Bertrand and Chamberlin meet Schumpeter*

Alexander Sidorov[†]

The paper studies the question what is an impact of concentration in industry on the Social welfare, measured as consumer's indirect utility. Is the Schumpeter hypothesis that this impact may be advantageous the truth, or concentration is always harmful for the end consumers? We study a general equilibrium model with two types of firms and imperfect price competition. Firms of the first type are monopolistic competitors with negligible impact to market statistics, subjected to typical assumptions, e.g., free entry until zero-profit cut-off. Unlike this, the firms of second type assumed to have non-zero impact to market statistics, in particular, to consumer's income via distribution of non-zero profit across consumers-shareholders. Moreover, these large firms (oligopolies) allow for dependence of profits on their strategic choice, generating so called Ford effect. The first result we present is that in case of CES utility the concentration effect is generically harmful for consumers' well-being. However, the result may be different for preferences, generating the demand with Variable Elasticity of Substitution (VES). We find the natural assumption on VES utilities, which hold for most of the commonly used classes of utility functions, such as Quadratic, CARA, HARA, etc., which allows to obtain the positive welfare effect, i.e., to vindicate Schumpeter hypothesis.

Keywords: Bertrand competition, monopolistic competition, additive preferences, Ford effect, Schumpeter hypothesis

JEL classification: D43, L11, L13.

*I owe special thanks to J. F. Thisse and M. Parenti for long hours of useful and discussions in CORE (Louvain-la-Neuve, Belgium). The study has been funded by the Russian Fund of Basic Research, Project No. 18-010-00728.

†Novosibirsk State University and Sobolev Institute of Mathematics (Russia). Email: alex.v.sidorov@gmail.com

1 Introduction

“The elegant fiction of competitive equilibrium” does not dominates now the frontier of theoretical microeconomics as stated by T. Marschak and R. Selten in [8] in early 1970-th, being replaced by also elegant monopolistic competitive Dixit-Stiglitz “engine”. The idea that firms are price-makers even if their number is “very large”, e.g., continuum, is a common wisdom. But what if the monopolistic competitive equilibrium conception, where firms has zero impact to market statistics and, therefore, treat them as given, is just a brand new elegant fiction? When firms are sufficiently large, they face demands, which are influenced by the income level, depending in turn on their profits. As a result, firms must anticipate accurately what the total income will be. In addition, firms should be aware that they can manipulate the income level, whence their “true” demands, through their own strategies with the aim of maximizing profits [5]. This feedback effect is known as the *Ford effect*. In popular literature, this idea is usually attributed to Henry Ford, who raised wages at his auto plants to five dollars a day in January 1914. Ford wrote “our own sales depend on the wages we pay. If we can distribute high wages, then that money is going to be spent and it will serve to make... workers in other lines more prosperous and their prosperity is going to be reflected in our sales”, see [4], p. 124–127. To make things clear, we have to mention that the term “Ford effect” may be used in various specifications. As specified in [2], the Ford effect may have different scopes of consumers income, which is sum of wage and a share of the distributed profits. The first (extreme) specification is to take a whole income parametrically. This is one of solutions proposed by Marschak and Selten [8] and used, for instance, by Hart [6]. This case may be referred as “No Ford effect”. Another specification (also proposed by Marschak and Selten [8] and used by d’Aspremont et al., [2]) is to suppose that firms take into account the effects of their decision on the total wage bill, but not on the distributed profits, which is still treated parametrically. This case may be referred as “Wage Ford effect” and it is exactly what Henry Ford meant in above citation. One more intermediate specification of The Ford effect is an opposite case to the previous one: firms take wage as given, but take into account the effects of their decisions on distributed profits. This case may be referred as “Profit Ford effect”. Finally, the second extreme case, Full Ford effect, assumes that firms take into account total effect of their decisions, both on wages and on profits. These two cases are studied in newly published paper [3]. In what follows, we shall assume that wage is determined. This includes the way proposed by Hart [6], in which the workers fix the nominal wage through their union. This assumption implies that only the Profit Ford effect is possible, moreover, firms maximize their profit anyway, thus being price-makers but not wage-makers, they have no additional powers at hand in comparison to No Ford case, with except the purely informational advantage — knowledge on consequences of their decisions. Nevertheless, as we show in [11], this advantage allows firms to get more market power, which vindicate the wisdom “Knowledge is Power”. It should be mentioned also that being close in ideas with paper [3], we have no intersections in results, because the underlying economy model of this paper differers from our one, moreover, that research focuses on existence and uniqueness of equilibria with different specifications of Ford effect and does not concern the aspects of market power and welfare.

It should be noted, however, an assumption that industry consists only of the large firms, oligopolies, is another extreme point of view, which hardly fits the reality. More naturally is to suggest that industry consists of both types of firms – “large” oligopolies and “small” monopolistic competitors. This is not a brand new idea, the mixed models of such type were studied in [10] and in [7]. However, our approach has some substantial differences. In paper [10] oligopolistic and monopolistic competitive sectors differ in their nature: large firms compete with quantities (Cournot-type competition), while small firms compete with prices in Dixit-Stiglitz style. As result, this heterogeneity generates counter-intuitive outcome: Social Welfare always increases with respect to number of oligopolies. This result seems even more unusual as we take into account that under CES utility the pure monopolistic competition provides social optimum. In working paper [7] consumers’ utility is quasi-linear quadratic function, which completely kills income effect, all the more – Ford effect. One of the main conclusions of this paper is that in equilibrium the large firm “mimic” the monopolistic competitors, i.e., set the same price and output as small firms, which seems to be completely unrealistic.

