Social security system and the complementarity of fiscal policy instruments[1]

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Many countries are launching pension reforms in order to secure the sustainability of the social security system in the future which is at risk because of increasing expenditures on pensions. The main drivers of these dynamics are demographic changes: fertility rates below the replacement level and higher life expectancy.

Although several reforms of social security systems have been implemented, federal transfers remain one of the key sources of balancing the budget of pension funds. Moreover, they are expected to rise steadily in the future: according to the OECD estimates, this part of fiscal expenditures will increase from 9.3% of the GDP in 2010 to 11.7% of the GDP in 2050.[3] In Russia, the deficit of the pension fund is also covered by a transfer from the federal budget. This transfer was 4.3% of the GDP in 2013 and 3.4% in 2014.[4] Financing pension fund deficits out of the federal budgets has become even more complicated after the financial crisis of 2008-2009 and the European debt crisis, which started in 2010.

In the case of increasing pension expenditures pension reforms (the introduction of a higher retirement age, higher social contributions, lower pensions) can be considered as an alternative to the traditional measures of the fiscal consolidation. This research defines the optimal combination of two fiscal instruments (rate of social contributions and income tax) chosen by the social planner and compares social welfare in case of balanced and unbalanced pension system. It also specifies how this policy mix changes with the retirement age, life expectancy, labor productivity and how it depends on the type of the pension system (balanced or unbalanced).

The research of pension reforms can be classified by the type of pension system under consideration. First group consists of research of PAYG reforms (e.g. Nickel et al., 2008; Karam et al., 2010; Kilponen et al., 2006; Castro et. al., 2016; Almeida et al., 2013; Pierrard-Snessens, 2009; Marchiori-Pierrard, 2012). Others consider the switch from PAYG to fully funded pension system (Borsch-Supan et al., 2006; McGrattan-Prescott, 2015). Both types of pension systems were analyzed in Marchiori et al. (2011) and de la Croix et al. (2013). This research falls into the first category.

The analysis is based on the overlapping generations model (OLG) initially developed by Yaari (1965) and Blanchard (1985) and extended further by Buiter (1988), Giovannini (1988), Weil (1989) and Bovenberg (1993). In order to investigate the optimal

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[3]OECD, Pensions at glance 2013

[4]Transfer has decreased due to the freeze of the accumulation part of the pension savings.

policy mix we extend the model of Heijdra and Bettendorf (2006), who analyzed the economic consequences of lower pensions and a higher retirement age in an open economy with traded and non-traded sectors. They, however, consider an exogenous interest rate along with a rudimentary pension system, which allows them to analyze intergenerational redistribution which is assumed to be balanced. We extend their model to investigate the unbalanced budget of a pension fund in a closed economy with an endogenous interest rate, which allows us to account for the effect of different economic characteristics (retirement age, life expectancy, labor productivity and population structure) on the capital accumulation. While Heijdra and Bettendorf (2006) consider the consequences of shocks to the welfare of each generation, we investigate the optimal subset of measures conducted by a benevolent government, which maximize the social welfare function.

Nickel et al. (2008) extend the framework of Nielsen (1994) and Heijdra and Bettendorf (2006) by considering an unbalanced pension system and assuming that firms issue equities and face adjustment costs in investment. They analyze three fiscal scenarios in an economy with decreasing population: the suspension of the public pension system and a decrease in lump-sum labor tax; the suspension of the public pension system and a decrease in distortionary corporate tax; or an increase in the retirement age. Their results suggest that the adverse consequences of pension reforms can be decreased by appropriate taxation policies. The main difference with our research is that Nickel et al. (2008) consider the government as a non maximizing entity and investigate how the predetermined changes in policy instruments would affect the transition of the main macroeconomic variables to the new equilibrium in an open economy, while we define socially optimal fiscal policy (social contributions and income tax) and compare the optimal set of policy instruments in equilibrium with both increasing and decreasing population in the closed economy.

