Optimal Income Taxation with Endogenous Prices*

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Abstract

We consider a Mirleesian model of optimal income taxation with endogenously product prices. In the presence of endogenous prices, the distribution of income effects social welfare not only directly, but also through its influence on the level of prices in the economy. To correct for this price externality, the optimal income tax schedule contains a new Pigouvian term. For competitive markets with increasing long-run market supply, the Pigouvian term is positive for normal goods, negative for inferior goods, increasing for luxury goods, and decreasing for necessity goods. Using a calibrated model of the US housing market, we quantify the price effect showing that it increases marginal income tax by 4-5 percentage points for most income levels in the optimum. For oligopostic markets, we also show that the price effect is increasing with additional market power. We also study the robustness of our findings to the presence of commodity and profit taxation.

Keywords: Optimal income taxation, endogenous pricing, externality, oligopolistic markets, housing market.

JEL Classification: H21, H23, D43.

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1 Introduction

In most of the literature on public economics (see Mirrlees and Adam, 2010), product prices are regarded as fixed – the consequence of perfectly competitive market assumption.[1] This assumption is convenient for analysis, but it is hardly an accurate description of major industries. According to recent studies (e.g., Azar et al., 2015; Francis and Knutson, 2015), one third of U.S. industries are highly concentrated and about two thirds of U.S. publicly traded companies currently operate in more concentrated markets than was the case 20 years ago. Similar market structures are observed in Europe, where the market share of the top three food retailers ranges from 30 to 50% for most countries (Data and Trends, 2012).

What are implications for optimal income taxation if markets are not perfectly competitive? Does the income effect on market demand and, hence, product prices matter empirically? The goal of this paper is to answer these questions and, thus, bridge an existing gap in public economics literature by incorporating endogenous product prices into the analysis of optimal income taxation.[2] When product markets are not perfectly competitive, the distribution of income effects social welfare not only directly, but also through its influence on the level of product prices in the economy.

We consider the standard Mirleesian framework Mirrlees (1971) in the presence of imperfectly competitive product markets. There is a continuum of agents who care about consumption of two goods. The first good is a numeraire good that is produced with a constant returns to scale technology and perfectly elastic supply function. The second good is produced with a decreasing returns to scale technology and strictly increasing supply function. As a consequence, the price of the first good remains constant and the price of the second good is endogenously determined by the interesection of the market supply and demand - the market equilibrium condition.[3]

We observe that a change in income distribution creates externality through its effects on the market demand and, hence, the equilibrium price level. Thus, when solving for the optimal marginal income taxes, we obtain an additional term – referred to as the Pigouvian term – the role of which is to correct for the above price externality. We show that the Pigouvioan term is positive for normal goods and negative for inferior goods. Intuitively, when the second good is normal, the public authority wants to discourage agents working too hard. Though this policy leads to a lower income level it also reduces the price of the second good. Combined with savings on labor cost it leads to a larger total welfare. More strikingly, the lump sum taxation is no longer first best, i.e., it is not longer optimal when agents' productivity is perfectly observable.

[1]Some important exceptions are Myles (1989, 1995) and Auberbach and Hines (2003) who study *commodity taxation* in the presence of imperfectly competitive markets. We call a market *perfectly competitive* if its long-run market supply is perfectly elastic. If long-run supply has a positive slope and firms are small such that they cannot influence the price level we call such market *competitive*.

[2]For a detailed discussion of this gap in public economics see Atkinson (2012).

[3]In Appendix B, we also support our framework with a general equilibrium model with agents supplying their labor for a fixed wage and the firms spending their profit solely on the numeraire good.

Another implication of endogenous prices is that the famous end-point result of zero marginal tax for the highest income bracket ceases to hold. In addition, we show that the Pigouvian term is increasing for luxury goods and decreasing for necessity goods.

To estimate the price effect on optimal income tax rates we use the US housing market as an example. This market is particularly suitable for this purpose because housing costs account for almost a third of average household expenditures in US. Using a calibrated model of US housing market due to Albouy et al. (2017) we estimate that the price effect leads to 4-5 percentage points increase in optimal marginal income tax for most income levels.

We also extend our results to markets with various forms of oligopolistic competition. The presence of market power leads to both an inefficient low quantities traded in the economy and high prices. To increase the amount of goods traded in equilibrium, the public authority stimulates labor income by decreasing the marginal income tax level in the optimum. Decreasing marginal income tax has an effect similar to a corrective subsidy in the case of commodity taxation (see Auberbach and Hines, 2003). At the same time, we show that the price effect on income taxes (the change in optimal marginal income tax keeping the price level fixed) is increasing with additional market power. For US housing example, the first effect dominates the second one leading to lower optimal marginal income taxes for more concentrated markets.

We also study whether commodity and profit taxation can help to alleviate the price externality. In competitive markets, the optimal commodity tax exactly offsets price externality eliminating the Pigouvian term in optimal marginal tax formula.[4] The introduction of 100% profit taxation also achieves the same outcome, which is in line with the production efficiency theorem of Diamond and Mirrlees (1971) saying that agents' productive efforts should not be distorted in the presence of optimal income tax schedule. However, the Pigouvian term continues to play an important role in correcting for the price externality if the market for the second good is oligopolistic and it is eliminated by neither commodity nor profit taxation. This result is similar to Auberbach and Hines (2003) who show that it is too costly for the public authority to fully eliminate the price externality using commodity taxation in the presence of imperfectly competitive markets.

Related Literature. In the vast literature on optimal income taxation our analysis is closest to papers on optimal income taxation in the presence of endogenous wages on labor markets. Stiglitz (1982) considers an economy with workers of two different talents who could earn potentially different wages. He shows that the optimal tax policy should subsidize high talent workers and tax low talent workers. Rothschild and Scheuer (2013) extend this analysis to a setting where workers have multidimensional skill characteristics. Ales, Kurnaz, and Sleet (2015) consider a similar talent assignment model and study a change in optimal income policy in response for a technical change. Sachs, Tsyvinskiy, and Werquin (2016) considers a general equilibrium model with occupation specific wages. They show the optimal labor supply

[4] In contrast to Atkinson and Stiglitz (1976), we obtain a coexistence of commodity and income taxation in the optimum because we assume that profits are not 100% taxed by the public authority.

is determined by quite complicated integral equation. Using CES and Translog production function, they analyze the incidence of any tax reform considering the actual U.S. tax code as a starting point. Compared to the literature on endogenous wages, the analysis of the price effect on optimal income taxation is more tractable because the prices are determined only by aggregate demand compared to occupation-specific wages. Hence, we were able to obtain an analytical expression for the Pigouvian term for each agent and analyze its properties for a wide spectrum of individual demand functions including normal and inferior goods.[5]

Our paper also related to a few papers analyzing *commodity taxation* in imperfectly competitive markets; Myles (1989, 1995) and Auberbach and Hines (2003). Similar to Myles (1989) we show that it is too costly to completely eliminate price externality in the presence of imperfect competition. As in Auerback and Hines (2003) we show that it is optimal to decrease optimal marginal income tax, which is parallel to providing a subsidy in case of commodity taxation.

Our paper is also closely related to the strand of literature that studies the effects of relativity concerns on optimal income taxation; see Boskin and Sheshinski (1978), Oswald (1983), Ireland (2001), Kanbur and Tuomala (2013).[6] These papers are typically motivated by empirical observations that people care not only about their absolute level of consumption, but also how it compares with that of others. The main message of this literature is that social comparisons typically add toward progressivity of income taxation. In our paper, relativity concerns arise because income distribution influence product prices. But unlike the previous papers, relativity concerns in our paper arise endogenously from the market equilibrium condition.

The remainder of the paper is organized as follows. In Section 2, we introduce the framework of our model. In Section 3, we consider the case of competitive markets, where we solve for the optimal marginal income tax schedule, analyze its properties, and provide a numerical example. In Section 4, we consider market structures with oligopolistic competition. In Section 5, we study the robustness of our results to the presence of commodity and profit taxation. Section 6 concludes. The omitted proofs are postponed to Appendix A. Appendix B contains a general equilibrium model that supports the main model of the paper. Appendix C contains simulation results for various supply elasticities and Champernowne distribution of agent productivity.

2 Model

In the economy, there is a continuum of agents indexed by their productivity type n. Agent n's gross income is given by $z = n\ell$, where ℓ is the number of hours worked. The labor cost function is given by an increasing and convex function $c(\ell)$. Productivity type n is distributed according to probability density function $f(n) > 0$ with support $[\underline{n}, \overline{n}]$.

[5]For other papers analyzing optimal income taxation in the presence of endogenous wages see also Johnes (1987), Slavk and Yazici (2014), Stantcheva (2014), Lockwood et al. (2015), and Ales and Sleet (2016).

[6]A similar idea that individuals may seek a more equal income distribution in order to improve their terms of trade is explored by Zubrickas (2012).

An agent's disposable income after tax $T(z)$ is given $y = z - T(z)$, which is spent on the consumption of two goods: a numeraire good and good X. The price of the numeraire good is fixed and normalized to 1. The numeraire good is produced with a homogeneous of degree one technology and traded in a competitive market which results in the fixed price and zero firm profits. Let p denote the price of good X, which is produced using a decreasing returns to scale technology that yields positive profits $\Pi(p)$.[7] We assume that these profits are spent solely on the consumption of the numeraire good.