The presented paper uses an additive separable utility function of general type. Unlike [10], large firms compete with prices (Bertrand-type competition) as well as small monopolistic competitors, but the main distinctive feature of oligopolies is that its impact to market statistics is not negligible. As result, oligopolies charge the higher equilibrium prices than monopolistic competitors. The main focus of this research is on welfare aspects, e.g., how many (if any) oligopolies are needed to foster the Social Welfare? Put it differently, are there any circumstances supporting Schumpeter hypothesis on positive effect of oligopolization on Social Welfare, or concentration is always harmful for consumers? It is shown that answer depends on consumers’ utility. In CES case the optimum structure of industry is pure monopolistic competition. The result changes significantly beyond CES world. We found sufficiently weak condition, which holds for many popular classes of utility function, including CARA, HARA (with exception of pure CES), separable quadratic, etc., and demonstrate the positive welfare effect, supporting Schumpeter hypothesis, provided that oligopolies keep its size in certain interval. As a natural policy implication of this result is a usual anti-trust procedure when “too large” mono/oligopoly, e.g., AT&T Corporation, have to be divided into “not too large” parts to increase the total Social Welfare, however, there is no need to “smash it into dust”, i.e., into mass of monopolistic competitive firms.

2 The model

2.1 Firms and consumers

The economy involves two sector with different competitions regimes supplying a horizontally differentiated good and one production factor - labor. There is a continuum $[0, 1]$ of identical consumers endowed with one unit of labor. The labor market is perfectly competitive and labor is chosen as the numéraire. There are two types of firms: a finite number $N \geq 2$ of the "big" oligopolistic firms and a continuum mass M of the "small" monopolistic competitive firms. Each variety is produced by a single firm and each firm produces a single variety, thus the horizontally differentiated good $\mathbf{x} = \big\{x_k \geq 0 \;\big|\; k \in \{1, \ldots, N\} \cup M\big\}$ consists of two parts – the finite dimensional oligopolistic part $(x_1, \ldots, x_N) \in \mathbb{R}^N_+$, and MC-produced bundle of varieties $\big\{x_j \geq 0 \;\big|\; j \in M\big\}$. To operate every monopolistic competitive firm needs a fixed requirement $f > 0$ and a marginal requirement $c > 0$ of labor. The same holds for oligopolies, but we will denote the corresponding labor costs as F and C, assuming that they *may* – not necessary, *will* – differ from monopolistic competitive labor requirements. Wage is also normalized to 1, then the cost of producing q_k units of variety $i \in \{1, \ldots, N\}$ is equal to $F + 1 \cdot C \cdot q_i$, while the monopolistic competitive production costs are $F + 1 \cdot C \cdot q_j$ for $j \in M$. Moreover, the relative share of labor hired by one oligopolistic firm is denoted as s, thus the total amount of oligopolistic employee is $N \cdot s$, while the monopolistic competitive share in labor market is $1 - Ns$. In what follows we discuss how s may be determined, but anyways it is naturally to assume that MC-sector treats both N and s parametrically.

Remark. It was noticed in [9] that the oligopoly i may be equivalently treated as a "cartel" of the non-atomic firms $[i - 1, i]$, where each non-atomic firm $j \in [i - 1, i]$ has negligible impact, but they act *in concord* due to cartel agreement. This means that mass of all non-atomic firms is actually a concatenation of two intervals on $\mathbb{R}$ – the cartel interval $[0, N]$ and interval M of monopolistic competitors. From this point of view, our model became "seamless" – the new oligopolistic firm is simply a new cartel agreement over the bunch of monopolistic competitors, and *vice versa*, instead of "destroyed" oligopoly i we obtain immediately the continuum $[i - 1, i]$ of monopolistic competitors. The possible changes in production cost $f \to F$, $c \to C$ after this side effect we can interpret as a *cost effect* of "cartelizing", which may be negative ($F > f$, $C > c$), positive ($F < f$, $C < c$) or ambiguous.

Consumers share the same additive preferences given by

$$U(\mathbf{x}, \mathbf{z}) = \sum_{i=1}^{N} u(x_i) + \int_M u(x_j)\mathrm{d}j \tag{1}$$

where $u(\cdot)$ is thrice continuously differentiable, strictly increasing, strictly concave, and such that $u(0) = 0$.

Following [12], we define the relative love for variety (RLV) as follows:

$$r_u(x) = -\frac{xu''(x)}{u'(x)}$$

which is strictly positive for all $x > 0$. Under the CES, we have $u(x) = x^\rho$ where ρ is a constant such that $0 < \rho \leq 1$, thus implying a constant RLV given by $1 - \rho$. Another example of additive preferences is provided by Behrens and Murata in [1] who consider the CARA utility $u(x) = 1 - \exp(-\alpha x)$ where $\alpha > 0$ is the absolute love for variety; the RLV is now given by αx. Very much like the Arrow-Pratt's relative risk-aversion, the RLV measures the intensity of consumers' variety-seeking behavior.