The fact that income tax rate and social contributions are substitutes is confirmed by the comparison of optimal policy mix in the case of balanced and unbalanced pension system. In the former case social contributions are strictly positive and decreases with the population growth, while income tax rate is constant and does not depend on it. In the case of unbalanced pension system, on the contrary, the corner solution is optimal: social contributions are at zero while income tax rate decreases with the population growth. This policy mix remains optimal in the steady states with the different levels of life expectancy and labor productivity. Unbalanced pension system leads to the optimal interest rate which changes with structural characteristics of the economy while social contributions are at zero.

1 The model

The model of Heijdra and Bettendorf (2006) was extended by introducing an unbalanced pension system with the deficit covered by a benevolent government, which conducts fiscal policy to maximize the social welfare. A closed economy with an endogenous interest rate is considered. Futhermore the model was extended to include two types of agents: Ricardian and non Ricardian, whose share in the population equals λ.

The economy consists of two types of households, firms, the government and the pension fund. Infinitely lived households maximize the present value of utility from consumption taking into account the life expectancy. The others consume all their after tax income each period. Both types of households work and pay income tax throughout the life, social contributions before the retirement age and receive pensions at the retirement. Pensions are paid by the pension fund, the deficit of which is covered by the government. Public debt is financed by bonds held by the households and income tax payments.

In the model upper case variables are aggregates, lower case variables with a line ($\bar{c}$) denote individual variables, while lower case variables without any notation are aggregates per unit of efficient labor.

1.1 Households

Individual households

The representative consumer of a Ricardian type is born at time υ maximizes the expected present value of instantaneous utility of consumption.

$$U(\upsilon, t) = \int_{t}^{\infty} [\ln \bar{c}(\upsilon, t)] \, \mathrm{e}^{(\rho+\beta)(t-\tau)} d\tau, \tag{1}$$

where $\bar{c}$ is personal consumption, $\rho > 0$ is the rate of time preference and $\beta \geq 0$ is the probability of death.

Following Bettendorf and Heijdra (2006) we consider PAYG pension system introduced by Nielsen (1994). The households pay income taxes during their lives, while paying social contributions t_W before the retirement age π and receiving pensions z at retirement. The threshold level π can be loosely considered as retirement age as the households continue to work after it.

Labor supply is non elastic: each household supplies one unit of labor. The households pay income tax on the labor income and receive an interest rate $r(t)$ on the financial wealth, $\bar{a}(\upsilon, t)$. The payment $\beta a(\upsilon, t)$ is the actuarially fair annuity paid by the life insurance company.[1] Interest and non-interest net labor income, $WI(\upsilon, t)$, are spent on consumption and saving. Household financial wealth consists of capital goods, $\bar{k}$, and government bonds, ($\bar{a}^G$), both denominated in terms of consumer goods.

[1]See Yaari (1965), Blanchard (1985)

The household budget constraint in terms of the consumer good is:[2]

$$\dot{\bar{a}}(\upsilon,t) = (r(\tau)+\beta)\bar{a}(\upsilon,t) + WI(\upsilon,t) - \bar{c}(\upsilon,t), \tag{2}$$

$$\bar{a}(\upsilon,t) = \bar{k}(\upsilon,t) + \bar{a}^G(\upsilon,t), \tag{3}$$

$$WI(\upsilon,t) = \begin{cases} (1-t_L)W^N(\upsilon,\tau) - t_W & \text{for } t-\upsilon \leq \pi, \\ (1-t_L)W^N(\upsilon,t) + z & \text{for } t-\upsilon > \pi. \end{cases}, \tag{4}$$

where $W^N(\upsilon,t)$ is the wage at time t of the worker born at time υ.

Labor productivity decreases with the age of the worker. The worker of generation υ at time t supplies $n(\upsilon,t)$ efficiency units of labor:

$$n(\upsilon,t) = E(t-\upsilon)\bar{l}(\upsilon,t), \tag{5}$$

where $\bar{l}(\upsilon,t) = 1$ is the labor hours and $E(t-\upsilon)$ is the efficiency index, which falls exponentially with the worker's age:[3]

$$E(t-\upsilon) = \omega_0 \mathrm{e}^{-\alpha(t-\upsilon)}, \tag{6}$$

where ω_0, a positive constant, equals to 1 and $\alpha > 0$ specifies the speed at which the efficiency falls with age.

