Anticipated expansions of life expectancy and their long-run growth effects[3]

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Endogenous growth model

Paul M. Romer (1990)
Endogenous Technological Change
Journal of Political Economy 98(5), Part 2: S71-S102

Fixed number of Ramsay consumers with discounted preferences

$$\max_{C(\cdot)} \int_0^\infty e^{-\rho t} U(C(t))dt, \qquad u(C) = \frac{C^{1-\theta}}{1-\theta}$$

Endogenous growth model with OLG

Klaus Prettner (2013)
Population aging and endogenous economic growth
Journal of Population Economics 26(2), pp 811–834

Klaus Prettner, Timo Trimborn (2017)
Demographic Change and R&D-based Economic Growth
Economica 84, pp 667–681

Agnieszka Gehringer, Klaus Prettner (2017)
Longevity and technological change
Macroeconomic Dynamics

Blanchard (1985) overlapping generations
with "perpetual youth" (constant mortality rate μ)

$$\max_{C(\cdot)} \int_{\tau}^{\infty} e^{-(\rho+\mu)t} U(C(t))dt, \qquad u(C) = \ln C$$

Endogenous growth model with OLG

Main theoretical prediction

Intuitively, a decrease in the rate of mortality implies that households live longer and therefore they discount the future less heavily. As a consequence, aggregate savings rise, exerting downward pressure on the long-term market interest rate. Since the expected profits of R&D investments are discounted with the market interest rate, the profitability of R&D rises. This implies that more resources are devoted to R&D activities with a positive impact upon technological progress and productivity growth.

Main extention

We assume that fertility and mortality rates are time dependent in general equilibrium.

Demographic dynamics with "rejuvenation" I

Mortality equation for cohort born at τ

$$\frac{\partial}{\partial t} n(\tau, t) = -\mu(t)\, n(\tau, t), \qquad \forall t \geq \tau,$$

Renewal equation

$$n(t, t) = \int_{-\infty}^{t} n(\tau, t)\, \beta(t) d\tau, \qquad \forall t \geq 0,$$

Initial conditions

$$n(\tau, 0) = n_0(\tau), \qquad \forall \tau < 0,$$

$\mu(t)$ and $\beta(t)$ are time-dependent mortality and fertility coefficients

Demographic dynamics with "rejuvenation" II

Number of agents

$$N(t) := \int_{-\infty}^{t} n(\tau, t) d\tau$$

Population dynamics

$$\frac{\dot{N}(t)}{N(t)} = \beta(t) - \mu(t), \quad N(0) = \int_{-\infty}^{0} n_0(\tau) d\tau,$$

Age profile

$$n(\tau, t) = e^{-\int_{\tau}^{t} \mu(\theta) d\theta} N(\tau)\, \beta(\tau).$$

Population aging: shock or not?

When both mortality and fertility decrease at the same value, when the difference $\beta(t) - \mu(t)$ is unchanged so is the dynamics of N, while the age profile n is shifting to older ages. This is called *population aging* – the issue for many industrialized countries.

Assumtion that fertility and mortality are time dependent in general equilibrium model, allows to study the effect of predicted population aging on economic growth.

Agent problem I

We assume that an agent born at time τ maximizes her discounted life-time utility with respect to $c(\tau, t)$ for $t \in [\tau, \infty)$

$$u(\tau) \equiv \int_{\tau}^{\infty} e^{-\rho(t-\tau)-\int_{\tau}^{t}\mu(\theta)d\theta} \log[c(\tau, t)]\, dt,$$

where $c(\tau, t)$ denotes consumption at time t of the agent along with her children.

Agent problem II

Each agent from cohort τ is endowed at time t with one unit of labor, which she inelastically supplies on the labor market to earn the going wage rate $w(t)$. We also assume that individuals are insured against the risk of dying with positive assets by a fair life insurance company that redistributes wealth of individuals who died amongst those who are still alive. Therefore the real rate of return $r(t)$ on assets $a(\tau, t)$ is augmented by the mortality rate $\mu(t)$. Taking final goods as numéraire, the wealth constraint of individuals belonging to cohort τ reads

$$\frac{\partial}{\partial t} a(\tau, t) = [r(t) + \mu(t)]\, a(\tau, t) + w(t) - c(\tau, t). \qquad (1)$$

Agent problem III

Utility of representative consumer born at time τ

$$\int_{\tau}^{\infty} e^{-\rho(t-\tau)-\int_{\tau}^{t}\mu(\theta)d\theta} \log[c(\tau,t)]\, dt \to \max_{c>0},$$

$c(\tau,t)$ – consumption.