All agents have identical preferences for consumption represented by an indirect utility function $v(p, y)$, which is concave and increasing in y. Agent's net utility is defined by

$$U(p, y, \ell) = v(p, y) - c(\ell).$$

The social welfare function is given by the aggregation of agents' net utilities together with the firm profits weighted by $\omega \geq 0$

$$W = \int H(U(p, y(n), \ell(n)))f(n)dn + \omega\Pi(p),$$

where H is an increasing and concave function. The public authority wants to maximize the social welfare function W subject to three constraints. The first one is resource constraint

$$\int T(z(n))f(n)dn = \int (n\ell(n) - y(n))\, f(n)dn \geq R, \tag{1}$$

which ensures that the public authority covers its own expenses $R \geq 0$ that are spent solely on the numeraire good. The second one is incentive compatibility constraint

$$U\left(p, n\ell(n) - T(n\ell(n)), \ell(n)\right) \geq U\left(p, m\ell(m) - T(m\ell(m)), m\ell(m)/n\right) \tag{2}$$

for all $n, m \in [\underline{n}, \overline{n}]$. Constraint (2) ensures that an agent with productivity n does not prefer to seek the income of an agent with a different type of productivity.

The third constraint is a *market equilibrium condition* that determines price p. This condition differs across various market structures that we analyze separately. In competitive markets (Section 3), the market equilibrium condition requires the market supply equal to the market demand for good X. In oligopolistic markets (Section 4), the market equilibrium condition is determined by the firm profit maximizing behavior.

Overall, the only difference between our framework and the model of Mirrlees (1971) is that we do not assume fixed prices in the economy. Instead, the price of one of the goods is endogenously determined by the market equilibrium condition. The effect of this condition on

[7] We do not model explicitly why firms that produce the numeraire good do not switch to a more profitable production of good X. One could think about many reasons why this happens including firms having a lack of technology, patents, or facing other barriers to entry. The high degree of persistent performance differences among firms is a well-documented phenomenon (see, for example, Syverson (2011) for a recent survey).

optimal income taxation is the main subject of our analysis presented in the following sections.

3 Competitive Market

In this section, we analyze the problem of optimal income taxation when good X is traded in a competitive market. The price of good X is determined by the market equilibrium condition

$$S(p) = \int x(p, y(n)) f(n) dn, \tag{3}$$

where on the left-hand side of the equation we have market supply $S(p)$ and on the right side the market demand for good X where $x(p, y)$ is the Walrasian demand function of an agent with disposable income y. We consider a non-decreasing supply function $S'(p) \geq 0$ and zero fixed costs so that firm profits coincide with total producers' surplus $\Pi(p) = \int_0^p S(\tilde{p}) d\tilde{p}$. The demand $x(p, y)$ is determined using the Roy's identity $x(p, y) = -v_p(p, y)/v_y(p, y)$ and satisfies the law of demand, i.e., $x_p < 0$. We also assume that the demand has the same curvature with respect to income, i.e. either $x_{yy} \geq 0$ or $x_{yy} < 0$ for any level of income. In Appendix B, we show how our framework can be supported with a general equilibrium model including labor markets. We also explain why we need only one condition (3) to cleare the product and labor markets in the economy.

The public authority's objective is to find the tax schedule $T(z)$ that maximizes the social welfare function W subject to the resource and incentive compatibility constraints and the market equilibrium condition. The public authority problem is then to find price p, levels of disposable income $y(n)$, and individual labor supply $\ell(n)$ that maximize

$$\max_{p, y(n), \ell(n)} \int H(U(p, y(n), \ell(n))) f(n) dn + \omega \Pi(p) \text{ subject to (1), (2), and (3).}$$

It is analytically more convenient to reformulate this maximization problem so that the choice variables are price p, labor supply $\ell(n)$, and agent utility $u(n) = U(p, y(n), \ell(n))$ (see Mirrlees (1976)). From the latter expression we can invert disposable income $y(n)$, which we express as a function $y = q(p, u, \ell)$. Using the new set of independent variables, we rewrite the market equilibrium condition as

$$S(p) = \int x(p, q(p, u, \ell)) f(n) dn \tag{4}$$

and the resource constraint as

$$\int \left(n\ell(n) - q(p, u, \ell) \right) f(n) dn = R. \tag{5}$$

Incentive compatibility constraints (2) can be written as $u(n) = \max_m U(p, y(m), m\ell(m)/n)$.

The envelope theorem then implies that[8]

$$u'(n) = \frac{dU}{dn} = \frac{\ell(n)c_\ell(\ell(n))}{n}. \tag{6}$$

Given the new formulation, the public authority chooses $u(n)$, $\ell(n)$, and price p to maximize

$$\max_{p,\, u(n),\, \ell(n)} \int H(u(n))f(n)dn + \omega\Pi(p) \text{ subject to (4), (5), and (6).}$$

The Lagrangian of the public authority's problem is given by

$$\mathcal{L} = \int [(H(u(n)) + \omega\Pi(p) + \lambda(n\ell(n) - q(p, u(n), \ell(n)) - R)$$
$$+ \gamma(S(p) - x(p, q(p, u(n), \ell(n)))))f(n) + \mu(n)(u'(n) - \ell(n)c_\ell(\ell(n))/n)]dn,$$

where γ, λ, and $\mu(n)$ are multipliers corresponding to constraints (4), (5), and (6) respectively. After integrating $\int \mu(n)u'(n)dn$ by parts, the first-order conditions can be written as

$$\begin{aligned}
u(n) &: [H'(u) - (\lambda + \gamma x_y)q_u]f(n) - \mu'(n) = 0, \\
\ell(n) &: [\lambda n - (\lambda + \gamma x_y)q_\ell]f(n) - \mu(n)(c_\ell(\ell) + \ell c_{\ell\ell}(\ell))/n = 0, \\
p &: \int [\omega\Pi'(p) - \lambda q_p + \gamma(S'(p) - x_p - x_y q_p)]f(n)dn = 0,
\end{aligned}$$

along with the transversality conditions $\mu(\underline{n}) = \mu(\overline{n}) = 0$. By implicit differentiation, we obtain the derivatives $q_u = 1/v_y,\ q_\ell = c_\ell/v_y,\ q_p = -v_p/v_y = x$, where the last expression follows from the Roy's identity. The first-order conditions then reduce to

$$u(n) : \left(H'(u) - \frac{\lambda + \gamma x_y}{v_y}\right) f - \mu'(n) = 0, \tag{7}$$

$$\ell(n) : \left(\lambda n - \frac{c_\ell}{v_y}(\lambda + \gamma x_y)\right) f - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{8}$$

$$p : \omega S(p) - \lambda S(p) + \gamma \left(S'(p) - \int (x_p + x_y x) f dn\right) = 0. \tag{9}$$

In deriving (9), we also use the market equilibrium condition (4) and that $\Pi'(p) = S(p)$.

To find the expression for the optimal marginal income tax $t(z) = T'(z)$ we note that individual maximization problem $u(n) = \max_m U(p, y(m), z(m)/n)$ implies that $nv_y y'(n) - c_\ell z'(n) = 0$. Given that $y = z - T(z)$ the optimal marginal tax rate must satisfy $t(z) = 1 - c_\ell/(nv_y)$. Furthermore, from the same individual utility maximization condition we can find that $1 + \ell c_{\ell\ell}/c_\ell = (1 + E^u)/E^c$, where E^c is the elasticity of compensated labor supply and E^u is the elasticity of uncompensated labor supply E^u (see the proof of Theorem 1).

[8]The second-order condition also requires $u(n)$ be a non-decreasing function.

Multiplying (8) by v_y/c_ℓ we obtain the following result.

Theorem 1. *In competitive markets with endogenous prices, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{1+E^u}{E^c}\frac{v_y \mu}{\lambda n f} + \frac{\gamma}{\lambda} x_y. \tag{10}$$

The optimal income tax formula in (10) has two terms. The first one is the standard Mirrleesian term that balances work incentives with the public authority's redistributive and budgetary motives. The role of the second term is to correct for the price externality that arises from endogenous pricing: a change in income distribution influences social welfare not only through the change of disposable income but also indirectly through the change of the price of good X. Given the externality interpretation, we call the second term the *Pigouvian term* of optimal income taxation.

Next, we analyze the properties of the Pigouvian term. We first observe that the multiplier corresponding to the budget constraint is positive, $\lambda > 0$. From (9) we find that the multiplier corresponding to the market equilibrium condition is given by

$$\gamma = \frac{S(p)(\lambda - \omega)}{S'(p) - \int (x_p + x_y x) f dn}. \tag{11}$$

From the Slutsky equation we observe that $x_p + x_y x = h_p$, where h is the compensated (Hicksian) demand function. The law of compensated demand $h_p \leq 0$ together with $S'(p) \geq 0$ imply that the denominator in the above equation is positive. Thus, multiplier γ is positive if the weight on the producer surplus is small enough, i.e., $\omega < \lambda$. In the latter case, the sign of the Pigouvian term is determined by the derivative of the demand with respect to income x_y. We obtain, therefore, a positive Pigouvian term for normal goods and negative for inferior goods. Furthermore, our assumption that the demand has the same curvature with respect to income also implies that x_y is increasing for luxury goods and decreasing for necessity goods. These observations are summarized in the following proposition.