At each moment t the household chooses the paths of consumption and financial assets so as to maximize the present value of lifetime utility (1) subject to budget constraint (2) and a transversality condition. The initial value of the financial assets $a(\upsilon,t)$ and the government consumption per household are taken as given.

The optimal path of household consumption is defined by the Euler equation:

$$\frac{\dot{\bar{c}}(\upsilon,t)}{\bar{c}(\upsilon,t)} = r(t) - \rho, \tag{7}$$

which specifies that in each moment consumption is proportional to the total wealth:

$$\bar{c}(\upsilon,t) = (\rho+\beta)(\bar{a}(\upsilon,t) + \bar{a}^H(\upsilon,t)), \tag{8}$$

where $\bar{a}^H$ is human wealth defined as the present value of the after-tax labor income:

$$\bar{a}^H(\upsilon,t) = \int_t^{\infty} WI(\upsilon,\tau)\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau. \tag{9}$$

where $R(t,\tau) = \int_t^{\tau} r(s)ds.$.

For non Ricardian agents their consumption is defined only by their non interest

[2]A dot above the variable stands for the variable's time derivative: $\dot{\bar{a}}(\upsilon,t) = d\bar{a}(\upsilon,t)/dt$.

[3]As in Blanchard (1985).

labor income, specified by the equation 4.

Demography

The framework allows us to consider non-zero population growth, by distinguishing the probability of death $\beta \geq 0$, and the probability of birth, $\eta > 0$.[4] The population size $L(t)$ grows with net growth rate n_L:

$$\frac{\dot{L}(t)}{L(t)} = \eta - \beta = n_L. \tag{10}$$

Taking into account the initial condition $L(0) = 1$, the population size is:

$$L(t) = \mathrm{e}^{n_L t}. \tag{11}$$

The size of the generation born at t is proportional to the current size of the population:

$$L(\upsilon, \upsilon) = \eta L(\upsilon). \tag{12}$$

The size of each generation falls exponentially with the probability of death β:

$$L(\upsilon, t) = \mathrm{e}^{\beta(\upsilon - t)} L(\upsilon, \upsilon), t \geq \upsilon \tag{13}$$

The current size of the generation born at time υ can be obtained by substituting (11) and (12) into (13):

$$L(\upsilon, t) = \eta \mathrm{e}^{\eta \upsilon} \mathrm{e}^{-\beta t} \tag{14}$$

Aggregate household sector

The aggregate variables are defined as the integral of the variable values, specific for each living generation, weighted by the size of that generation. Aggregate consumption, for example, can be defined as follows:

$$C(t) = \int_{-\infty}^{t} L(\upsilon, t) \bar{c}(\upsilon, t) d\upsilon, \tag{15}$$

where $L(\upsilon, t)$ and $\bar{c}(\upsilon, t)$ are given by (14) and (8), respectively.

In can be shown that aggregate consumption of the Ricardian agents is proportional to the household's wealth, where $A(t)$ is aggregate financial wealth and $A^H(t)$ is aggregate human wealth:

$$C^R(t) = (\rho + \beta) \left[A(t) + A^H(t) \right]. \tag{16}$$

The growth rate of the aggregate consumption is obtained from (15), taking into

[4] This framework was developed by Buiter (1988).

account (7) and (14):

$$\frac{\dot{C^R}(t)}{C^R(t)} = [r(t) - \rho] + \frac{\eta L(t)\bar{c}^R(t,t) - \beta C^R(t)}{C^R(t)}, \tag{17}$$

where $r(t) - \rho$ is the growth of individual consumption, while the second term represents the so-called generational turnover (Bettendorf and Heijdra, 2006), which depends on the demographic parameters. Aggregate consumption increases with the arrival of new agents and decreases with the death of the older generation.