Asset dynamics

$$\frac{\partial}{\partial t} a(\tau,t) = [r(t)+\mu(t)]\, a(\tau,t) + w(t) - c(\tau,t),$$

$w(t)$ – wage, $r(t)$ – real interest rate.

No-Bequest and No-Ponzi-Game conditions

$$a(\tau,\tau) = 0, \qquad \lim_{t\to\infty} e^{-\int_{\tau}^{t}[r(\theta)+\mu(\theta)]d\theta} a(\tau,t) \geq 0.$$

Agent problem solution I

Note that the solution to the optimization problem has to obey the no-bequest condition and the no-Ponzi game condition

$$a(\tau, \tau) = 0, \quad \lim_{t \to \infty} R(\tau, t)\, a(\tau, t) \geq 0, \tag{2}$$

where we define the discounting factor for convenience as

$$R(\tau, t) := e^{-\int_\tau^t [r(\theta) + \mu(\theta)]\, \mathrm{d}\theta}. \tag{3}$$

The latter condition in (2) results in transversality condition

$$\lim_{t \to \infty} R(\tau, t)\, a(\tau, t) = 0, \tag{4}$$

that implies that the solution of (1) has the form

$$a(\tau, t) = \frac{1}{R(\tau, t)} \int_t^\infty R(\tau, z)\, [c(\tau, z) - w(z)]\, \mathrm{d}z. \tag{5}$$

Agent problem solution II

Assuming convergence of the following improper integral

$$h(\tau) := \int_{\tau}^{\infty} R(\tau, z)\, w(z)\, \mathrm{d}z, \tag{6}$$

we introduce life-time human wealth $h(\tau)$, that equals discounted life-time consumption

$$\int_{\tau}^{\infty} R(\tau, z)\, c(\tau, z)\, \mathrm{d}z = h(\tau), \tag{7}$$

due to the no-bequest condition $a(\tau, \tau) = 0$ in (2). Notice that, since the wage income $w(t) \cdot 1$ does not depend on date of birth τ, according to definitions (3) and (6) assets (**??**) of each agent born at τ

$$a(\tau, t) = \int_{t}^{\infty} R(t, z)\, c(\tau, z)\, \mathrm{d}z - h(t),$$

Agent problem solution III

are her discounted remaining consumption minus the human wealth of a newly born agent, $h(t)$.
Maximization w.r.t. $c(\tau, t)$ is equivalent to the maximization of the following functional

$$\int_{\tau}^{\infty} e^{-\rho(t-\tau)-\int_{\tau}^{t}\mu(\theta)\,\mathrm{d}\theta} \log[c(\tau,t)]\,\mathrm{d}t \to \max_{c(\tau,\cdot)}, \tag{8}$$

subject to (7). Optimal consumption

$$c(\tau,t) = \frac{e^{-\rho(t-\tau)+\int_{\tau}^{t} r(\theta)\,\mathrm{d}\theta}}{\int_{\tau}^{\infty} e^{-\rho(s-\tau)-\int_{\tau}^{s}\mu(\theta)\,\mathrm{d}\theta}\,\mathrm{d}s}\,h(\tau) = e^{-\rho(t-z)+\int_{z}^{t} r(\theta)\,\mathrm{d}\theta} c(\tau,z) \tag{9}$$

is proportional to human wealth $h(\tau)$.The last equality yields

$$c(\tau,t)\int_{t}^{\infty} e^{-\rho(z-t)-\int_{t}^{z}\mu(\theta)\,\mathrm{d}\theta}\,\mathrm{d}z = a(\tau,t) + h(t). \tag{10}$$

Optimal consumption and asset profiles

Consumption and assets

$$c(\tau, t) = e^{-\rho(t-\tau)+\int_\tau^t r(\theta)\,\mathrm{d}\theta}\sigma(\tau)\,h(\tau), \qquad a(\tau, t) = \frac{c(\tau, t)}{\sigma(t)} - h(t).$$

Human wealth

$$h(\tau) := \int_\tau^\infty R(\tau, z)\,w(z)\,\mathrm{d}z, \qquad R(\tau, t) := e^{-\int_\tau^t [r(\theta)+\mu(\theta)]\,\mathrm{d}\theta}.$$

Notation

$$\sigma(\tau) := \frac{1}{\int_\tau^\infty e^{-\rho(z-\tau)-\int_\tau^z \mu(\theta)\,\mathrm{d}\theta}\,\mathrm{d}z}.$$

if $\mu = const$, then $\sigma = \rho + \mu$.