Proposition 1. *In competitive markets with endogenous prices when the social weight on firm profits is small, the Pigouvian term is positive for normal goods and negative for inferior goods, increasing for luxury goods and decreasing for necessity goods.*

Next, we observe that the Pigouvian term is proportional to the price externality resulting from a change in income distribution. To illustrate this, let us consider an increase in the level of income from $y(n')$ to $y(n') + \phi$ for all types $n' \in [n - \delta/2, n + \delta/2]$ for some n. Using the Implicit Function Theorem the market equilibrium condition (3) implies

$$\frac{dp}{d\phi} = \frac{x_y(p, y(n)) f(n) \delta}{S'(p) - \int x_p(p, y(n)) f(n) dn}. \tag{12}$$

Observing that the denominator is positive, we obtain the following result.

Proposition 2. *In competitive markets with endogenous prices, the Pigouvian term is proportional to the price externality resulting from a local change in income distribution.*

The immediate implication of Theorem 1 is that the seminal end-point results of Sadka (1976) and Seade (1977) no longer hold with price endogeneity. In the standard model with fixed prices the optimal tax formula has only the incentive term and the transversality condition implies that the optimal marginal tax for the most productive agents is zero. Intuitively, if the marginal tax were positive, the public authority could instead impose zero tax on any income in excess of the current income of the most productive agents. Then, these agents would respond by exerting an additional effort to achieve the previously not feasible level of utility. Since the amount of tax collected would not change while the most productive agents would obtain a higher level of utility, the overall welfare would increase. With endogenous prices, however, the aforementioned tax relief would change the income distribution in the economy with implications for product prices. Therefore, the public authority cannot increase the utility of the most productive agents without influencing the rest of the agents.

The latter observation actually extends even further. Suppose that the public authority observes productivity types and can condition taxes T both on earned income z and productivity n ($\mu(n) = 0$). When prices are fixed ($\gamma = 0$) it is well-known result that the optimal outcome is lump sum taxation. It also follows from (10) noting that the latter equation reduces to zero when if $\gamma = 0$ and $\mu(n) = 0$. In the presence of price externality ($\gamma \neq 0$) effort-distorting taxation becomes superior to lump sum taxation. Intuitively, each agent does not internalize externality that it imposes on others by earning high income level, which leads to high market demand and high equilibrium prices. By imposing effort-distorting taxation the public authority decreases agent income, improves agent terms of trade, and safes on labor costs.

Corollary 1. *In competitive markets with endogenous prices, lump sum tax is not optimal.*

Price Effect Estimate

To estimate the size of the effect of endogenous prices on optimal income taxation, we turn to the US housing market. This market suits particularly well for the purpose of quantifying our results obtained in the previous section because housing costs account for 32.9% of the total household expenditures (see Consumer Expenditure Survey, 2015).[9]

To model the demand side of the housing market, we use the model of Albouy et al. (2016) who estimate a non-homothetic constant elasticity of substitution (NH-CES) utility function

[9] More precisely, Davis and Ortalo-Magné (2011) estimate that housing costs account for 37% of expenditures at the bottom and for 17% at the top quartiles of the US income distribution.

for the US housing market. If we denote the consumption of housing by x and of other goods by g, the NH-CES utility function can be written as

$$u(x,g) = \left(\frac{\eta x^{\frac{\sigma-1}{\sigma}} + \theta_1}{\theta_2 - (1-\eta) g^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{\sigma}{\beta(\sigma-1)}}, \tag{13}$$

where σ is a substitution parameter, β is a non-homotheticity parameter, η is a distribution parameter, $\theta_1 = \frac{1-\sigma-\beta\eta}{\beta\sigma}$, and $\theta_2 = \frac{1-\sigma-\beta(\eta-1)}{\beta\sigma}$.[10] This function becomes a standard CES function (Arrow et al. (1961)) when $\beta \to 0$ and Cobb-Douglas when $\sigma \to 1$. Albouy et al. (2016) calibrate the model and show that this utility function fits well the patterns of housing consumption in the US and passes tests imposed by rationality and household mobility. The parameters that most accurately fit the data are $\sigma = 2/3$, $\eta = 1/8$, $\beta = 4/3$, which yields the following utility specification

$$u(x,g) = \left(\frac{27 - 14g^{-1/2}}{2x^{-1/2} + 3}\right)^{3/2}. \tag{14}$$

Following Kanbur and Tuomala (2013) (also see Saez et al. (2012) for a review), we assume the following cost function $c(\ell) = \ell^4/4$ leading to constant labor supply elasticity equal to 1/3.

On the supply side of the housing market, we consider the standard constant price elasticity function $S(p) = sp^{\varepsilon}$, where s is a scale parameter and ε is the price elasticity of supply.[11] We calibrate scale parameter $s = 0.0021$ to match the average share of US household expenditure on housing equal to 32.9% (see Consumer Expenditure Survey, 2015). In the literature, the estimates of the price elasticity of supply ε vary significantly across countries and even across cities within the same country. In particular, Saiz (2010) show that ε highly depends on geographical and regulatory constraints within US metropolitan areas. Following his estimates for the average US metropolitan area, we consider $\varepsilon = 1.75$. In Appendix C, we also present the results for inelastic supply function with $\varepsilon = 0$ that better describes the housing supply in major UK cities,[12] and the results for a quite elastic supply function with $\varepsilon = 3$ that is closer to the estimates obtained in Green et al. (2005) and Epple and Romer (1991).

For the distribution of agent productivities $F(n)$, we consider the lognormal function $\ln(m, \sigma)$ that matches well the empirical pattern of US income. Following Kanbur and Tuomala (2013), we consider mean $m = e^{-1}$ and standard deviation $\sigma = 0.7$. In Appendix C, we also present results for the Champernowne distribution that fits better an upper tail of income distribution. Finally, we set the level of public expenditures at $R = 0$, the weight on producer surplus at $\omega = 0$, and assume that social welfare function is linear $H(u) = u$.

[10] We omit household size and neutral shifts in quality of life in agent utility as in Albouy et al. (2016).

[11] This functional form corresponds to a Cobb-Douglas production function (see, e.g., Epple and Romer (1991), Epple et al. (2010)).

[12] See "How to solve Britain's housing crisis, " *The Economist*, 2017, August 5.