A consumer's income is equal to her wage plus her share in total profits. Since we focus on symmetric equilibria, consumers must have the same income, which means that profits have to be uniformly distributed across consumers. In this case, a consumer's income y is given by

$$y = 1 + \sum_{i=1}^{N} \Pi_i + \int_M \pi_j \mathrm{d}j \geq 1 \tag{2}$$

where the profit made by the oligopoly selling amount q_i of variety $i \in \{1, \ldots, N\}$ at price p_i is given by

$$\Pi_i = (p_i - C)q_i - F \tag{3}$$

while the profit of monopolistic competitive firm $j \in M$ is equal to

$$\pi_j = (p_j - f)q_j - f \tag{4}$$

Evidently, the income level varies with firms' strategies.

A consumer's budget constraint is given by

$$\sum_{i=1}^{N} p_i x_i + \int_M p_j x_j \mathrm{d}j = y \tag{5}$$

The first-order condition for utility maximization yields

$$u'(x_k) = \lambda p_k, \tag{6}$$

where λ is the Lagrange multiplier

$$\lambda = \frac{\sum_{i=1}^{N} u'(x_i)x_i + \int_M u'(x_j)x_j \mathrm{d}j}{y} > 0, \tag{7}$$

which implies that the inverse demand

$$p_k = \frac{yu'(x_k)}{\sum_{i=1}^{N} u'(x_i)x_i + \int_M u'(x_j)x_j \mathrm{d}j} \tag{8}$$

for all varieties $k \in \{1, \ldots, N\} \cup M$.

Let $\mathbf{p} = \{p_k \geq 0 \mid k \in \{1, \ldots, N\} \cup M\}$ be a price profile. In this case, consumers' demand functions $x_i(\mathbf{p})$ are obtained by solving the system of equations (8) where consumers' income y is now defined as follows:

$$y(\mathbf{p}) = 1 - NF - Mf + \sum_{i=1}^{N} (p_i - C)x_i(\mathbf{p}) + \int_M (p_j - c)x_j(\mathbf{p})\mathrm{d}j.$$

It follows from (7) that the marginal utility of income λ is a market aggregate that depends on the price profile $\mathbf{p}$. Indeed, the budget constraint

$$\sum_{j=1}^{N} p_j x_j(\mathbf{p}) = y(\mathbf{p})$$

implies that

$$\lambda(\mathbf{p}) = \frac{1}{y(\mathbf{p})} \left[\sum_{i=1}^{N} x_i(\mathbf{p})u'\left(x_i(\mathbf{p})\right) + \int_M x_j(\mathbf{p})u'\left(x_j(\mathbf{p})\right)\mathrm{d}j \right]. \tag{9}$$

Since $u'(x)$ is strictly decreasing, the demand function for variety i is thus given by

$$x_k(\mathbf{p}) = \xi(\lambda(\mathbf{p})p_k), \tag{10}$$

where ξ is the inverse function to u'. Moreover, firm i's profits can be rewritten as follows:

$$\Pi_i(\mathbf{p}) = (p_i - C)x_i(\mathbf{p}) - F = (p_i - C)\xi(\lambda(\mathbf{p})p_i) - F, \tag{11}$$

$$\pi_j(\mathbf{p}) = (p_j - c)x_j(\mathbf{p}) - f = (p_j - c)\xi(\lambda(\mathbf{p})p_j) - f. \tag{12}$$

2.2 Market equilibrium

The market equilibrium is defined by the following conditions:

(i) each consumer maximizes her utility (1) subject to her budget constraint (5),

(ii) each firm k maximizes its profit (3, (4)) with respect to p_k ,

(iii) product market clearing:

$$x_k = q_k \qquad \text{for all } k \in \{1, \ldots, N\} \cup M,$$

(iv) labor market clearing:

$$NF + C\sum_{i=1}^{N} q_i = Ns,\ \ Mf + c\int_M q_j \mathrm{d}j = 1 - ns.$$

Conditions (iii) and (iv) imply that

$$\bar{x} \equiv \frac{s-F}{C},\ \ \hat{x} \equiv \frac{1}{c}\left(\frac{1-Ns}{M} - f\right) \tag{13}$$

are the only candidate symmetric equilibrium demands for "oligopolistic" and "monopolistic competitive" varieties, respectively.

Remark. In what follows we can refer s as a *size of oligopoly*, which is equivalent to the widely used determination of firm's size in terms of output, due to (13).

2.3 Free Entry

In equilibrium, profits must be non-negative for firms to operate. Moreover, if profit is strictly positive, this causes an enter of new firms, which will stop when profit become zero. One of the main difference between "big" and "small" firms in our model is that "small" firms are free to enter into industry, while formation of oligopolies is typically subjected to more sophisticated laws, e.g., to some kind of antitrust legislation.

