m_0 rises but m_1 and m_2 remain fixed, the threshold $\widehat{\sigma}_D$ falls and the numbers of entrants $\overline{N}$ and active firms N rise. If m_2 rises but m_0 and m_1 remain fixed, the threshold $\widehat{\sigma}_D$ increases and the numbers of entrants $\overline{N}$ and active firms N fall.

2 Discussion of selection

We now discuss the selection of firms. Under the above conditions, we can make the following comparative statics exercise.

2.1 No heterogeneity

First let us fix the covariance between conformity and taste to zero. Then,

$$\hat{\sigma}_D = 2\frac{\sqrt{\frac{\beta}{M}f} - \frac{\alpha_0\beta}{2\beta+\gamma}\vartheta}{(\alpha_0\mu - \eta)\left(1 - \frac{k}{k-1}\cdot\frac{\gamma}{(2\beta+\gamma)}\right)}.$$

The cutoff $\hat{\sigma}_D$ rises with higher fixed cost of production and smaller market size. Hence, a higher fixed cost means more selection. The cutoff $\hat{\sigma}_D$ is a decreasing function of the average shifter ϑ of taste parameter ϑ_h. Indeed, if consumers have higher idiosyncratic willingness to pay ϑ_h of all varieties, the firm demand should increase for each variety and more firms survive. The cutoff $\hat{\sigma}_D$ also falls with higher μ, which expresses the average of each consumer's sensitivity to quality μ_h. A higher μ_h shifts the willingness to pay upward for all varieties. As it makes low quality varieties more desirable, more varieties survive. For the same reason, the cutoff $\hat{\sigma}_D$ also falls with the average willingness to pay parameter α_0 and increase with the marginal cost η. The cutoff $\hat{\sigma}_D$ also falls when the taste distribution is more concentrated (higher k). There are less much higher quality varieties for which consumers substitute. Low quality firms then face a less intensive competition and more of them survive. The cutoff $\hat{\sigma}_D$ finally falls with higher product differentiation (lower γ). This is because the intensity of competion also diminishes in this case.

2.2 Heterogeniety in ϑ_h

Suppose $\mu_h = \mu_0$ so that $\theta_{ih} \equiv \sigma_i + \vartheta_h$. For the sake of conciseness, suppose that high valuation consumers are trend leaders so that $\text{cov}_h(\vartheta_h,\, \delta_h) < 0$. We get

$$\hat{\sigma}_D = 2\frac{\sqrt{\frac{\beta}{M}f} - \frac{\alpha_0\beta}{2\beta+\gamma}\left[\vartheta - \frac{1}{\beta+\gamma+\delta}\text{cov}_h(\vartheta_h,\, \delta_h)\right]}{(\alpha_0\mu - \eta)\left(1 - \frac{k}{k-1}\cdot\frac{\gamma}{(2\beta+\gamma)}\right)}.$$

Less firms survive in the market if $\hat{\sigma}_D$ increases. That is, for lower values of ϑ and δ and lower absolute values of covariances $\text{cov}_h(\vartheta_h, \delta_h)$.

Consider a parallel upward shift in the distribution of taste ϑ_h such that ϑ_h becomes $\vartheta_h + \varepsilon$, $\varepsilon > 0$ so that ϑ rises and the difference $(\vartheta_h - \vartheta)$ and therefore $\text{cov}_h(\vartheta_h,\, \delta_h)$ are constant. So, $\hat{\sigma}_D$ falls so that more low quality firms stay in the market. Intuitively, all consumers get higher valuations for both low and high quality goods. Low quality good producers then get higher demands and stay in the market. This effect does not depend on conformism (δ_h).

Consider a parallel upward shift in the distribution of taste δ_h such that $\delta_h := \delta_h + \varepsilon$, $\varepsilon > 0$ so that δ rises and the difference $(\delta_h - \delta)$ and therefore $\text{cov}_h(\vartheta_h,\, \delta_h)$ are constant. When $\text{cov}_h(\vartheta_h,\, \delta_h) < 0$, this raises $\hat{\sigma}_D$ so that low quality firms exit the market. Intuitively, all consumers are subject to stronger conformism. The intuition can be that low willingness to pay consumers reshuffle their consumption baskets towards high quality goods more than what the high willingness to pay consumer do towards low quality goods. Indeed, the latter have already high consumption of low quality goods anyway. As a result the average consumption shifts towards higher quality goods and makes low quality firms unprofitable.