The growth rate of the aggregate consumption can be simplified to:

$$\frac{\dot{C^R}(t)}{C^R(t)} = r(t) - \rho + \alpha + n^L - (\rho + \beta)\frac{\eta\gamma L(t) + (\alpha + \eta)A(t)}{C^R(t)}, \tag{18}$$

$$\gamma(t) = \frac{-d(t)}{r(t) + \beta} + (r(t) + \alpha + \beta)\left(\frac{\mathrm{e}^{-\beta\pi}}{1 - \mathrm{e}^{-\eta\pi}}\right)\left(\frac{z - d(t)}{r(t) + \beta}\right)\left(\frac{\mathrm{e}^{-r(t)\pi} - \mathrm{e}^{-n_L\pi}}{n^L - r(t)}\right). \tag{19}$$

The aggregate consumption growth, therefore, exceeds the growth of individual consumption if the net population growth is positive ($n_L > 0$) and the labor productivity decreases over time ($\alpha > 0$). It can be lower if newborns consume less or due to the redistribution from the young to the old through the pension system. In contrast to Bettendorf and Heijdra (2006) γ depends on the deficit of the pension fund, defined below.

Bettendorf and Heijdra (2006) point out that $\eta\gamma/(r + \alpha + \beta)$ can be considered as per capita deficit of the pension system. When $r > n_L$ social contributions are perceived by working households as tax on the labor income, as they are forced to save at the rate n_L which is lower then the market rate r. In case of retirees the opposite affect takes place.

Aggregate financial wealth is defined as follows:

$$A(t) = \int_{-\infty}^{t} L(v,t)\bar{a}(v,t)dv. \tag{20}$$

The definition of aggregate savings can be found by differentiating equation (20) for the aggregate financial wealth with respect to time and taking into account that the newborn generation does not have any financial wealth, $\bar{a}(t,t) = 0$:

$$\dot{A}(t) = -\beta A(t) + \int_{-\infty}^{t} L(v,t)\dot{\bar{a}}(v,t)dv \tag{21}$$

By substituting (2) in (21) we get:

$$\dot{A}(t) = r(t)A(t) + WI(t) - C^R(t), \tag{22}$$

$$WI(t) = \frac{\eta\omega_0}{\alpha+\eta}(1-t_L)F_N(k_N(t),1))L(t) + D(t), \tag{23}$$

where $F_N(k_N(t),1)$ is the marginal product of labor and $D(t)$ is the deficit of pension system.

The aggregate labor supply at time t measured in efficiency units is proportional to the population size in the corresponding period and is obtained from (5), (6), (11) and (14):

$$N(t) = \int_{-\infty}^{t} L(v,t)\bar{n}(v,t)dv = \frac{\eta\omega_0}{\alpha+\eta}L(t). \tag{24}$$

1.2 Firms

As opposed to Bettendorf and Heijdra (2006) we consider a closed economy with endogenous interest rate, important in the estimation of pensions. The output is produced according to the Cobb-Douglas technology:

$$Y = F(K,N) = K^{\varepsilon}N^{1-\varepsilon}, \tag{25}$$

where K and N represent capital and efficiency labor units. Producers maximize profit, choosing the optimal level of capital and labor:

$$\Pi(t) = Y(t) - \int_{-\infty}^{t} W^N(v,t)L(v,t)dv - W^K(t)K(t), \tag{26}$$

where $W^K(t)$ is a capital rent and $W^N(v,t)$ is the wage at time t of the worker of generation v.

The first order conditions are:

$$W^K(t) = F_K(k_N(t),1), \tag{27}$$

$$W^N(t) \equiv \frac{W^N(v,t)}{E(\tau-v)} = F_N(k_N(t),1), \tag{28}$$

where $F_K = \partial F/\partial K_N$ and $F_N = \partial F/\partial N$. $W^N(t)$ is the wage per efficiency unit of labor and $k_N(t) = K(t)/N(t)$ is the capital efficiency unit of labor.

The produced output is allocated to private consumption, investment I and government expenditures G.

$$Y(t) = C^T(t) + I(t) + G(t). \tag{29}$$

The optimal investment decision is based on the maximization of the net present value of cash flows from the investor's capital stock subject to the capital accumulation identity:

$$V(t) = \int_t^\infty \left[W^K(\tau)K(\tau) - I(\tau)\right] \mathrm{e}^{-R(t,\tau)} d\tau, \tag{30}$$

$$\dot{K}(t) = I(t) - \delta K(t), \tag{31}$$

where $R(t,\tau) = \int_t^\tau r(s)ds$ is a discount factor. First order condition, (32), specifies that the rental rate W^K equals the return on the capital $r(t)$ taking into account the amortization rate δ.