Consider symmetric price/quantity profile, i.e., $p_i = \bar{p}$, $x_i = \bar{x} = \frac{s-F}{C}$ for all $i = 1, \ldots, n$, $p_j = \hat{p}$, $x_j = \hat{x} = \frac{1}{c}\left(\frac{1-Ns}{M} - f\right)$ for all $j \in M$, then the Zero-Profit Condition $\pi_j = 0$ takes the form

$$(\hat{p} - c)\hat{x} = f \iff \frac{\hat{p} - c}{\hat{p}} = \frac{f}{\hat{p}\hat{x}},$$

while the symmetric budget constraint

$$N\bar{p}\bar{x} + M\hat{p}\hat{x} = y = 1 - NF + N(\bar{p} - C)\bar{x} + 0 \iff \hat{p}\hat{x} = \frac{1 - NF - NC\bar{x}}{M} = \frac{1 - NF - Ns + NF}{M} = \frac{1 - Ns}{M},$$

taking into account (13). Substituting this equation into previous one, we obtain the following form of Zero-Profit Condition

$$\mu \equiv \frac{\hat{p} - c}{\hat{p}} = \frac{Mf}{1 - Ns} \iff M = \frac{(1 - Ns)\mu}{f}, \tag{14}$$

where μ is monopolistic competitive markup, which implies

$$\hat{x} \equiv \frac{1}{c}\left(\frac{1 - Ns}{M} - f\right) = \frac{f}{c}\frac{1 - \mu}{\mu} > 0, \tag{15}$$

provided that μ satisfies $0 < \mu < 1$.

2.4 When Bertrand meets Ford

As shown by (6) and (7), the income level influences firms' demands, whence their profits. As a result, firms must anticipate accurately what the total income will be. In addition, firms should be aware that they can manipulate the income level, whence their "true" demands, through their own strategies with the aim of maximizing profits (see e.g. [5]). This feedback effect is known as the *Ford effect* (see [2]). Note that these considerations may concern only the "big" oligopolistic firms $i \in \{1, \ldots, N\}$. The "non-atomic" monopolistic competitive firms $j \in M$ have negligible effect on market statistics, such that consumer's income $y(\mathbf{p})$ or marginal utility of money $\lambda(\mathbf{p})$, in other words, we obtain that

$$\frac{\partial \lambda}{\partial p_j} = \frac{\partial y}{\partial p_j} = 0, \; j \in M$$

The generalized *Bertrand equilibrium* is a vector $\mathbf{p}^*$ such that p_i^* maximizes $\Pi_i(p_i, \mathbf{p}_{-i}^*)$ for all $i \in \{1, ..., N\}$. Applying the first-order condition to (12) yields

$$\frac{p_i - C}{p_i} = -\frac{\xi(\lambda p_i)}{\lambda p_i \xi'(\lambda p_i)\left(1 + \frac{p_i}{\lambda}\frac{\partial \lambda}{\partial p_i}\right)}, \tag{16}$$

which involves $\partial\lambda/\partial p_i$ because λ depends on $\mathbf{p}$. The monopolistic competitive firms, however, get a less complicated form of the FOC

$$\frac{p_j - c}{p_j} = -\frac{\xi(\lambda p_j)}{\lambda p_j \xi'(\lambda p_j)} = -\frac{x_j u''(x_j)}{u'(x_j)} = r_u(x_j). \tag{17}$$

Indeed, by definition of function $\xi = (u')^{-1}$, we obtain $\xi(\lambda p_j) = \xi(u'(x_j)) = x_j$, $\lambda p_j = u'(x_j)$ is a first-order condition in consumer's problem, while $\xi'(\lambda p_j) = \dfrac{1}{u''(x_j)}$ easily follows from the formula of derivative of inverse function. As result, in symmetric solution we obtain that equilibrium markup in monopolistic competitive sub-sector satisfies the following equation

$$\mu = \frac{\hat{p} - c}{\hat{p}} = r_u(\hat{x}) = r_u\left(\frac{f}{c} \cdot \frac{1-\mu}{\mu}\right), \tag{18}$$

which depends only on function $u(x)$ and fraction f/c and *does not depend* on number of oligopolies N and their share on labor market s. It was proved in [12] (18) has unique solution $0 < \mu < 1$ provided that function $u(x)$ satisfies the following additional assumption

$$r_u(x) = -\frac{xu''(x)}{u'(x)} < 1, \; r_{u'}(x) = -\frac{xu'''(x)}{u''(x)} < 2 \tag{19}$$

Remark. These assumptions provide the Second Order Condition for profit function Π_k at symmetric solutions (for income-taking firms it was proved in [9], for firms, accounting for the Ford effect, proof is quite similar), though in CES case we obtain that Π_k is strictly concave everywhere.

2.5 FOC for oligopolies

Now let's focus on oligopolies $k \in \{1, ..., N\}$, which have non-zero influence on market statistics, in particular, we can expect that $\partial\lambda/\partial p_k \neq 0$. But how does firm k determine $\partial\lambda/\partial p_k$? By the standard interpretation, the Lagrange multiplier λ is a marginal utility of money, therefore "big" firms understands that the demand functions (10) must satisfy the budget constant as an identity. The consumer budget constraint, before symmetrization, can be rewritten as follows:

$$\sum_{i=1}^{N} p_i\xi(\lambda(\mathbf{p})p_i)+\int_M p_j\xi(\lambda(\mathbf{p})p_j)\mathrm{d}j = 1-NF-Mf+\sum_{i=1}^{N}(p_i-1)\xi(\lambda(\mathbf{p})p_i)+\int_M (p_j-1)\xi(\lambda(\mathbf{p})p_j)\mathrm{d}j,$$

which boils down to

$$\sum_{i=1}^{N} \xi(\lambda(\mathbf{p})p_i) + \int_M \xi(\lambda(\mathbf{p})p_j)\mathrm{d}j = 1 - NF - Mf. \tag{20}$$