Consider now a mean-preserving spread in the taste distribution, i.e. an increase in the variance of the distribution of taste ϑ_h such that ϑ and δ remain constant. In this case, this diminishes $\hat{\sigma}_D$ if it increases the absolute value of $\mathrm{cov}_h(\vartheta_h, \delta_h)$ (more negative $\mathrm{cov}_h(\vartheta_h, \delta_h)$). So, more low-quality firms remain in the market when the trend leaders get higher willingness to pay consumers and trend followers lower willingness to pay.

Proposition: *Suppose $\theta_{ih} = \sigma_i + \vartheta_h$. Then, if high valuation consumers are trend leaders, a rise in willingess to pay ϑ_h and/or a rise in conformism δ_h entice low quality firms to exit the market.*

2.3 Heterogeneity in μ_h

Assume now that $\theta_{ih} = \mu_h \sigma_i + \vartheta$, which means that consumers differ not in their propensity to shopping, but in the marginal value they assign to higher quality. The cutoff level of quality (13) then becomes

$$\hat{\sigma}_D = \frac{\sqrt{\frac{\beta}{M} f} - \frac{\alpha_0 \beta}{2\beta+\gamma}\vartheta}{\frac{\alpha_0\mu-\eta}{2}\left(1 - \frac{k}{k-1}\cdot\frac{\gamma}{(2\beta+\gamma)}\right) + \frac{\alpha_0}{k-1}\cdot\left(\frac{k\beta}{2\beta+\gamma}\mathrm{cov}_h\left(\mu_h, \omega_h^1\right) - \frac{1}{2}\mathrm{cov}_h(\mu_h, \omega_h^0)\right)}.$$

Combining this with (7), we get

$$\hat{\sigma}_D = \frac{2\left(\sqrt{\frac{\beta}{M} f} - \frac{\alpha_0 \beta}{2\beta+\gamma}\vartheta\right)}{(\alpha_0\mu - \eta)\left(1 - \frac{k}{k-1}\cdot\frac{\gamma}{2\beta+\gamma}\right) + \frac{\alpha_0}{k-1}\left(\frac{1}{\beta+\delta} - \frac{2\beta k}{2\beta+\gamma}\frac{1}{\beta+\gamma+\delta}\right)\mathrm{cov}\left(\mu_h, \delta_h\right)}. \tag{15}$$

We now come to studying the impact of conformity distribution on the cutoff quality level. Consider first an increase "on average" in communication intensities:

$$\delta_h \to \delta_h + \varepsilon_h, \tag{16}$$

where ε_h are such that

$$\int_0^M \varepsilon_h \mathrm{d}h > 0, \qquad \mathrm{cov}(\mu_h, \varepsilon_h) = 0. \tag{17}$$

In particular, (16) – (17) describes a parallel upward shift in the distribution of δ_h if $\varepsilon_h = \varepsilon$. The impact of such shocks in communication intensities on the cutoff quality level may be summarized as follows.

Proposition . *If either product differentiation is high enough, or quality dispersion is low enough, then an increase in communication intensities on average, described by* (16) – (17)*, leads to an increase* (*decrease*) *in the cutoff quality level* $\hat{\sigma}_D$ *when communication intensity is negatively* (*positively*) *correlated with propensity to quality. Otherwise, the relationship between* δ *and* $\hat{\sigma}_D$ *is U-shaped* (*bell-shaped*)*.*

Proof. A shock in $(\delta_h)_{h\in\mathcal{H}}$ described by (16) – (17) induces an increase in δ and leaves the covariance $\mathrm{cov}_h(\mu_h,\, \delta_h)$ unchanged. Differentiating $\hat{\sigma}_D$ in δ yields

$$\frac{\partial \hat{\sigma}_D}{\partial \delta} = \frac{-2\left(\sqrt{\frac{\beta}{M}f} - \frac{\alpha_0\beta}{2\beta+\gamma}\vartheta\right)\left[\frac{2k\beta}{2\beta+\gamma}\left(\frac{\beta+\delta}{\beta+\gamma+\delta}\right)^2 - 1\right]\mathrm{cov}_h(\mu_h,\, \delta_h)}{\left[(\alpha_0\mu-\eta)\left(1-\frac{k}{k-1}\cdot\frac{\gamma}{2\beta+\gamma}\right)+\frac{\alpha_0}{k-1}\left(\frac{1}{\beta+\delta}-\frac{2\beta k}{2\beta+\gamma}\frac{1}{\beta+\gamma+\delta}\right)\mathrm{cov}\left(\mu_h,\delta_h\right)\right]^2},$$

which has the same sign as

$$-\mathrm{cov}_h(\mu_h,\, \delta_h)\cdot\left[\frac{2k\beta}{2\beta+\gamma}\left(\frac{\beta+\delta}{\beta+\gamma+\delta}\right)^2 - 1\right].$$