$$W^K(t) = r(t) + \delta \tag{32}$$

1.3 Public sector and the benevolent government

Government budget identity defines the accumulation path of public debt $A^G(t)$, which depends on the current government expenditures $G(t)$, labor tax revenues and additional expenditures coming from the deficit of the pension fund $D(t)$. It can be written as follows:

$$\dot{A^G}(t) = r(t)A^G(t) + G(t) - t_L W^N(t)N(t) + D(t). \tag{33}$$

Taking into account the transversality condition:

$$\lim_{\tau \longrightarrow \infty} A^G(\tau)\mathrm{e}^{-R(t,\tau)} = 0, \tag{34}$$

public debt is:

$$A^G(t) = \int_t^\infty \left[t_L W^N(\tau)N(\tau) - G(\tau) - D(\tau)\right] \mathrm{e}^{-R(t,\tau)} d\tau. \tag{35}$$

The key difference with the paper of Heijdra and Bettendorf (2006) is the assumption that PAYG pension system can be run on an unbalanced-budget basis, with a deficit $D(t)$.

$$t_W(1 - e^{-\eta\pi})L(t) = z\mathrm{e}^{-\eta\pi}L(t) - D(t) \tag{36}$$

The left-hand side of (36) represents the total social contributions paid by the young, while on the right-hand side are total pensions paid to the old and the surplus (or deficit) of the pension fund if the sum of social contributions and pensions do not match.

For the easier comparison we assume that the government determine the value of social contributions setting the value of ψ, which is the share of social contributions in the median wage:

$$t_W = \psi\omega_0 F_N(k(t), 1)\mathrm{e}^{-\alpha\left(\pi - \frac{1}{\alpha}\ln\left(\frac{1+\mathrm{e}^{\alpha\pi}}{2}\right)\right)}. \tag{37}$$

Social contributions, therefore, depend on the retirement age.

We define the social welfare function as the present value of the utility of all currently living and future generations weighted by their share in the population. The first term in (38) represents the welfare of young generations, while the second is the welfare of the retirees.

$$SW(t) = \int_{t}^{\infty} \int_{-\infty}^{\tau-\pi} L(\upsilon,\tau)[\ln \bar{c}(\upsilon,\tau)] e^{(\rho+\beta)(t-\tau)} d\upsilon d\tau + \tag{38}$$

$$+ \int_{t}^{\infty} \int_{\tau-\pi}^{\tau} L(\upsilon,\tau)[\ln \bar{c}(\upsilon,\tau)] e^{(\rho+\beta)(t-\tau)} d\upsilon d\tau$$

The individual consumption of the young and elder generations can be expressed as functions of their individual human wealth (a_y^H and a_o^H, respectively), which depends on the capital per efficiency units of labor.

1.4 Model summary

The key equations in per capita terms are presented in Table 1 below. The endogenous variables are k, y, c, a, a^G, r, W^N,W^K, γ, n. Parameters are β, η, n_L, α, λ and ρ. Policy instruments are π, z, t_W, t_L.