Differentiating (20) with respect to p_k yields

$$\xi'(\lambda p_k)\lambda + \frac{\partial\lambda}{\partial p_k}\left(\sum_{i=1}^{N} \xi'(\lambda p_i)p_i + \int_M \xi'(\lambda p_j)p_j\mathrm{d}j\right) = 0$$

or, equivalently,

$$1 + \frac{p_k}{\lambda}\frac{\partial\lambda}{\partial p_k} = \frac{\sum_{i\neq k}^{N} \xi'(\lambda p_i)\lambda p_i + \int_M \xi'(\lambda p_j)\lambda p_j\mathrm{d}j}{\sum_{i=1}^{N} \xi'(\lambda p_i)\lambda p_i + \int_M \xi'(\lambda p_j)\lambda p_j\mathrm{d}j}. \tag{21}$$

Substituting (21) into (16) and symmetrizing the resulting expression yields the candidate equilibrium markup:

$$m \equiv \frac{\bar{p} - C}{\bar{p}} = r_u(\bar{x})\frac{N\frac{u'(\bar{x})}{u''(\bar{x})} + M\frac{u'(\hat{x})}{u''(\hat{x})}}{(N-1)\frac{u'(\bar{x})}{u''(\bar{x})} + M\frac{u'(\hat{x})}{u''(\hat{x})}} = r_u(\bar{x})\frac{N\bar{x} + M\hat{x}\frac{r_u(\bar{x})}{r_u(\hat{x})}}{(N-1)\bar{x} + M\hat{x}\frac{r_u(\bar{x})}{r_u(\hat{x})}}.$$

Substituting (13), (14), (15), we obtain

$$m = r_u\left(\frac{s-F}{C}\right)\frac{N\cdot(s-F) + (1-Ns)\frac{C(1-\mu)}{c\mu}r_u\left(\frac{s-F}{C}\right)}{(N-1)\cdot(s-F) + (1-Ns)\frac{C(1-\mu)}{c\mu}r_u\left(\frac{s-F}{C}\right)}. \tag{22}$$

We didn't specified yet how the oligopoly share in labor market s and number of oligopolies N are determined. As an extreme case we may assume that oligopolies, as well as monopolistic competitive firms, may freely enter to industry and/or freely choose size s, which maximizes their profit. The natural outcome of these assumption will be the devourment of monopolistic competitive sector by oligopolies, i.e., the resulting $s = 1/N$. This case was studied in papers [11] and [9], with and without Ford effect. Moreover, it was noticed in [11] that under natural assumption on utility function $u(x)$, oligopolization is harmful for consumers' welfare.

This extreme assumption on freely acting oligopolies, however, doesn't fits well to the real life. If Social Welfare is valued, the society (or government) will establish mechanisms, which "adjust" behavior of large firms. In the next section we shall not specify any adjustment mechanism. Our main research aim is comparative statics of Social Welfare with respect to parameters N and s and implications of their adjustment by Social Planner.

3 Welfare considerations

Consider the following Social Welfare function (actually, an indirect utility)

$$V(s, N) = Nu(\bar{x}) + Mu(\hat{x}) = A + N \cdot B(s), \tag{23}$$

where

$$A \equiv \frac{\mu}{f} u\left(\frac{f}{c}\frac{1-\mu}{\mu}\right)$$

does not depend on N and s, while

$$B(s) \equiv u\left(\frac{s-F}{C}\right) - s\frac{\mu}{f}u\left(\frac{f}{c}\frac{1-\mu}{\mu}\right)$$

is a strictly concave function of s, which does not depend on N. This implies the two following conclusions.

Proposition 1. Let

$$u'(0) > \frac{\mu C}{f} u\left(\frac{f}{c}\frac{1-\mu}{\mu}\right) > u'(\infty)$$

(e.g., $u'(0) = +\infty,\ u'(\infty) = 0$), then there exists the "optimum size" of oligopolistic firm s^*, which does not depend on N.

Proof. Indeed, for any given $n > 0$ the indirect utility function V is strictly concave with respect to s, the First-Order Condition

$$\frac{\partial V}{\partial s} = 0 \iff \frac{\partial B}{\partial s} = 0 \iff u'\left(\frac{s-F}{C}\right) = \frac{\mu C}{f} u\left(\frac{f}{c}\frac{1-\mu}{\mu}\right),$$

implies that the optimum size is given by

$$s^* = F + C\xi\left(\frac{\mu C}{f} u\left(\frac{f}{c} \cdot \frac{1-\mu}{\mu}\right)\right), \tag{24}$$

where $\xi \equiv (u')^{-1}$ is an inverse function.

Proposition 2. Let the oligopolistic firm size s is given, whether optimal or not, then there are the following options for optimum number N^* of oligopolies