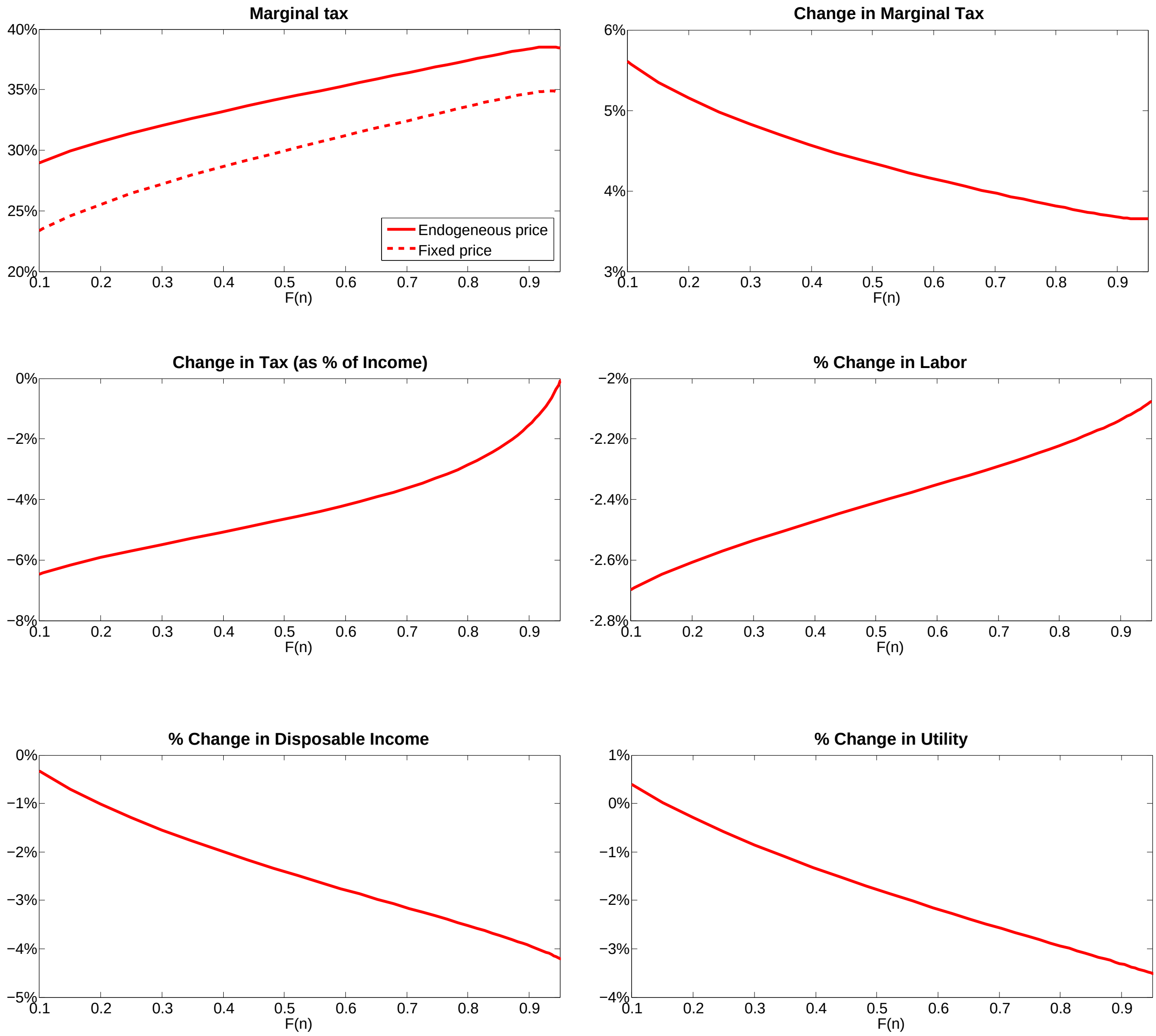

Figure 1. The graphs show the optimal marginal taxes, change in the optimal marginal tax, % change in labor supply, and % change in agent utility between the optimal tax regimes in economies with endogeneous and fixed price.

Figure 1 presents our main simulation results. The top left figure shows the optimal marginal tax for the economy with endogeneous prices (solid line) and for the economy with fixed prices (dashed line). For the latter economy we take the price of housing equal to the equilibrium price in the economy with endogeneous prices. The simulations show that the change in marginal tax $t_{endogeneous} - t_{fixed}$ (the top right figure) is positive for all income levels as for any normal good (see Proposition 1). In addition, the change in marginal tax decreases in income as for any necessity good (see Proposition 1). In response to higher marginal income taxes, we also observe a decrease in labor supply, as shown in the middle right figure.

Higher marginal taxes discourage labor supply (the middle right figure). This leads to both smaller income and smaller disposable income with an exception of the bottom 10% of the distribution (the low left figure). The existence of price externalities prompts the public authority to impose higher marginal taxes to subdue the negative effect of higher market demand on housing prices. While agents pay smaller absolute taxes (the middle left figure), the total change in agent utility is still negative for most income levels (the low right figure)

4 Oligopolistic Competition

In this section, we relax the assumption that firms are price-takers and consider instead markets with a varying degree of oligopolistic competition. In particular, we consider $M \geq 1$ firms that have the same convex cost function $K(x)$ of producing x units of good X. Denoting the inverse aggregate demand function by $p(X)$, where $X = \int x(p, y(n))f(n)dn$, we write firm i's profit as $x_i p(X) - K(x_i)$.[13] The first order condition for profit maximization can be expressed by

$$p(X) - K'(x_i) + (1+\theta)x_i \; p'(X) = 0, \tag{15}$$

where $\theta = dX/dx_i - 1$ is the conjectural variation corresponding to the firm's belief about how the competitors react to the variation in its output (we borrow this exposition from Auerbach and Hines (2001)). Parameter θ can capture a wide variety of types of oligopolistic competition. For instance, in the Cournot-Nash model we have $\theta = 0$, i.e., each firm considers the output of other firms as given, and in a competitive market we have $\theta = -1$. Various forms of Stackelberg competition correspond to various values of $\theta \in (-1, 0]$.

For subsequent analysis, it is convenient to express the market equilibrium condition in terms of market price rather than quantity. In what follows, we also limit attention to symmetric equilibria so that $x_i = X/M$. Hence, the market equilibrium condition reduces to

$$J(X, X_p, p) \equiv p - K'\left(\tfrac{X}{M}\right) + (1+\theta)\frac{X}{MX_p} = 0, \tag{16}$$

where $X_p = \int x_p(p, y(n))f(n)dn$.

To streamline our exposition, henceforth we impose a zero weight, $\omega = 0$, on firm profits. The next theorem establishes the formula of the optimal marginal income tax for oligopolistic markets. Its derivation is similar to that of Theorem 1 and, therefore, all the details are relegated to Appendix A.

Theorem 2. *In oligopolistic markets with endogenous prices, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{1+E^u}{E^c}\frac{v_y \mu}{\lambda n f} + \frac{\gamma}{\lambda}(-J_1 x_y - J_2 x_{py}), \tag{17}$$

where J_i is the partial derivative with respect to ith argument.

[13] The assumption of $x_p < 0$ ensures that the inverse aggregate demand function $p(X)$ is well defined.

As in Theorem 1, the tax formula consists of the standard Mirleesian term and the Pigouvian term. However, the Pigouvian term has now two terms reflecting the fact that the equilibrium price under oligopolistic competition depends not only on market demand but also on its slope. Note that in a competitive market when $\theta = -1$ the second part of the Pigouvian term disappears rendering the tax formula as in (10).[14] Clearly, the result of Corollary 1 that lump sum taxation is not first best also holds with oligopolistic markets because of price externalities as in the case of competitive markets.

With oligopolistic markets, we do not have a simple characterization of properties of the Pigouvian term unlike in the case of competitive markets presented in Proposition 1. Yet, we can demonstrate that the absolute value of the Pigouvian term remains proportional to the price externality resulting from a change in income distribution. Specifically, let us consider a small local increase in the levels of income from $y(n')$ to $y(n') + \phi$ for all types $n' \in [n - \delta/2, n + \delta/2]$ for some n. Taking into account the market equilibrium condition (16) and using $X_{pp} = \int x_{pp}(p, y(n)) f(n) dn$, we obtain the absolute value of price change as

$$\left| \frac{dp}{d\phi} \right| = \left| \frac{(J_1 x_y + J_2 x_{py}) f(n) \delta}{dJ/dp} \right|, \tag{18}$$

where $dJ/dp = J_1 X_p + J_2 X_{pp} + J_3$. Thus, similarly to the case of competitive markets (Proposition 2), we have

Proposition 3. *In oligopolistic markets with endogenous prices, the Pigouvian term is proportional to the price change resulting from a local change in income distribution.*

To establish the sign of the Pigouvian term, consider a situation when the public authority can propose a small unit rebate r to consumers of good X. Let us assume that the equilibrium price increase in response to the rebate is smaller than the rebate itself, which can be related to the "no tax overshifting" condition of Seade (1985) and Myles (1987). Proposition A1 in Appendix A shows that with no tax overshifting, the sign of the Pigouvian term coincides with the sign of the price change resulting from a change in income distribution.

Price Effect Estimate: Oligopolistic Competition

We now turn to the analysis of the optimal income taxation in the presence of endogeneous prices for various forms of oligopolistic competition. In particular, using again the calibrated model of US housing market due to Albouy et al. (2016) we obtain the optimal marginal income tax schedule and the size of Pigouvian term for markets with the extreme values of the conjectural variation parameter.

Let us consider an economy with $M = 2$ firms and two forms of oligopolistic competition: competitive market with $\theta = -1$ and Cournot-Nash model with $\theta = 0$. In competitive markets

[14] Note that $-J_1$ does not depend on n and, thus, can be included in Lagrange multiplier γ.

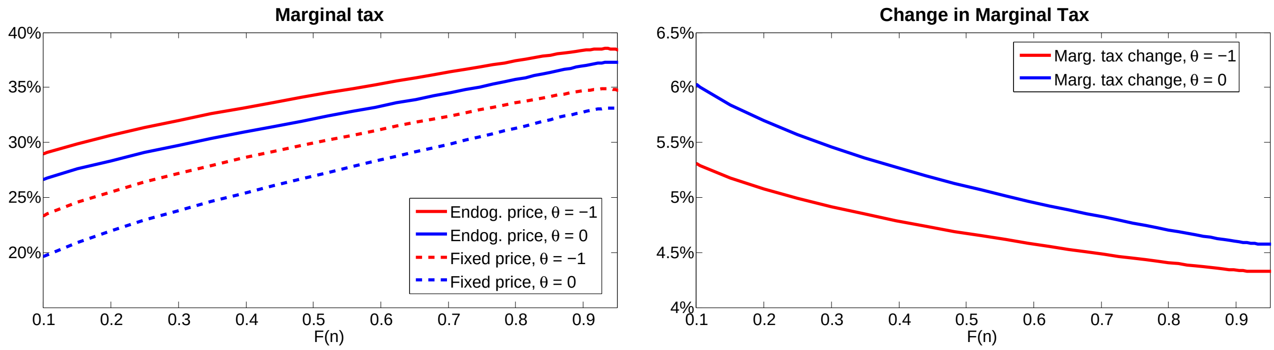

Figure 2. The graphs illustrate the optimal marginal income taxation for various income percentiles for competitive market $\theta = -1$ and Cournot-Nash model with 2 firms and $\theta = 0$. The left figure shows the optimal marginal income tax level for competitive market with endogeneous prices (thick solid red line) and fixed price (thick dashed red line) and for Cournot-Nash model with endogeneous price (thin solid blue line) and fixed price (thin dashed blue line). The right figure presents the change in the optimal marginal income tax for competitive market (thick red line) and Cournot-Nash model (thin blue line).

the firm maximization problem reduces to $p - K'(\frac{X}{M}) = 0$, where X equals market supply $S(p)$ in market equilibrium. To make sure that our analysis reduces to competitive market sturcture of Section 3 where market supply $S(p) = sp^{\varepsilon}$ with $s = 0.0021$ and $\varepsilon = 1.75$, we assume that firm's costs satisfy

$$K'(x) = \left(\frac{xM}{s}\right)^{\frac{1}{\varepsilon}}.$$

For our simulations, we again consider Albouy et al. (2016) utility (13) calibarated on the US housing market. In Appendix C, we show how one obtains indirect utility $v(p, y)$, demand $x(p, y)$, and demand slope $x_p(p, y)$ for this utility specification.