Aggregation

Aggregated consumption and assets

$$A(t) := \int_{-\infty}^{t} a(\tau, t)\, n(\tau, t)\, \mathrm{d}\tau, \quad C(t) := \int_{-\infty}^{t} c(\tau, t)\, n(\tau, t)\, \mathrm{d}\tau.$$

$$\dot{A}(t) = r(t)\, A(t) + w(t)\, N(t) - C(t), \tag{11}$$

$$\dot{C}(t) = [r(t) - \rho + \beta(t) - \mu(t)]\, C(t) - \beta(t)\, \sigma(t)\, A(t). \tag{12}$$

Final good sector with perfect competition

Production function

$$Y(t) = [L_Y(t)]^{1-\alpha} \int_0^{Q(t)} [x(t,q)]^{\alpha} \, \mathrm{d}q, \tag{13}$$

L_Y – labor used in final goods production, $Q(t)$ – technological frontier, $x(t,q)$ is the amount of intermediate good $q \in (0, Q(t)]$.

Wage rate paid in the final goods sector

$$w_Y(t) = (1-\alpha) \frac{Y(t)}{L_Y(t)}, \tag{14}$$

Prices paid for intermediate inputs

$$p(t,q) = \alpha \, [L_Y(t)]^{1-\alpha} \, [x(t,q)]^{\alpha-1}, \tag{15}$$

Intermediate goods sector with monopolistic competition

Linear one-to-one production function

$$x(t,q) = k(t,q).$$

Profits of firm q

$$\begin{aligned}\pi(t,q) &= p(t,q)\,k(t,q) - r(t)\,k(t,q) - \delta\,k(t,q)\\ &= \alpha\,[L_Y(t)]^{1-\alpha}\,[k(t,q)]^{\alpha} - [r(t)+\delta]\,k(t,q) \to \max_{k(t,q)}\end{aligned}$$

δ – depreciation rate of machines.

Capital and prices of intermediate good

$$k(t) := \left[\frac{\alpha^2}{r(t)+\delta}\right]^{\frac{1}{1-\alpha}} L_Y(t), \qquad p(t) := \frac{r(t)+\delta}{\alpha}.$$

Final and intermediate goods sectors I

Optimal profit

$$\pi(t,q) = (1-\alpha)\, p(t)\, k(t) = (1-\alpha)\,\alpha \left[\frac{\alpha^2}{r(t)+\delta}\right]^{\frac{\alpha}{1-\alpha}} L_Y(t). \tag{16}$$

Aggregation of capital

$$K(t) := \int_0^{Q(t)} k(t,q)\,\mathrm{d}q = \left[\frac{\alpha^2}{r(t)+\delta}\right]^{\frac{1}{1-\alpha}} L_Y(t)Q(t).$$

Capital and labor take constant shares of final outcome

$$p(t)K(t) = \alpha Y(t), \qquad w(t)L_Y(t) = (1-\alpha)\, Y(t).$$

Final and intermediate goods sector equations

Labor employed in production, price of intermediate goods, output, and interest rate

$$L_Y(t) = \frac{1-\alpha}{\alpha}\frac{K(t)}{w(t)}\,p(t), \quad p(t) = \alpha\left[(1-\alpha)\,\frac{Q(t)}{w(t)}\right]^{\frac{1-\alpha}{\alpha}},$$
$$r(t) = \alpha p(t) - \delta, \qquad Y(t) = p(t)K(t)/\alpha \tag{17}$$

as functions of the capital K, the wage rate w, and the technological frontier Q.

Total operating profit of firms

$$\Pi(t) := \int_0^{Q(t)} \pi(t,q)\,\mathrm{d}q = (1-\alpha)\,p(t)\,K(t) = \alpha\,w(t)\,L_Y(t). \tag{18}$$

R&D sector with perfect competition

Poduction function

$$\dot{Q}(t) = \lambda\, Q(t)\, L_Q(t), \tag{19}$$

$L_Q(t)$ – labor of scientists employed at time t in R&D sector.