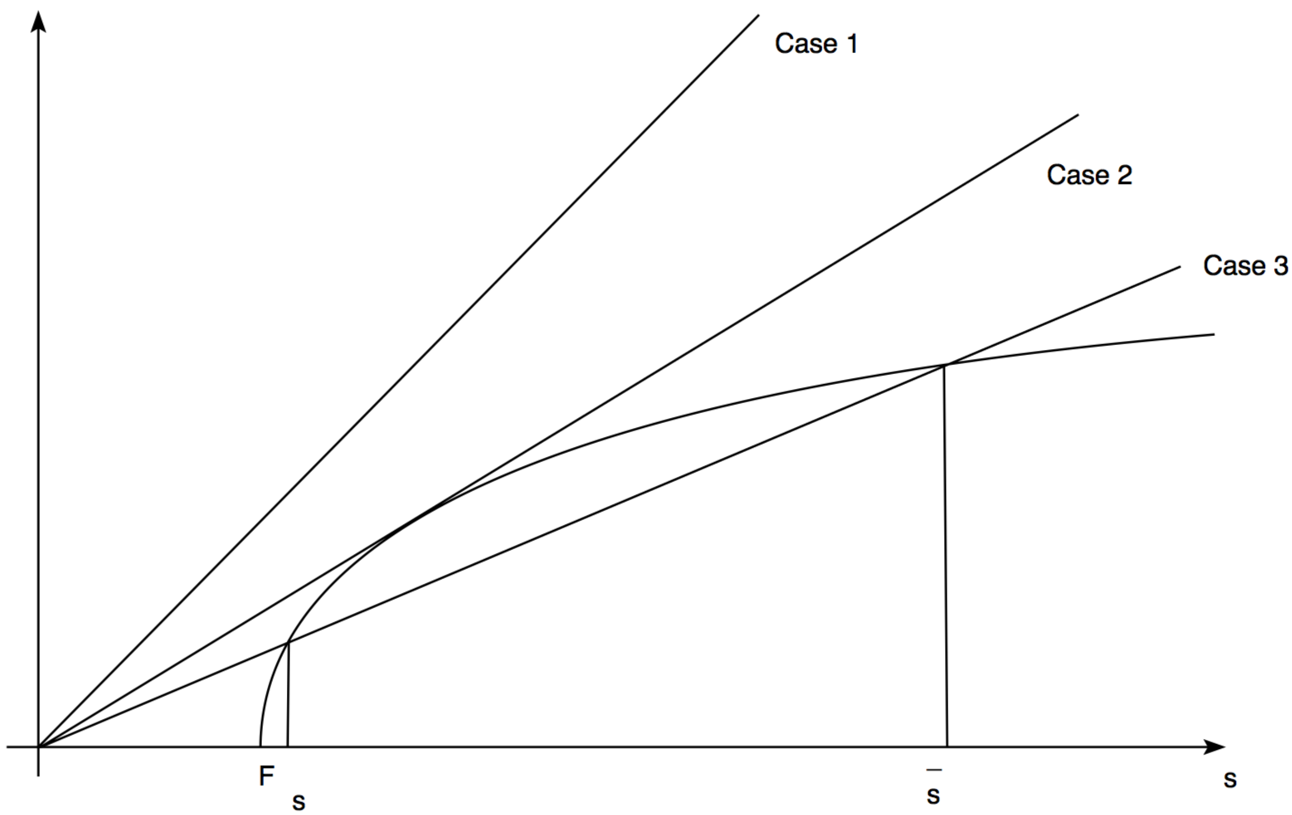
1. $B(s) < 0 \Rightarrow N^* = 0$, i.e., the optimum structure of industry is pure monopolistic competitive

2. $B(s) = 0 \Rightarrow N^*$ may by arbitrary, i.e., consumers' welfare is indifferent to the structure of industry

3. $B(s) > 0 \Rightarrow N^* = [s^{-1}]$, where $[x]$ denotes an integer part of number x, i.e., the optimum structure of industry may be achieved under the maximum concentration of production.

Statements of the Proposition are obvious due to linearity of V with respect to N.

For any given model primitives c, f, C, F and utility function $u(\cdot)$ all these conditions may be verified with direct calculation (μ is determined as solution of equation (18).) Moreover, left-hand side of condition does not depend on f, c, while the right-hand side does not depend on F, C, which implies that all three cases are highly probable for appropriate values of the model primitives. To fix ideas, let's consider the following diagram, which shows the possible cases of the two function incidence: linear with respect to s function $\frac{\mu}{f} u\left(\frac{f}{c}\frac{1-\mu}{\mu}\right) s$ and strictly concave function $u\left(\frac{s-F}{C}\right)$.



It is obvious, that if angular coefficient $\frac{\mu}{f} u\left(\frac{f}{c}\frac{1-\mu}{\mu}\right)$ is sufficiently large (Case 1), we obtain that the pure monopolistic competition is best best outcome. It is obvious that $u\left(\frac{s-F}{C}\right) < \frac{\mu}{f} u\left(\frac{f}{c}\frac{1-\mu}{\mu}\right) s$ holds for all $s \leq F$, thus Case 3 implies that there is $\underline{s}$, such that $B(\underline{s}) = 0$ and $B(s) < 0$ for all $s < \underline{s}$. Because s is bounded from above by $1/N$, we ensure existence of finite upper bound $\bar{s}$.