Figure 2 presents our main simulation results. The left figure shows the optimal marginal income tax schedules for competitive market $\theta = -1$ with endogeneous prices (thick solid red line) and fixed price (thick dashed red line) and for Cournot-Nash model $\theta = 0$ with endogeneous price (thin solid blue line) and fixed price (thin dashed blue line). The right figure presents the Pigouvian term (the difference between endogenous price and fixed price structures) for competitive market (thick red line) and Cournot-Nash model (thin blue line).

The graphs illustrate that the Pigouvian term is higher in markets where firms have larger market power (larger θ). For these markets, the government needs to ensure a more equal society to guarantee a lower price in the economy and that more agents have access to the oligopolistic product. At the same time, the level of the marginal income tax itself decreases with additional market power. This happens because an additional market power leads to a larger distortion from the efficient production level. To offset this distortion the government needs to increase the market demand using the tax schedule. This decreases the government's redistributional concerns leading to a smaller level of the optimal marginal income tax.

5 Commodity and Profit Taxation

The main result of this paper is the emergence of an additional term in the optimal income tax formula that is aimed at correcting for price externality arising from changes in income distribution. In this section, we study the question how robust our main result is in the presence of other forms of taxation – commodity and profit, in particular. Arguably, and similarly to the Pigouvian term, commodity and profit taxation could also play a role in offsetting price externalities arising from changes in income distribution.

Before we present our main findings, we make the following technical assumption imposed henceforth.

Assumption 1. *Let the Marshallian demand function $x(p, y)$ satisfy that $x_{py} \neq \rho x_y + \nu$, where $\rho, \nu \in \mathbb{R}$.*

We require the cross derivative x_{py} of the Marshallian demand function be not a linear transformation of its partial derivative x_y or, in other words, the Marshallian demand and its price slope follow different dynamics with respect to income.

First, we consider the case of commodity taxation that takes the form of an excise tax imposed on good X. Let the tax be paid by producers and resultant revenues be used to fund public expenditures. We show that

Theorem 3. *With optimal commodity taxation the Pigouvian term of optimal income taxation is (i) zero if the market is competitive and (ii) non-zero if the market is oligopolistic.*

As shown earlier, in competitive markets the price externality and, accordingly, Pigouvian term are proportional to the change in demand x_y resulting from a change in income. We observe that the additional amount paid in commodity tax that arises from additional income is also proportional to the change in demand x_y. Thus, we argue that it is possible to design the excise tax such that it fully corrects for the price externality of income taxation, thus, rendering the Pigouvian term obsolete. Intuitively, our finding suggests that the optimal solution to the price externality problem is not to restrict workers from earning too much but make them pay for the externality they create (and use the additional revenue for redistributive or budgetary purposes).

However, commodity taxation can only partially correct for price externality in oligopolistic markets because price externality arises not only from changes in demand but also from oligopolistic competition. This implies that in the economy with oligopolistic markets income taxation continues to play a role in correcting for price externality.

Next, we consider the problem of optimal income taxation in the presence of profit taxation. In our model, the optimal profit tax is 100%, which follows from the linearity of profits in the maximization problem and is in line with the related theoretical literature on taxation (Diamond and Mirrlees (1971); Atkinson and Stiglitz (1976)). In our exposition of the effects of profit taxation, we abstract from commodity taxation which, as argued by Atkinson and Stiglitz (1976), is superfluous under 100% profit taxation. The next result shows that with profit taxation we obtain the same conclusions as in Theorem 3.

Theorem 4. *With 100% profit taxation the Pigouvian term of optimal income taxation is (i) zero if the market is competitive and (ii) non-zero if the market is oligopolistic.*

In competitive markets 100% profit taxation eliminates the Pigouvian term from optimal income taxation. This finding can be linked with the production efficiency result of Diamond and Mirrlees (1971). Their result says that optimal taxation should not distort aggregate production efficiency, which is not the case with an effort-distorting Pigouvian term. With 100% profit taxation, it is welfare enhancing for the public authority to maximize total produce and redistribute it in the population. Importantly, however, with oligopolistic markets the Pigouvian term does not disappear even with 100% profit taxation. There is no production efficiency under oligopolistic pricing which can imply that better work incentives will translate into disproportionately higher prices rather than more goods produced. In this case, it also remains optimal to restrain work incentives by the means of additional income taxation.[15]

Finally, from the practical perspective we note that 100% profit taxation is more a theoretical construct than an accurate description of the reality. But absent 100% profit taxation, there is also room for a non-zero Pigouvian term in competitive markets even when commodity taxes are allowed. In particular, when extended to multiple goods with variable marginal costs of production our analysis suggests differentiated commodity taxation with different goods taxed in accordance with the extent of price externality. If such differentiation is infeasible, the solution to price externalities is the extension of income taxation with the Pigouvian term that aggregates all price externalities over all goods.

6 Conclusion

This paper is centered on the observation that an income tax policy can affect people's welfare not only through changes in levels of disposable income but also indirectly through changes in consumer prices. In particular, income distribution determines aggregate demand for consumption goods and, therefore, can affect price formation in the markets. Consequently, a tax policy with implications for income distribution can create price externality, neglecting which may render the welfare assessment of the policy biased.

We demonstrate, theoretically and empirically, that the effect of price externality on optimal income taxes is significant. In theoretical analysis, this effect appears in the form of an additional term in the optimal marginal income tax formula. This term is referred to as *Pigouvian* to reflect that it is proportional to the price externality that a marginal change in

[15]The inefficiency of oligopolistic pricing has also an effect on optimal commodity taxation as discussed in Myles (1987) and Auerbach and Hines (2001).

income creates. We show that the presence of this term is robust to different market structures. Furthermore, in oligopolistic markets it is also robust to profit and commodity taxation. An immediate implication of our finding is that the famous outcome of zero tax rate for the top income bracket is no longer optimal with price externality. But more strikingly, our analysis also demonstrates that in the presence of price externalities effort-distortive income taxation can be more efficient than lump sum taxation. Empirically, we estimate the price effect of the US housing market on marginal income tax rates, which is 4-5% for most income levels.

Policy implications that arise form this work go beyond optimal income taxation. We argue that the welfare assessment of any policy with an effect on income distribution is biased unless the policy's price effects are accounted for. For instance, this argument pertains to social policies related to subsidies, welfare benefits, pensions but it even pertains to minimum wage regulation.

Appendix A – Proofs

Proof of Theorem 1. The statement follows from the argument presented in the main text before Theorem 1. Below, we derive the elasticities of uncompensated and compensated labor supply, E^u and E^c, respectively.

Observing that $y'(n) = z'(n)(1-t)$, we can express the individual utility maximization condition as

$$v_y(p,y)\ n(1-t) - c_\ell(\ell) = 0 \tag{A.1}$$

or, using $y = w\ell + \overline{y}$, where $w = n(1-t)$ is a net wage rate and $\overline{y}$ is non-labor income,

$$v_y(p, w\ell + \overline{y})\ w - c_\ell(\ell) = 0. \tag{A.2}$$

Implicitly differentiating (A.2) we obtain

$$\frac{\partial \ell}{\partial w} = -\frac{v_{yy} w\ell + v_y}{v_{yy}w^2 - c_{\ell\ell}} = \frac{v_{yy} w\ell + v_y}{c_{\ell\ell} - v_{yy}w^2}.$$

Then, the elasticity of uncompensated labor supply $E^u = \partial\ell/\partial w\ (w/\ell)$ is equal to

$$E^u = \frac{v_{yy} w\ell + v_y}{c_{\ell\ell} - v_{yy}w^2}\ \frac{w}{\ell} = \frac{v_{yy}(c_\ell/v_y)^2 + c_\ell/\ell}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2},$$

where we use $w = c_\ell/v_y$ from (A.2).

To obtain the elasticity of compensated labor supply, E^c, we employ the Slutsky equation $E^c = E^u - E^m$, where $E^m = w(\partial\ell/\partial\overline{y})$ is the income effect parameter:

$$E^m = w\ \frac{\partial \ell}{\partial \overline{y}} = \frac{v_{yy}(c_\ell/v_y)^2}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2}.$$

Thus, the elasticity of compensated labor supply, E^c, is given by

$$E^c = E^u - E^m = \frac{v_{yy}(c_\ell/v_y)^2 + c_\ell/\ell}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2} - \frac{v_{yy}(c_\ell/v_y)^2}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2} = \frac{c_\ell/\ell}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2}.$$

Finally, we observe that $1 + \ell c_{\ell\ell}/c_\ell = (1+E^u)/E^c$.

