Hyper graph network models for stock market

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Problem statement. Network model of a complex system is a complete weighted graph where weights of edges are defined by a measure of pairwise similarity of nodes. Network structures are subgraphs of this graph. Popular network structures in stock market network are market graph (MG) and maximum spanning tree (MST). Market graph is constructed from network model by removing edges with weights less than a given threshold. Clique of the market graph is a complete subgraph of the graph. Hub of the MG or of the MST is vertex which has greatest degree. However, pairwise similarity does not reflect all information about complex system. More information can be obtained if one can use similarity between sets of nodes. This leads to a hyper graph network model. Such models are not well investigated yet. The present paper is a contribution to fill this gap.

Methodology

In this paper, the general framework of the random variables network (Kalyagin at al, 2020) is further developed for the hyper graph networks. Nodes of a hyper graph network are associated with random variables. The weight of a hyper edge (set of nodes) is defined by a measure of similarity (dependence) for the random variables associated with the nodes of the set. If only hyper edges with k nodes are considered one get a k-hyper graph network model. In particular, 2-hyper graph network model is usual random variable network model with pairwise similarity. Hyper graph network models give an important additional information in network analysis. All these models have a clear interpretation for the stock market network.

Hypergraph network model

A simple measure of similarity, sign coincidence (Kalyagin at al, 2013), is used to define a hyper graph networks. The weight of a hyper edge (set of nodes) is defined by the probability of sign coincidence More precisely, let $X_1, X_2, \ldots, X_N$ be

the random variables associated with N nodes of the network, let I be a subset of the set {1, 2,…, N} (hyper edge). The weight of the hyper edge I is defined by

$$\text{Weight}(I) = P((X_i - E(X_i) \text{ have the same sign for all } i \text{ in } I)$$

for the random variables associated with the nodes of the set I.

Results

Hyper graph network models with sign similarity are investigated. The following problems are investigated with applications to stock market networks: clustering in hyper graph networks, uncertainty of hyper graph network structure identification, and hyper edge specific properties in stock market network. It is shown that the use of hyper graph models for clustering gives some advantages in cluster interpretation. Robust network structure identification in random variable networks (Kalyagin at al, 2017) is generalized to robust identification in hyper graph random variable networks. pairwise dependence is investigated. It is shown that the joint dependence of several random variables is quickly decreased with respect to their number. Specific properties of hyper edges in market networks are investigated. In particular, it is shown that the weights of hyper edges are rapidly decreasing when number of nodes in the hyper edge is increasing.

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