CES Case

Theorem 1. Let $u(x) = x^\rho,\ 0 < \rho < 1$, then

1. $f^{1-\rho}c^\rho < F^{1-\rho}C^\rho \Rightarrow B(s^*) < 0$, which implies that for all admissible s the optimum industry structure is pure monopolistic competition

2. $f^{1-\rho}c^\rho = F^{1-\rho}C^\rho \Rightarrow B(s^*) = 0$, which implies that for all $s \neq s^*$ the optimum industry structure is pure monopolistic competition, while at $s = s^*$consumers' welfare does not depend on structure of industry

3. $f^{1-\rho}c^\rho > F^{1-\rho}C^\rho \Rightarrow B(s^*) > 0$, which implies that there exists interval $(\underline{s}, \overline{s})$, such that for all $\underline{s} < s < \overline{s}$ optimum industry structure is achieved at maximum concentration, otherwise the optimum industry structure is pure monopolistic competition

Proof The direct calculations show, that $\mu = 1 - \rho$ and

$$s^* = F + \left(\frac{c}{C}\right)^{\frac{\rho}{1-\rho}} \frac{f\rho}{1-\rho},$$

which implies

$$B(s^*) = u\left(\frac{s^* - F}{C}\right) - \frac{\mu C}{f} u\left(\frac{f}{c}\frac{1-\mu}{\mu}\right) s^* = \left[\frac{f\rho}{c}\right]^\rho (1-\rho)^{1-\rho} \left(\left(\frac{c}{C}\right)^{\frac{\rho}{1-\rho}} - \frac{F}{f}\right).$$

The rest is a simple algebra.

Remark. Let condition $f^{1-\rho}c^\rho = F^{1-\rho}C^\rho$ holds, then the output of oligopolistic firm of merit size s^* is

$$\bar{x} = \frac{s^* - F}{C} = \frac{c^{\frac{\rho}{1-\rho}}}{C^{\frac{\rho}{1-\rho}}} \frac{f\rho}{1-\rho} = \frac{(f^{1-\rho}c^\rho)^{\frac{1}{1-\rho}}}{C^{\frac{\rho}{1-\rho}}} \frac{\rho}{1-\rho} = \frac{F}{C}\frac{\rho}{1-\rho}.$$

Comparing this with corresponding output $\hat{x} = \frac{f}{c}\frac{1-\mu}{\mu} = \frac{f}{c}\frac{\rho}{1-\rho}$ of monopolistic competitive firm we obtain that "optimum" oligopoly produces at the same level as monopolistic competitor facing the production costs F, C. In particular, if $F = f$, $C = c$ the "optimum" oligopoly does not differ from monopolistic competitor in terms of output.

Non-CES Case

When consumer preferences differ from CES the model become more complicated, but the computer simulation shows that results are, in general, the same: all three cases of Theorem 1 are possible and depend on relations between oligopolistic and monopolistic competitive labor requirements. As before, larger F, C tends to Case 1, while for $F < f$, $C < c$ we typically obtain Case 3. The continuity considerations imply that Case 2 is possible for specific relation between F, C and f, c, which is not so simple as in case of CES utility. In what follows we assume that $F = f$,

$C = c$, which allows us to to disregard the obvious cost effects of oligopolization, both positive and negative. In CES case these conditions imply that concentration industry, i.e., transformation of myriads of monopolistic competitors into small number of oligopolistic 'cartels' generically leads to decreasing of social welfare, the only exception is the very specific case when oligopolies 'mimic' the monopolistic competitors, see Remark to Theorem 1. In non-CES case the outcome changes significantly.

In what follows we assume that the elementary utility function $u(x)$ satisfies the following additional

Assumption. *Derivatives* $0 < |u^{(k)}(0)| < \infty$, *for* $k = 1, 2, 3$.

Obviously CES preferences don't meet this requirement, while many other classes of preferences, for example, HARA $u(x) = (x + \alpha)^\rho - \alpha^\rho$ with $\alpha > 0$, CARA $u(x) = 1 - e^{-\alpha x}$, quadratic $u(x) = \alpha x - x^2/2$, fit it well. Our purpose is to show that under this assumption only Case 3 of Proposition 2 take place, at least when fixed labor requirement f is sufficiently small with respect to total labor supply $L = 1$.

Theorem 2. Let Assumption holds and fraction f/c is sufficiently small, then an inequality $B(s^*) > 0$ holds, and there exists interval $(\underline{s}, \overline{s})$, such that for all $\underline{s} < s < \overline{s}$ optimum industry structure is achieved at maximum concentration, otherwise the optimum industry structure is pure monopolistic competition.

Proof. It is obvious that Assumption imply the following equalities

$$r_u(0) = r_{u'}(0) = 0,\ r'_u(0) = -\frac{u''(0)}{u'(0)}.$$

The last equality is a simple consequence of identity

$$r'_u(x) = \frac{r_u(x)}{x}\left(1 + r_u(x) - r_{u'}(x)\right)$$

proved in [12]. Let' denote fraction f/c as φ, then the optimum size of oligopoly is equal to

$$s^* = f + c\xi\left(\frac{\mu c}{f}u\left(\frac{f}{c}\cdot\frac{1-\mu}{\mu}\right)\right) = c\left[\varphi + \xi\left(\frac{\mu}{\varphi}u\left(\varphi\cdot\frac{1-\mu}{\mu}\right)\right)\right].$$

Note that μ is actually an implicit function $\mu(\varphi)$ determined by equation

$$\mu = r_u\left(\varphi\frac{1-\mu}{\mu}\right) \tag{25}$$

as well as equilibrium output of the monopolistic competitive firms

$$x(\varphi) \equiv \varphi\frac{1-\mu(\varphi)}{\mu(\varphi)},$$

thus the Case 3 $B(s^*) > 0$ of Proposition 2 is equivalent to inequality

$$\Delta(\varphi) \equiv u\left(\xi\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)\right) - \frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\left[\varphi + \xi\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)\right] > 0. \tag{26}$$