Proof of Proposition 1. Regarding normal and inferior goods, the statement of the proposition follows from the argument presented in the main text before Proposition 1. Next, we show that the assumption that x_{yy} does not change its sign implies $x_{yy} \geq 0$ for luxury goods. Consider $x_{yy}(p,y)$ at $y = 0$, which can be expressed as the limit

$$x_{yy}(p,0) = \lim_{h\to 0} \frac{x(p,2h) - 2x(p,h) + x(p,0)}{h^2}$$

or, observing that $x(p,0)=0$,

$$x_{yy}(p,0)=\lim_{h\to 0}\frac{x(p,2h)-2x(p,h)}{h^2} \tag{A.3}$$

By the definition of the luxury good, i.e., $p(x(p,y))/y$ is increasing in y, we have that

$$\frac{x(p,2h)}{2h}\geq\frac{x(p,h)}{h}$$

or $x(p,2h)\geq 2x(p,h)$. Hence, the limit in A.3 is non-negative, which implies that $x_{yy}(p,0)\geq 0$. Then, by the assumption that x_{yy} does not change its sign we have $x_{yy}\geq 0$ for all y, which proves the proposition for luxury goods. Using the analogous argument, one can show that the proposition also holds for necessity goods.

Proof of Proposition 2. The statement follows from the argument presented in the main text.

Proof of Corollary 1. With observable productivity, the public authority's tax policy takes the form of $T(z,n)$, which conditions tax payments on gross income z and type n. As before, let $t(z,n)=\partial T/\partial z$ denote the marginal income tax. An agent's individual maximization problem $\max_z v(p,z-T(z,n))-c(z/n)$ implies $v_y(1-t(z,n))-c_\ell/n=0$ or $t(z,n)=1-c_\ell/(nv_y)$ as in Theorem 1. The observability of types removes the incentive compatibility constraint from the public authority's problem which otherwise remains intact. Setting $\mu(n)=0$ in (10) renders the optimal marginal income tax formula, from which we obtain that $t(z,n)\neq 0$ if $\gamma\neq 0$ or lump sum taxation is not optimal.

Proof of Theorem 2. In oligopolistic markets, the public authority's problem is

$$\max\int H(u(n))f(n)dn$$

$$s.t.\begin{cases}\frac{du}{dn}=\frac{\ell c_\ell}{n} & (\mu(n),\ \text{incentice compatibility})\\ \int\left[n\ell(n)-y(n)\right]f(n)dn\geq R & (\lambda,\ \text{resource constraint})\\ J(X,X_p,p)=0 & (\gamma,\ \text{oligopolistic market equilibrium})\\ X-\int x(p,y(n))f(n)dn=0 & (\alpha_0,\ \text{market demand})\\ X_p-\int x_p(p,y(n))f(n)dn=0 & (\alpha_1,\ \text{market demand slope})\end{cases}$$

with Largrange multipliers introduced next to their corresponding constraints. The Lagrangian of this problem is given by

$$\mathcal{L} = \int [(H(u(n)) + \mu(n)(u'(n) - \ell(n)c_\ell(\ell(n))/n) + \lambda(n\ell(n) - y(n) - R) + \gamma J(X, X_p, p)$$

$$+ \alpha_0(X - x(p,y)) + \alpha_1(X_p - x_p(p,y)))f(n)]dn$$

Using $y = q(u,\ell,p)$ with $q_u = 1/v_y,\ q_\ell = c_\ell/v_y$, and $q_p = x$, we obtain the first-order conditions as

$$u(n) : \left(H'(u) - \frac{\lambda + \alpha_0 x_y + \alpha_1 x_{py}}{v_y}\right) f - \mu'(n) = 0, \tag{A.4}$$

$$\ell(n) : \left(\lambda n - \frac{c_\ell}{v_y}(\lambda + \alpha_0 x_y + \alpha_1 x_{py})\right) f - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{A.5}$$

$$p : -\lambda X + \gamma J_3 - \int (\alpha_0(x_p + x_y x) + \alpha_1(x_{pp} + x_{py}x))f(n)dn = 0, \tag{A.6}$$

$$X : \alpha_0 + \gamma J_1 = 0, \tag{A.7}$$

$$X_p : \alpha_1 + \gamma J_2 = 0. \tag{A.8}$$

where $J_1,\ J_2$, and J_3 are partial derivatives of J with respect to its first, second, and third arguments respectively. Using equations (A.7) and (A.8) we obtain

$$u(n) : \left(H'(u) - \frac{\lambda - \gamma J_1 x_y - \gamma J_2 x_{py}}{v_y}\right) f - \mu'(n) = 0, \tag{A.9}$$

$$\ell(n) : \left(\lambda n - \frac{c_\ell}{v_y}(\lambda - \gamma J_1 x_y - \gamma J_2 x_{py})\right) f - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{A.10}$$

$$p : -\lambda X + \gamma \left(J_1 X_p + J_2 X_{pp} + J_3 + \int (J_1 x_y + J_2 x_{py}) x f(n)dn\right) = 0, \tag{A.11}$$

where we denote $X_{pp} = \int x_{pp}(p, y(n))f(n)dn$. Following the same steps when deriving the optimal income taxes in Theorem 1, we obtain the marginal income tax formula given in (17). Lastly, equation (A.9) gives the expression for multiplier $\mu(n) = \int_n^{\overline{n}} \left(\frac{\lambda+\gamma(-J_1x_y - J_2x_{py})}{v_y} - H'(u)\right) f(n)dn$.

Proposition A1. *Suppose that in response to a small unit rebate r given for good* X *its equilibrium price p increases by less than the rebate itself. The Pigouvian term for agent type n is positive if and only if the change in price in response to an increase in income levels in the neighborhood of $y(n)$ is positive.*

Proof. Let us denote $dJ/dp = J_1 X_p + J_2 X_{pp} + J_3$. We first show that the rebate condition implies that the sign of γ coincides with the sign of dJ/dp. We can rewrite the first-order condition (A.11) as

$$\frac{dJ}{dp}\left(1+\frac{\int(J_1x_y+J_2x_{py})xf(n)dn}{dJ/dp}\right)=\frac{\lambda}{\gamma}X(p). \tag{A.12}$$

Since $\lambda X(p)$ is positive the sign of γ coincides with the sign of the leftside, which we denote by A. Now consider a small unit rebate r, which increases an agent's income by $rx(p,y)$. Applying the Implicit Function Theorem to the equilibrium condition $J(X,X_p,p)=0$, we obtain that the effect of rebate on the equilibrium price is given by

$$\left.\frac{dp}{dr}\right|_{r=0}=-\frac{\int(J_1x_y+J_2x_{py})xf(n)dn}{dJ/dp}. \tag{A.13}$$

Taking into account the proposition hypothesis that $\left.\frac{dp}{dr}\right|_{r=0}<1$, we conclude from (A.33) and (A.13) that the sign of γ coincides with the sign of dJ/dp.

We now consider a change in income level from $y(n')$ to $y(n')+\phi$ for all types $n'\in[n-\delta/2,n+\delta/2]$ for some n. The resultant change in price is given by

$$\frac{dp}{d\phi}=-\frac{(J_1x_y+J_2x_{py})f(n)\delta}{dJ/dp}=\frac{-\gamma(J_1x_y+J_2x_{py})f(n)}{\gamma dJ/dp}\delta. \tag{A.14}$$

Since the sign of γ coincides with sign of (dJ/dp), which implies a positive denominator, the numerator – Pigouvian term for agent with productivity n – is positive if and only if $dp/d\phi>0$.

Proof of Theorem 3. Let b be an excise tax imposed on good X. Then, the equilibrium condition for market price changes to

$$J(X,X_p,p)-b=0, \tag{A.15}$$

where $J(X,X_p,p)=p-K'\left(\frac{X}{M}\right)+(1+\theta)\frac{X}{MX_p}$ as defined in (16). The firm profits are given by $\Pi(X,p,b)=(p-b)X-MK(\frac{X}{M})$. We impose one more constraint ensuring that firms receive non-negative profit

$$\Pi(X,p,b)\geq 0. \tag{A.16}$$

The Lagrangian of the public authority's problem is given by

$$\begin{aligned}\mathcal{L}=\int&[(H(u(n))+\lambda(n\ell(n)+bX-y(n)-R)+\gamma(J(X,X_p,p)-b)\\&+\alpha_0(X-x(p,y))+\alpha_1(X_p-x_p(p,y)))f(n)\\&+\alpha_2\Pi(X,p,b)+\mu(n)(u'(n)-\ell(n)c_\ell(\ell(n))/n)]dn,\end{aligned}$$

where multiplier α_2 corresponds to condition (A.16), with the other multipliers defined earlier.