Present value of the real profit flow firm $q \in [0, Q(t)]$

$$v(t, q) = \int_t^{\infty} \exp\left[-\int_t^s r(\theta)\,\mathrm{d}\theta\right] \pi(s, q)\,\mathrm{d}s. \tag{20}$$

Zero-profit condition

$$v[t, Q(t)]\, \dot{Q}(t) = w(t)\, L_Q(t), \tag{21}$$

R&D sector I

No-arbitrage condition (time-derivative of (20) with $q = Q(t)$)

$$\frac{\dot{v}[t, Q(t)]}{v[t, Q(t)]} + \frac{\pi[t, Q(t)]}{v[t, Q(t)]} = r(t), \qquad \lim_{T \to \infty} e^{-\int_t^T r(\theta)\, \mathrm{d}\theta} v[T, Q(T)] = 0, \tag{22}$$

Rate of capital gain (valuation gains of shares)

$$\frac{\dot{v}[t, Q(t)]}{v[t, Q(t)]} = \frac{\dot{w}(t)}{w(t)} - \frac{\dot{Q}(t)}{Q(t)}. \tag{23}$$

R&D sector II

Operating profit and price of patent, assuming $L_Q(t) > 0$

$$v[t, Q(t)] = \frac{w(t)\, L_Q(t)}{\dot{Q}(t)}, \qquad \pi[t, Q(t)] = \alpha\, \frac{w(t)\, L_Y(t)}{Q(t)}. \tag{24}$$

Profit rate (dividend payments per share)

$$\frac{\pi[t, Q(t)]}{v[t, Q(t)]} = \alpha\, \frac{L_Y(t)}{L_Q(t)}\, \frac{\dot{Q}(t)}{Q(t)}. \tag{25}$$

No-arbitrage condition (22)

$$\frac{\dot{w}(t)}{w(t)} - \frac{\dot{Q}(t)}{Q(t)} + \alpha\, \frac{L_Y(t)}{L_Q(t)}\, \frac{\dot{Q}(t)}{Q(t)} = r(t),$$

R&D sector equations

Technological frontier dynamics

$$\dot{Q}(t) = \lambda\, Q(t)\, L_Q(t),$$

No-arbitrage condition

$$\frac{\dot{w}(t)}{w(t)} + \lambda\, \{\alpha\, L_Y(t) - \varphi\, L_Q(t)\} = r(t). \tag{26}$$

subject to

$$\lim_{T\to\infty} e^{-\int_t^T r(\theta)\,\mathrm{d}\theta}\,\frac{w(T)}{Q(T)} = 0.$$

Balances

Labor market clearing

$$L_Q(t) + L_Y(t) = N(t). \tag{27}$$

Product market clearing

$$\dot{K}(t) = Y(t) - C(t) - \delta K(t), \quad K(0) = K_0 > 0. \tag{28}$$

Balance of asset flows

$$\dot{A}(t) = \dot{K}(t) + \dot{V}(t), \qquad V(t) := \int_0^{Q(t)} v(t,q)\,\mathrm{d}q. \tag{29}$$

No "bubbles" in asset market

Balance of assets

$$\dot{A}(t) = \dot{K}(t) + \dot{V}(t) \quad \Rightarrow \quad A(t) = K(t) + V(t), \qquad (30)$$

4

4

$$V(t) = v[t, Q(t)]\, Q(t) = \frac{w(t)\, L_Q(t)}{\dot{Q}(t)}\, Q(t) = \frac{w(t)}{\lambda}. \qquad (31)$$

We combine

$$\dot{A}(t) = r(t)\, A(t) + w(t)\, N(t) - C(t),$$
$$\dot{K}(t) = r(t)\, K(t) + Y(t) - C(t) - (r(t) + \delta)\, K(t),$$
$$\dot{V}(t) = r(t)\, V(t) + w(t)\, L_Q(t) - \Pi(t),$$

with the use of $Y(t) = p(t)K(t) + w(t)L_Y(t)$, $r(t) + \delta = \alpha p(t)$ to get

$$\dot{A}(t) - \dot{K}(t) - \dot{V}(t) = r(t)\,(A(t) - K(t) - V(t))\,.$$

Provided that $r(t) \neq 0$ we have (30) due to implicit assumption in (20):

$$\lim_{t\to\infty} v(t, q) \exp\left[-\int_0^t r(\theta)\, \mathrm{d}\theta\right] = 0.$$