We shall prove that

$$\Delta(0) \equiv \lim_{\varphi\to 0}\Delta(\varphi) = 0, \quad \Delta'(0) \equiv \lim_{\varphi\to 0}\Delta'(\varphi) > 0,$$

which implies that for all sufficiently small $\varphi = f/c$

First we calculate a series of more simple limits, then we obtain (26) by combining these limits.

$$\lim_{\varphi\to 0}\mu(\varphi) = 0, \ \lim_{\varphi\to 0}\frac{\varphi}{\mu(\varphi)} = 0, \ \lim_{\varphi\to 0}x(\varphi) = 0.$$

These statements were proved in [9] and they don't require Assumption to hold. In what follows, it will be crucial that there are exist finite limit values $u'(0)$ and $u''(0)$, provided by Assumption. As result, it also provides existence and finiteness of the all following limits. We start with the obvious equation

$$\lim_{\varphi\to 0}\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right) = \lim_{\varphi\to 0}\frac{u\left(x(\varphi)\right)}{x(\varphi)}(1-\mu(\varphi)) = u'(0).$$

Taking into account that by definition ξ is inverse function to u', we obtain

$$\lim_{\varphi\to 0}\xi\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right) = \xi\left(u'\left(0\right)\right) = 0.$$

Using (25) results in

$$\lim_{\varphi\to 0}\frac{\mu^2(\varphi)}{\varphi} = \lim_{\varphi\to 0}\frac{\mu(\varphi)}{\varphi}r_u(x(\varphi)) = \lim_{\varphi\to 0}\frac{r_u(x(\varphi))}{x(\varphi)}(1-\mu(\varphi)) = r_u'(0) = -\frac{u''(0)}{u'(0)}.$$

This implies

$$\lim_{\varphi\to 0}\mu'(\varphi)\mu(\varphi) = \frac{1}{2}\lim_{\varphi\to 0}\frac{(\mu^2(\varphi))'}{\varphi'} = \frac{1}{2}\lim_{\varphi\to 0}\frac{\mu^2(\varphi)}{\varphi} = -\frac{u''(0)}{2u'(0)},$$

and

$$\lim_{\varphi\to 0}x'(\varphi)\mu(\varphi) = \lim_{\varphi\to 0}\left(1-\mu(\varphi) - \frac{\varphi}{\mu^2(\varphi)}\mu'(\varphi)\mu(\varphi)\right) = 1 - \left(-\frac{u'(0)}{u''(0)}\right)\left(-\frac{u''(0)}{2u'(0)}\right) = \frac{1}{2}.$$

This allows to calculate more complicated limits:

$$\lim_{\varphi\to 0}\left(\frac{u\left(x(\varphi)\right)}{x(\varphi)}\right)'\mu(\varphi) = \lim_{\varphi\to 0}\frac{x(\varphi)u'\left(x(\varphi)\right) - u'\left(x(\varphi)\right)}{x^2(\varphi)}x'(\varphi)\mu(\varphi) =$$
$$= \lim_{x\to 0}\frac{xu'\left(x\right) - u'\left(x\right)}{x^2}\cdot\lim_{\varphi\to 0}x'(\varphi)\mu(\varphi) = \frac{u''(0)}{4},$$

and

$$\lim_{\varphi\to 0}\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)'\mu(\varphi)=\lim_{\varphi\to 0}\left(\frac{u\left(x(\varphi)\right)}{x(\varphi)}(1-\mu(\varphi))\right)'\mu(\varphi)=$$

$$=\lim_{\varphi\to 0}\left[-\mu(\varphi)\mu'(\varphi)\frac{u\left(x(\varphi)\right)}{x(\varphi)}+(1-\mu(\varphi))\left(\frac{u\left(x(\varphi)\right)}{x(\varphi)}\right)'\mu(\varphi)\right]=\frac{3u''(0)}{4}.$$

Taking into account that $\xi=(u')^{-1}$ and thus $\xi'=1/u''$, we obtain

$$\lim_{\varphi\to 0}\frac{\xi\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)}{\mu(\varphi)}=\lim_{\varphi\to 0}\xi'\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)\frac{\mu(\varphi)\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)'}{\mu(\varphi)\mu'(\varphi)}=-\frac{3u'(0)}{2u''(0)}.$$

These calculations imply

$$\lim_{\varphi\to 0}u\left(\xi\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)\right)=u\left(0\right)=0$$

and

$$\lim_{\varphi\to 0}\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\left[\varphi+\xi\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)\right]=u'(0)\cdot 0=0,$$

which means that $\Delta(0)=0$. Moreover, by definition $u'(\xi(z))=z$, therefore,

$$\begin{aligned}\Delta'(\varphi)&=u'\left(\xi\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)\right)\xi'\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)\left[\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right]'-\\&-\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)'\left[\varphi+\xi\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)\right]-\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)-\\&-\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\xi'\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)\left[\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right]'=\\&=-\mu(\varphi)\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)'\frac{\varphi}{\mu(\varphi)}-\mu(\varphi)\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)'\frac{\xi\left(\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\right)}{\mu(\varphi)}-\frac{\mu(\varphi)}{\varphi}u\left(x(\varphi)\right)\end{aligned}$$

which implies

$$\lim_{\varphi\to 0}\Delta'(\varphi)=-\frac{3u''(0)}{4}\cdot 0-\frac{3u''(0)}{4}\left(-\frac{3u'(0)}{2u''(0)}\right)-u'(0)=\frac{1}{8}u'(0)>0.$$

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