Hence, when $\mathrm{cov}_h(\mu_h,\, \delta_h) < 0$, which is the "plausible" case that individuals who have higher valuations of quality are non-conformists, $\hat{\sigma}_D$ increases if

$$k\left(\frac{\beta+\delta}{\beta+\gamma+\delta}\right)^2 > \frac{2\beta+\gamma}{2\beta}. \tag{18}$$

Define

$$\overline{k} \equiv \left(1+\frac{\gamma}{\beta}\right)^2\left(1+\frac{\gamma}{2\beta}\right).$$

Two cases may arise: (i) if $k > \overline{k}$, i.e. if the quality distribution across firms is not very dispersed, then (18) holds for all $\delta \geq 0$; (ii) if $k < \overline{k}$, then there exists a positive threshold value

$$\overline{\delta} \equiv \frac{\gamma}{\sqrt{\frac{2\beta k}{2\beta+\gamma}} - 1} - \beta$$

of δ, such that (18) holds only when $\delta > \overline{\delta}$. This corresponds to a U-shaped relationship between δ and $\hat{\sigma}_D$. Note that, because $k > 2$, (i) always holds under $\overline{k} \leq 2$, which is the case if and only if $\gamma/\beta < \xi_0$, where $\xi_0 \approx 0.314596$ is the unique solution of the equation $(1+2\xi)^2(1+\xi) = 2$, i.e. when the degree of product differentiation is sufficiently high.

Finally, when $\mathrm{cov}_h(\mu_h,\, \delta_h) > 0$, the results become the mirror image of those for $\mathrm{cov}_h(\mu_h,\, \delta_h) < 0$.

Proposition 2.3 states that when communication intensity is negatively correlated with propensity to quality and firms are heterogeneous enough, more communication leads to more care on quality, hence tougher competition and vanishing lower-quality firms from the market.

Mean-preserving spread. Next, consider what happens when communication intensity distribution becomes "more dispersed" in the following sense: $\delta_h \to \delta_h + \epsilon_h$, where ϵ_h are such that

$$\int_0^M \epsilon_h \mathrm{d}h = 0, \qquad \frac{\int_0^M \epsilon_h \delta_h \mathrm{d}h}{\int_0^M \epsilon_h^2 \mathrm{d}h} = \frac{\mathrm{cov}(\delta_h, \epsilon_h)}{\mathrm{var}(\epsilon_h)} > -\frac{1}{2}. \tag{19}$$

It is readily verified that (19) fully characterize shocks ϵ_h in communication intensity pattern which (i) are mean-preserving; (ii) increase $\mathrm{var}\,(\delta_h)$.

Since δ remains unchanged, the impact of a mean-preserving spread on $\hat{\sigma}_D$ is channelled through a change in the covariance term $\mathrm{cov}\,(\mu_h,\, \delta_h)$. Clearly, $\mathrm{cov}\,(\mu_h,\, \delta_h)$ increases (decreases) if and only if $\mathrm{cov}\,(\mu_h, \epsilon_h)$ is positive (negative).

Consider the case when $\mathrm{cov}\,(\mu_h, \epsilon_h) > 0$. In this case, $\mathrm{cov}\,(\mu_h, \delta_h)$ increases, whence a hike (drop) in the cutoff quality $\hat{\sigma}_D$ occurs if and only if

$$\frac{2\beta + \gamma}{2\beta k} < \frac{\beta + \delta}{\beta + \gamma + \delta}.$$

This inequality holds for all $\delta > 0$ when

$$k > \widetilde{k} \equiv \left(1 + \frac{\gamma}{\beta}\right)\left(1 + \frac{\gamma}{2\beta}\right).$$

Otherwise, there is positive selection if and only if δ exceeds a threshold value $\widetilde{\delta}$ defined by

$$\widetilde{\delta} \equiv \frac{(\beta + \gamma)(2\beta + \gamma) - 2\beta^2 k}{2\beta(k-1) - \gamma}.$$

When $\mathrm{cov}\,(\mu_h, \epsilon_h) < 0$, negative selection takes place under $k > \widetilde{k}$, otherwise positive (negative) selection occurs under small (large) values of δ.