Equations of general equilibrium I

We have 9 unknown functions Q, C, K, L_Y, L_Q, A, w, r, p

$$\frac{\dot{Q}(t)}{Q(t)} = \lambda\, L_Q(t), \qquad Q(0) = Q_0 > 0,$$

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho + \beta(t) - \mu(t) - \beta(t)\,\sigma(t)\,\frac{A(t)}{C(t)},$$

$$\frac{\dot{K}(t)}{K(t)} = \frac{r(t) + \delta\left(1-\alpha^2\right)}{\alpha^2} - \frac{C(t)}{K(t)}, \qquad K(0) = K_0 > 0,$$

$$\frac{\dot{A}(t)}{A(t)} = r(t) + N(t)\,\frac{w(t)}{A(t)} - \frac{C(t)}{A(t)},$$

$$\frac{\dot{w}(t)}{w(t)} = r(t) - \lambda\left[\alpha\, L_Y(t) - \varphi\, L_Q(t)\right],$$

$$L_Q(t) = N(t) - L_Y(t), \qquad r(t) = \alpha\, p(t) - \delta,$$

$$L_Y(t) = \frac{1-\alpha}{\alpha}\,\frac{K(t)}{w(t)}\,p(t), \quad p(t) = \left[(1-\alpha)\,\frac{Q(t)}{w(t)}\right]^{\frac{1-\alpha}{\alpha}} \quad A(t) = K(t) + \frac{w(t)}{\lambda}$$

Equations of general equilibrium in intensive form

We have 3 unknown functions p, $\frac{C}{K}$, $\frac{K}{w}$ and 3 differential equations

$$\frac{\dot{p}(t)}{p(t)} = \frac{1-\alpha}{\alpha}\left(\left[\lambda\,(1-\alpha)\,\frac{K(t)}{w(t)} - \alpha\right] p(t) + \delta\right) \tag{32}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\log\left(\frac{C(t)}{K(t)}\right) = {} & \frac{C(t)}{K(t)} - \frac{1-\alpha^2}{\alpha}\,p(t) - \rho + \beta(t) - \mu(t) \\ & - \frac{\beta(t)\,\sigma(t)}{\frac{C(t)}{K(t)}}\left(1 + \frac{1}{\lambda\,\frac{K(t)}{w(t)}}\right),\end{aligned} \tag{33}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\log\left(\frac{K(t)}{w(t)}\right) = \left[\lambda\,\frac{K(t)}{w(t)} + 1\right]\frac{1-\alpha^2}{\alpha}\,p(t) - \lambda N(t) - \frac{C(t)}{K(t)}, \tag{34}$$

Balanced Growth Path with constant mortality and replacement fertility $\mu = \beta$

$$N = const, \quad \sigma(t) \equiv \rho + \mu$$

$$\frac{\dot{Q}(t)}{Q(t)} = \frac{\dot{C}(t)}{C(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{A}(t)}{A(t)} = \frac{\dot{w}(t)}{w(t)} = \frac{\dot{p}(t)}{p(t)} := g.$$

Consider the case of $\varphi = 1$ and $\delta = 0$:

$$\lambda\,(1-\alpha)\,\frac{K(t)}{w(t)} - \alpha = 0, \quad \Rightarrow \quad \lambda\frac{K(t)}{w(t)} = \frac{\alpha}{1-\alpha}$$

$$\frac{C(t)}{K(t)} - \frac{1-\alpha^2}{\alpha}\,p - \rho - \frac{\beta\,\sigma}{\frac{C(t)}{K(t)}}\left(1 + \frac{1}{\lambda\,\frac{K(t)}{w(t)}}\right) = 0,$$

$$\left[\lambda\,\frac{K(t)}{w(t)} + 1\right]\frac{1-\alpha^2}{\alpha}\,p - \lambda N - \frac{C(t)}{K(t)} = 0,$$

We obtain the quadratic equation with one positive root

$$\alpha\,\frac{C(t)}{K(t)} - (1-\alpha)\,\lambda N - \rho - \frac{\beta\,\sigma}{\alpha\,\frac{C(t)}{K(t)}} = 0$$

$$\frac{C(t)}{K(t)} = \frac{\rho}{2} + \frac{(1-\alpha)\,\lambda\,N}{2} + \sqrt{\left(\frac{\rho}{2} + \frac{(1-\alpha)\,\lambda\,N}{2}\right)^2 + \beta\,\sigma}$$