The first-order conditions are

$$u(n):\left(H'(u)-\frac{\lambda+\alpha_0x_y+\alpha_1x_{py}}{v_y}\right)f-\mu'(n)=0, \tag{A.17}$$

$$\ell(n):\left(\lambda n-\frac{c_\ell}{v_y}(\lambda+\alpha_0x_y+\alpha_1x_{py})\right)f-\mu(n)(c_\ell+\ell c_{\ell\ell})/n=0, \tag{A.18}$$

$$p:-\lambda X+\gamma J_3+\alpha_2\Pi_2-\int(\alpha_0(x_p+x_yx)+\alpha_1(x_{pp}+x_{py}x))f(n)dn=0, \tag{A.19}$$

$$b:\lambda X-\gamma+\alpha_2\Pi_3=0, \tag{A.20}$$

$$X:\alpha_0+\lambda b+\gamma J_1+\alpha_2\Pi_1=0, \tag{A.21}$$

$$X_p:\alpha_1+\gamma J_2=0, \tag{A.22}$$

where J_i and Π_i denote partial derivatives with respect to their argument i, respectively. Taking into account that $J_3=1$ and $\Pi_2=-\Pi_3$, we use the last three conditions to transform the first three conditions as

$$u(n):\left(H'(u)-\frac{\lambda-(\gamma J_1+\lambda b+\alpha_2\Pi_1)x_y-\gamma J_2x_{py}}{v_y}\right)f-\mu'(n)=0, \tag{A.23}$$

$$\ell(n):\left(\lambda n-\frac{c_\ell}{v_y}(\lambda-(\gamma J_1+\lambda b+\alpha_2\Pi_1)x_y-\gamma J_2x_{py})\right)f-\mu(n)(c_\ell+\ell c_{\ell\ell})/n=0, \tag{A.24}$$

$$p:\int((\gamma J_1+\lambda b+\alpha_2\Pi_1)(x_p+x_yx)+\gamma J_2(x_{pp}+x_{py}x))\,f(n)dn=0. \tag{A.25}$$

Condition (A.25) implies that the optimal excise tax equals

$$b=-\frac{\alpha_2\Pi_1}{\lambda}-\frac{\gamma\int(J_2(x_{pp}+x_{py}x)+J_1(x_p+x_yx))f(n)dn}{\lambda\int(x_p+x_yx)f(n)dn}. \tag{A.26}$$

First, we observe that $\gamma\neq0$. Otherwise, condition (A.26) implies that $b\leq0$ because of $\Pi_1\geq0$ and $\alpha_2\geq0$. But $b\leq0$ implies that $\alpha_2=0$ because of positive profits $\Pi>0$, which in turn implies that $\alpha_2=0$. Then the first-order condition (A.20) is violated. We arrive at a contradiction.

Condition (A.24) implies that the Pigouvian term equals to

$$\frac{(-\gamma J_1-\lambda b-\alpha_2\Pi_1)x_y-\gamma J_2x_{py}}{\lambda}. \tag{A.27}$$

Condition (A.26) then implies that the Pigouvian term can be written as

$$\frac{\gamma J_2}{\lambda}\left(\frac{\int(x_{pp}+x_{py}x)f(n)dn}{\int(x_p+x_yx)f(n)dn}-x_{py}\right). \tag{A.28}$$

We notice that $J_2=0$ for competitive markets ($\theta=-1$), which yields a zero Pigouvian term proving part (i) of the theorem. But for $\theta>-1$ we have that the derivative $J_2\neq0$. Furthermore, the expression in the brackets cannot be equal to zero for all n because x_{py} is a non-constant function as implied by Assumption 1. Hence, the Pigouvian term does not reduce to 0 in oligopolistic markets with commodity taxation.

Proof of Theorem 4. With 100% profit taxation, the Lagrangian of the public authority's problem is given by

$$\mathcal{L} = \int [(H(u(n)) + \lambda(n\ell(n) + pX - MK(\tfrac{X}{M}) - y(n) - R) + \gamma J(X, X_p, p)$$
$$+ \alpha_0(X - x(p, y)) + \alpha_1(X_p - x_p(p, y)))f(n)$$
$$+ \mu(n)(u'(n) - \ell(n)c_\ell(\ell(n))/n)]dn.$$

The first-order conditions with respect to p and X are

$$p : \gamma J_3 - \int (\alpha_0(x_p + x_y x) + \alpha_1(x_{pp} + x_{py}x))f(n)dn = 0, \tag{A.29}$$
$$X : \alpha_0 + \lambda(p - K'(\tfrac{X}{M})) + \gamma J_1 = 0, \tag{A.30}$$

whereas those with respect to $u(n)$, $\ell(n)$, and $X_p(p)$ remain as in (A.17), (A.18), and (A.22), respectively. The Pigouvian term remains the same as in Theorem 2 :

$$\frac{-\gamma(J_1 x_y + J_2 x_{py})}{\lambda}. \tag{A.31}$$

Observing that $p - K'(\frac{X}{M}) = -(1+\theta)\frac{X}{MX_p}$ from (16), we obtain from (A.30) that the multiplier α_0 is equal to

$$\alpha_0 = \frac{\lambda(1+\theta)X(p)}{MX_p(p)} - \gamma J_1. \tag{A.32}$$

Thus, we can express the first-order condition (A.29) as

$$\gamma \int (J_3 + J_1(x_p + x_y x) + J_2(x_{pp} + x_{py}x))f(n)dn = \frac{\lambda(1+\theta)X}{MX_p} \int (x_p + x_y x)f(n)dn. \tag{A.33}$$

If $\theta = -1$ (competitive market) condition (A.33) implies that $\gamma = 0$ and, thus, a zero Pigouvian term, which proves part (i) of the theorem. With $\theta > -1$ (oligopolistic market), we respectively obtain $\gamma \neq 0$. In addition, conditions (A.22) and (A.32) imply that at least one of the multipliers α_0 or α_1 must be non-zero. Since by Assumption 1 we have that x_{py} is not a linear transformation of x_y, the Pigouvian term must be non-zero, which proves the theorem.

Appendix B – General Equilibrium Model

In this section, we show that our model of competitive markets can be supported with a labor market and a consumer's utility maximization problem. In particular, we consider two competitive industries: one producing the numeraire good G and the other producing good X. We label these industries as G and X respectively. We assume that agents can earn wage w for effective labor hours supplied (i.e., $n\ell(n)$) in both industries, the price for the numeraire good is normalized to $p_g = 1$, and the price for good X equals to p.

Industry G has a homogeneous of degree one production technology $F_g(L_g) \equiv L_g$, where L_g is the amount of labor used in production of good G. Since the price of numeraire good is normalized to 1, the profit maximization condition implies that $w = p_g = 1$, *zero profits*, and *any level of the equilibrium labor demand* L_g^d in industry G.

Industry X has a production technology with decreasing returns to scale $F_x(L_x)$, e.g., $F_x(L_x) = AL_x^a$, where L_x is the amount of labor supplied and $0 < A$, $0 < a < 1$ are constants. Hence, firm profit maximization problem

$$\max_{L_x} p \cdot F_x(L_x) - wL_x$$

The solution to this maximization problem leads to the equilibrium labor demand L_x^d in industry X. Taking into account that $w = 1$ for the production function mentioned we have $L_x^d(p) = (aAp)^{\frac{1}{1-a}}$. The equilibrium market supply of good X then equals $S_x(p) = pF(L_x^d(p))$. We also assume that firms spend all their profits $\Pi_x(p) = pS_x(p) - L_x^d(p)$ on the numeraire good G. The government also spends all its revenue R on numeraire good G.

On the demand side of the economy, we assume that agent's preferences can be summarized by utility function $u(x, g) - c(\ell)$, where (x, g) is the amount of good X and the numeraire G consumed by the agent. Utility u is a continuous function representing locally non-satiated preferences.

Consider an agent with productivity n who works ℓ hours. Taking into account that equilibrium wage $w = p_g = 1$ and tax schedule $T(n\ell)$ her income equals $n\ell - T(n\ell)$. Hence, agent's maximization problem is

$$\max_{x,\,g,\,\ell} u(x, g) - c(\ell) \qquad \text{(B.1)}$$
$$s.t. \;\; p \cdot x + g \leq n\ell - T(n\ell)$$

The solution to the above problem is labor supply $\ell^*(n, p)$ and consumption bundle $(x^*(n, p), g^*(n, p))$. Overall, aggregate labor supply and consumer demand equal

$$L^s(p) = \int n\ell^*(n, p)f(n)dn, \quad X(p) = \int x^*(n, p)f(n)dn, \quad G(p) = \int g^*(n, p)f(n)dn.$$

The economy must satisfy three market clearing conditions:

$$S_x(p) = X(p) \tag{B.2}$$

$$S_g(p) = G(p) + \Pi_x(p) + R \tag{B.3}$$

$$L^s(p) = L_x^d(p) + L_g^d(p), \tag{B.4}$$

where the market clearing condition (B.3) for numeraire G demands that the market supply equals the market demand for G plus profits of industry X and government spending R.