Table 1. *Summary of the Model*

Description	Analytical representation	
Dynamic equations:		
Capital	$\dot{k}(t) = ny(t) - c^T(t) - g(t) - (n_L + \delta)k(t)$	$(T1.1)$
Private consumption	$\dot{c}^R(t) = (r(t) - \rho + \alpha)c^R(t) - (\rho + \beta)(\eta\gamma(t) + (\alpha + \eta)a(t))$	$(T1.2)$
Public debt	$\dot{a}^G(t) = (r(t) - n_L)a^G(t) + g(t) - nt_L W^N(t) + d(t)$	$(T1.3)$
Private savings	$\dot{a}(t) = (r(t) - n_L)a(t) - c^R(t) + n(1 - t_L)W^N(t) + d(t)$	$(T1.4)$
Static equations:		
non Ricardian consumption	$c^K(t) = WI(t) = \dfrac{\eta\omega_0}{\alpha + \eta}(1 - t_L)F_N(k_N(t, 1) - d(t)$	$(T1.5)$
	$\gamma(t) = \frac{-d(t)}{r(t)+\beta} + \left(\frac{\mathrm{e}^{-\beta\pi}}{1-\mathrm{e}^{-\eta\pi}}\right)\left(\frac{z-d(t)}{r(t)+\beta}\right)(r(t) + \alpha + \beta)\left(\frac{\mathrm{e}^{-r(t)\pi}-\mathrm{e}^{-n_L\pi}}{n^L-r(t)}\right)$	$(T1.6)$
Pension fund	where $t_W(1 - \mathrm{e}^{-\eta\pi}) = z\mathrm{e}^{-\eta\pi} - d(t)$	
Rental rate	$W^K(t) = \varepsilon k_N(t)^{\varepsilon-1} = \varepsilon \left(\dfrac{k(t)}{n}\right)^{\varepsilon-1}$	$(T1.7)$
Interest rate	$r(t) = W^K(t) - \delta$	$(T1.8)$
Wage	$W^N(t) = (1 - \varepsilon)y(t)$	$(T1.9)$
Output	$y(t) = k_N^{\varepsilon}(t) = \left(\dfrac{k(t)}{n}\right)^{\varepsilon}$	$(T1.10)$
Supplied efficiency units	$n = \frac{\eta\omega_0}{\alpha+\eta}$	$(T1.11)$
Total consumption	$c^T(t) = (1 - \lambda)c^R(t) + \lambda c^K(t)$	$(T1.12)$
Savings	$a(t) = k(t) + a^G(t)$	$(T1.13)$

The Eq.T1 corresponds to the accumulation of capital per capita, and it is obtained by combining (31) and (32). Eq.T2 stands for the optimal path of per capita consumption, obtained from (18) in per capita terms. Eq.T3 is the government budget constraint expressed in per capita terms, derived from the government budget constraint (33).The last dynamic equation, Eq.T4, represents the accumulation of per capita assets and is obtained from (22), taking into account (23) and (36).

Definition 1. *Given the set of policy variables* $\{g_t, t_L, t_W, z, \pi\}$ *that satisfy the government budget constraint, the set* $\{W_t^K, r_t, y_t, c_t^T, k_t, a_t, a_t^G, d_t\}$ *defines equilibrium, if it satisfies the optimal conditions of households and firms, (7) and (27)-(28) and equilibrium conditions for goods and capital markets.*

$$y(t) = c^T(t) + i(t) + g(t) \tag{40}$$

$$a(t) = k(t) + a^G(t) \tag{41}$$

As the system of dynamic equations is non-linear and cannot be solved analytically we are solving numerically the system of equations T1.1-T1.3, taking into account T1.12.[5]

To distinguish the socially optimal policy mix of measures (income tax and social contributions) we check if the resulting set of possible equilibria satisfies the stability condition of the equilibrium and the condition on the limit of public debt.[6] In the maximization problem the level of pensions and the retirement age were fixed in order to analyze optimal choice of income taxes and social contributions in the existence of pension obligations. It is known that in the OLG model with dynamically efficient equilibrium PAYG pension system worsens social welfare. If the level of pensions is chosen optimally from the maximization of social welfare, it would be optimal to set pensions and social contributions which are equal zero.

It is worth mentioning that the equilibrium with diminishing labor productivity can be dynamically inefficient if the speed of the decrease in productivity α is large enough. In this case labor income is high during the youth and falls rapidly with age, so agents save a lot during youth so the capital stock can be too large and there is overaccumulation of capital. Necessary condition for dynamic inefficiency is $\alpha < \rho$, which corresponds to the positive interest rate. The calibration used in the paper satisfies the condition of dynamic efficiency.

[5] All calculations were conducted using Matlab. Restricting $\dot{k}(t) = 0$, and $\dot{a}^G(t) = 0$ we use a root-finding method (the bisection method) to define the level of capital per capita to bring the growth of per capita consumption to zero, $\dot{c}(t) = 0$. All possible combinations of fixed and variable parameters on the initially set intervals are considered to determine the steady-state level of k^* and the corresponding combinations of parameters which bring $\dot{c}(t) = 0$.

[6] The stability condition insures that the determinant of the Jacobian matrix of the log-linearized dynamic system of equations T1.1, T1.2 and T1.4 is less than zero. In this case model is locally saddle-point stable.