Anticipated aging I

If abrupt change in mortality and fertility at time t^*

$$\mu(t) = \beta(t) = \begin{cases} \mu_1, & t < t^*, \\ \mu_2, & t \geq t^*, \end{cases}$$

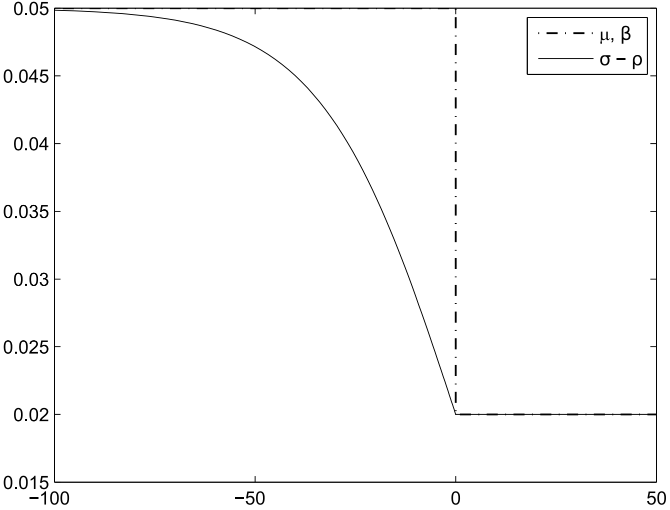
is anticipated, then for all $\tau \geq t^*$

$$\sigma(\tau) := \frac{1}{\int_\tau^\infty e^{-\rho(z-\tau) - \int_\tau^z \mu(\theta)\,\mathrm{d}\theta}\,\mathrm{d}z} = \mu_2 + \rho,$$

while for all $\tau \leq t^*$ we have

$$\sigma(\tau) = \frac{\rho + \mu_1}{1 + \frac{\mu_1 - \mu_2}{\rho + \mu_2} e^{-(\rho+\mu_1)(t^* - \tau)}} \to \mu_1 + \rho,$$

as $\tau \to -\infty$.


0.05
0.045
0.04
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−100
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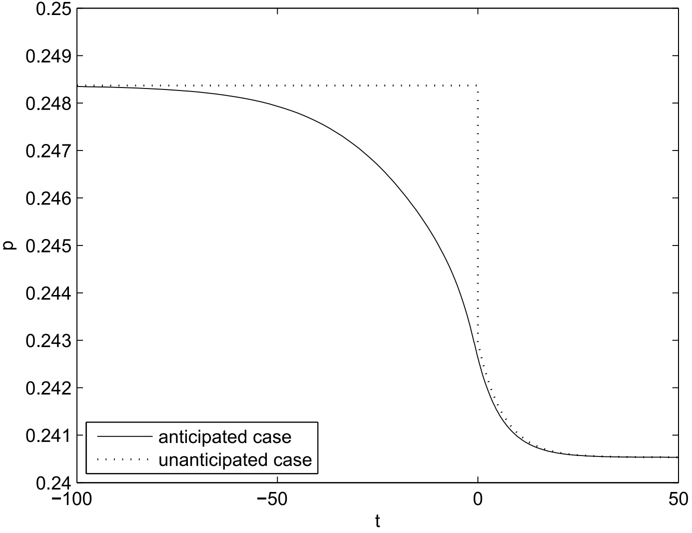
0
50
μ, β
σ − ρ



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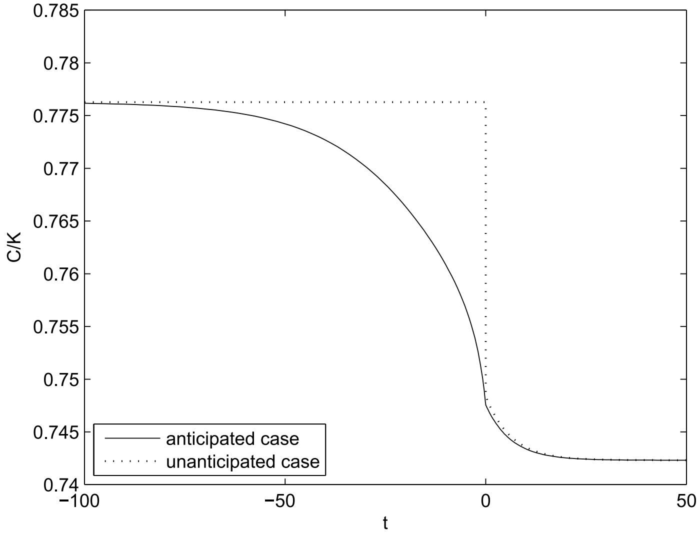
anticipated case
unanticipated case