Let us show that condition (B.2) is the only one that we should consider in optimal income taxation problem. Since any level $L_g^d(p)$ satisfies the maximization problem of firms producing numeraire good G (see above), we are free to choose $L_g^d(p) = L^s(p) - L_x^d(p)$ to clear the labor market. Taking into account that $S_g(p) = L_g(p)$ and $\Pi_x(p) = pS_x(p) - L_x^d(p)$ condition (B.3) can be equivalently rewritten as

$$L_g^d(p) + L_x^d(p) = G(p) + pS_x(p) + R$$

Given conditions (B.2) and (B.4) this is equivalent to

$$\int n\ell^*(n,p)f(n)dn = G(p) + pX(p) + R.$$

This condition follows from the budget constraint of agent's maximization problem (B.1) when the government spending constraint is satisfied as equality $\int T(n\ell^*(n,p))f(n)dn = R$ (as we assume). Overall, the only independent market clearing condition that we should take into account in the optimal income problem is (B.2) – the market clearing condition for good X.

Appendix C US Housing Market

In this appendix, we present some additional simulation results for the US housing market that we used in the main text to quantitatively estimate the size of price effect on optimal income taxation. Our calibrations solve the following problem

$$\max_{p,\,y(n),\,l(n)} \int \left(v(p, y(n)) - c(\ell(n))\right) f(n)dn \tag{C.1}$$

$$s.t. \begin{cases} \int \left(n\ell(n) - y(n)\right) f(n)dn = R. \\ v(p, y(n)) - c(\ell(n)) \geq v(p, y(n-1)) - c((n-1)\ell(n-1)/n) \\ S(p) = \int x(p, y(n)) f(n)dn \end{cases}$$

where we consider only adjacent and downward binding incentive compatibility constraints because agent's utility function satisfies the single-crossing condition (see Mirrlees (1976)). As described in the main text, to obtain the indirect utility function we consider a non-homothetic constant elasticity of substitution (NH-CES) utility function of Albouy et al. (2016)

$$u(x, g) = \left(\frac{27 - 14g^{-1/2}}{2x^{-1/2} + 3}\right)^{3/2}.$$

The corresponding expenditure function equals to

$$e(p, u) = \frac{4(p^{1/3} + 7^{2/3}u^{4/9})^3}{9(-9 + u^{2/3})^2}.$$

Hence, the indirect utility function $v(p, y)$ is implicitly determined by the following equation

$$y = \frac{4(p^{1/3} + 7^{2/3}v(p, y)^{4/9})^3}{9(-9 + v(p, y)^{2/3})^2}.$$

We use this equation together with the resource constraint, incentive compatibility constraint, and the market equilibrium constraint in maximization problem (C.1). Following Kanbur and Tuomala (2013), we also assume that agent costs equal $c(\ell) = \ell^4/4$.

Section 3 in the main text presents our simulation results for competitive market with supply function $S = sp^{\varepsilon}$, where $\varepsilon = 1.75$ corresponds to the price elasticity of the average US metropolitan area (Saiz (2010)) and $s = 0.0021$ is a constant calibrated to match the average share of US housing expenditure of 32.9% in equilibrium (see Albouy et al. (2016)). Since the housing supply elasticity differs widely for various countries and regions, we reestimate the change in optimal income tax for a median citizen for additional elasticity parameters. Table 1 presents our results for perfectly inelastic supply function $\varepsilon = 0$, which describes better a short-run housing supply or the housing supply in major UK cities (The Economist, 2017, Aug. 5), and a more elastic supply function with $\varepsilon = 3$, which is closer to the estimates of the price

	$\varepsilon = 0$	$\varepsilon = 1.75$	$\varepsilon = 3$
Δt	17%	4.5%	3%

Table 1: The change in optimal marginal income tax Δt between endogeneous and fixed price regimes for a median citizen for various elasticities of housing supply.

elasticity of US housing supply obtained by Green et al. (2005) and Epple and Romer (1991).[16] The results suggest that the price effect could be significantly higher for markets with more inelastic supply functions.

In addition to extending our results to various forms of housing supply elasticities, we consider an alternative distribution of agent abilities. The lognormal distribution does not match well the upper tail of the income distribution (at least for the US data). As an alternative, we consider the Champernowne distribution that could be a better approximation of the income distribution for some countries and approaches a Pareto distribution for the upper tail of income. The probability density function of the Champernowne distribution is

$$f(n) = \frac{m^{\theta} n^{\theta - 1}}{(m^{\theta} + n^{\theta})^2}$$

where θ is a shape parameter and m is a scale parameter. We consider parameters $\theta = 3$ and $m = 2$ that fit well the US data (see Kanbur et al., 2013).

Figure 3 presents our results. Qualitatively the results are quite similar to the simulation results using the lognormal distribution. The change in marginal tax between endogeneous and fixed price economies is positive for all agents, equals around 6% for a median citizen, and decreasing with agent abilities (as well as agent's income). The absolute level of tax as percentage of income did not change compared to negative change for the case of lognormal distribution. The change in labor supply, disposable income, and agent utility exhibit similar patterns to the case of the lognormal distribution.

[16]For both elasticities we calibarated parameter s to match the average share of US housing expenditure of 32.9% in equilibrium ($s = 0.07$ for $\varepsilon = 0$, $s = 0.0021$ for $\varepsilon = 1.75$, and $s = 0.00015$ for $\varepsilon = 3$).

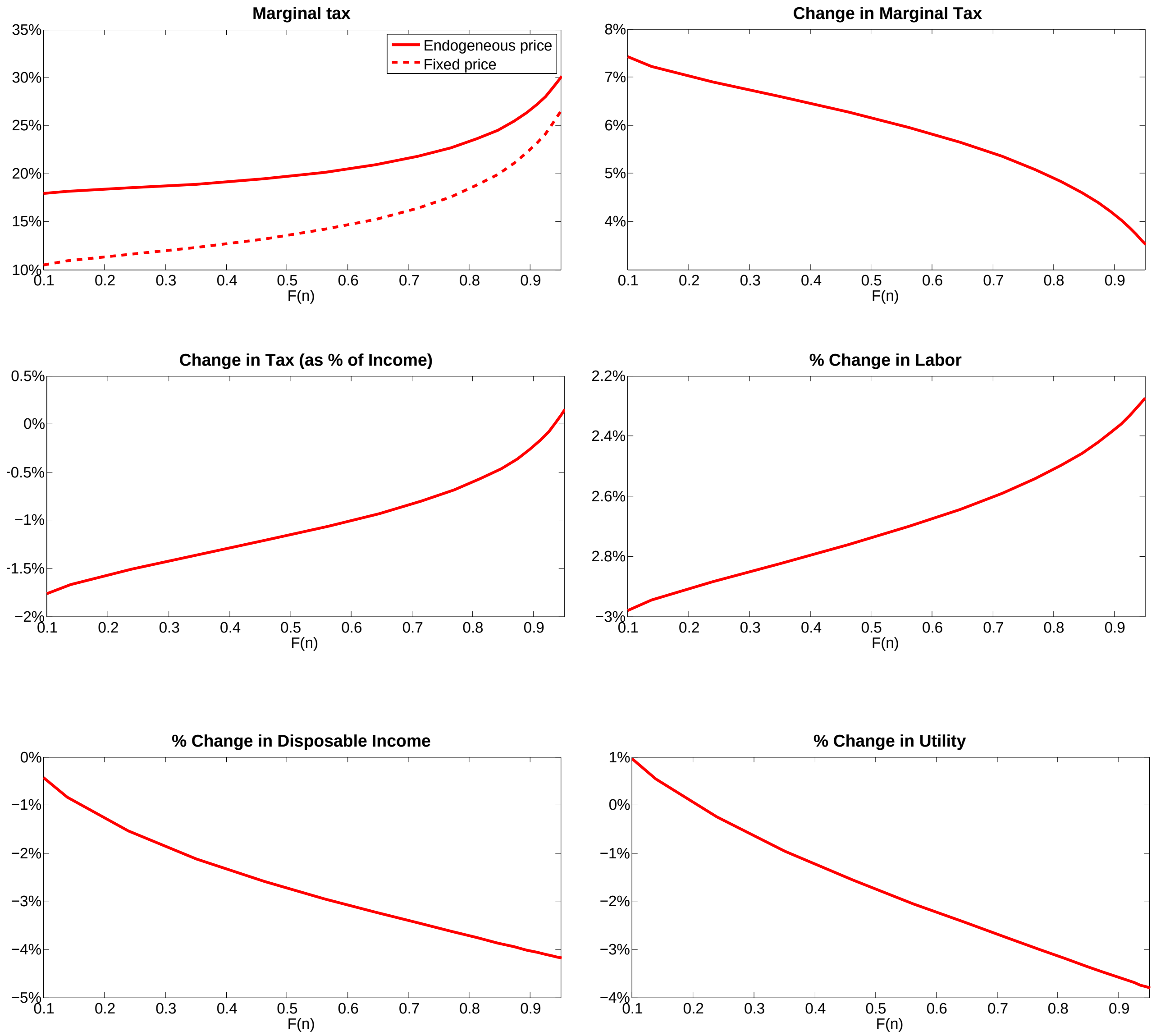

Figure 3. The figure presents the results of our simulations for Champernowne distribution of agent abilities. The graphs show the optimal marginal taxes, change in the optimal marginal tax, % change in labor supply, and % change in agent utility between the optimal tax regimes in economies with endogeneous and fixed price.

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