0.785
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0.765
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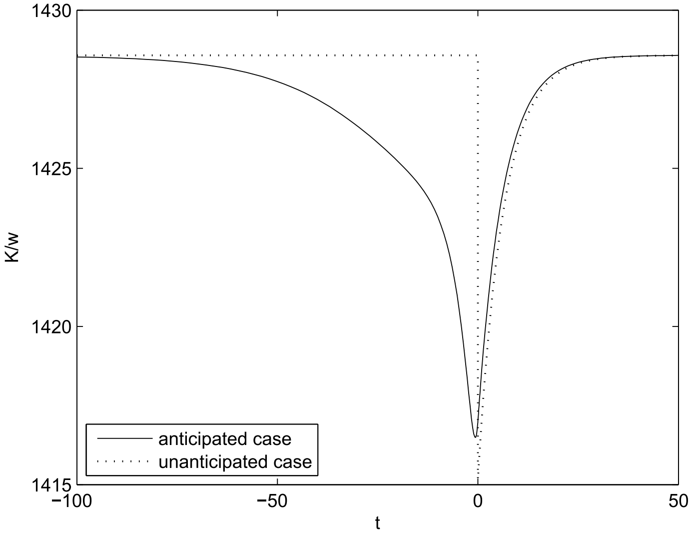
anticipated case
unanticipated case


1430
1425
1420
1415
K/w
−100
−50
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t
anticipated case
unanticipated case

The GDP growth rate

It follows from $p(t)K(t) = \alpha Y(t)$ that the GDP growth rate has the expression $g(t) \equiv \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} + \frac{\dot{p}(t)}{p(t)}$, which with $r(t) = \alpha\, p(t) - \delta$,

$$\frac{\dot{K}(t)}{K(t)} = \frac{r(t) + \delta\left(1 - \alpha^2\right)}{\alpha^2} - \frac{C(t)}{K(t)},$$

and

$$\frac{\dot{p}(t)}{p(t)} = \frac{1-\alpha}{\alpha}\left(\left[\lambda\,(1-\alpha)\,\frac{K(t)}{w(t)} - \alpha\right] p(t) + \delta\right)$$

yields

$$g(t) = \left(\lambda\,(1-\alpha)^2\,\frac{K(t)}{w(t)} + 1 - \alpha + \alpha^2\right)\frac{p(t)}{\alpha} - \frac{C(t)}{K(t)} + \frac{1-2\alpha}{\alpha}\delta.$$

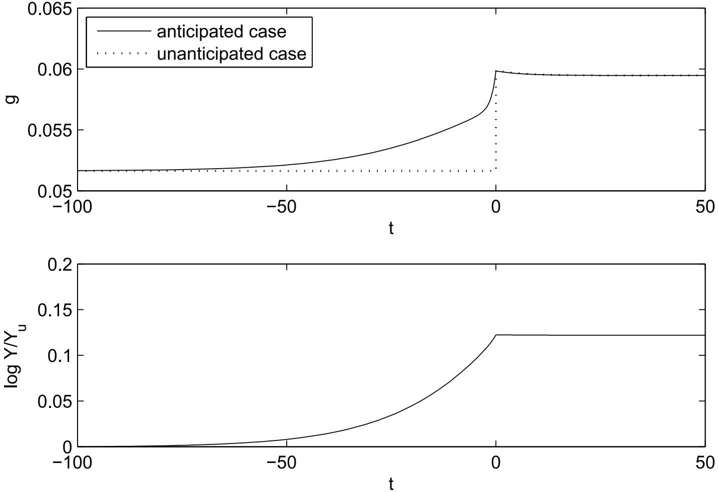

Figure: Comparison of the GDP growth rates between unanticipated aging and anticipated aging.

Conclusions

- Population aging, i.e., a decrease in β and μ, implies faster economic growth along the balanced growth path.
- Anticipated aging is associated with faster economic growth during the transition to the long-run balanced growth path and, hence, a higher level of per capita GDP in the long run.
- Overall, the accurate information of the population leads to economic gains that could be valuable for a society in the transition to an older population.