Should Monetary Authorities Prick Asset Price Bubbles? Evidence from a New Keynesian Model with an Agent-Based Financial Market

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Abstract: We develop the approach based on the synthesis of New Keynesian macroeconomics and agent-based models, and build a model, allowing for the incorporation of behavioral and speculative factors in financial markets in a New Keynesian model with a financial accelerator, à la Bernanke et al. (1999). Using our model, we, for the first time in the literature, study the optimal strategy of central banks in pricking asset price bubbles for the maximization of social welfare and preserving financial stability. Our results show that pricking asset price bubbles can be a policy that enhances social welfare, and reduces the volatility of output and inflation; especially, in the cases when asset price bubbles are caused by credit expansion, or when the central bank conducts effective information policy, for example, effective verbal interventions. We also argue that pricking asset price bubbles with the lack of the effectiveness of information policy, only by raising the interest rate, leads to negative consequences to social welfare and financial stability.

Keywords: Optimal monetary policy; Asset price bubble; New Keynesian macroeconomics; Agent-based financial market.

JEL-Classification: E03, E44, E52, E58, G01, G02

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1 Introduction

Crashes and bubbles in financial markets that lead to instability and volatile dynamics in the economy have been the subject of hot debate in the macroeconomic literature for a long time. The Global Financial Crisis of 2008–2009, and the recent bubble on the stock market in China, have only increased the interest in these phenomena, stressing several drawbacks in existing macroeconomic frameworks. These include issues related to the influence of behavioral and speculative factors in financial markets on the economy and to the necessary response of regulatory authorities to financial bubbles.

One stream of research that arose after the Great Recession is the incorporation of agent-based financial markets, which may reproduce speculative phenomena in financial markets in traditional macroeconomic frameworks. The further development of this approach is the first contribution of our paper. For this purpose, we construct a more complex model than the models from previous literature in this field. This complication allows us to make a second contribution, the most significant contribution of our paper. Using our constructed model, for first time in the literature (to the best of our knowledge), we study the optimal strategy of central banks in pricking asset bubbles for the maximization of social welfare and preserving financial stability.

The hot debate about the necessary response of monetary policy on asset price bubbles, among policy makers and in academia, is known as the "clean" versus "lean" debate. Following the "clean" approach, central banks should not respond to asset price bubbles before the bubble bursts, above the necessary reaction for the stabilization of inflation and employment, but just clean up the consequences of the bubble. This approach may be more preferable for monetary policy, because of several possible reasons: generally, bubbles are hard to detect; a bubble may exist only in a small market; and raising interest rates may not sufficiently affect bubbles or, sometimes, may cause the bubble to burst more severely. The "clean" approach prevailed in central banks and academia before the Global Financial Crisis of 2008–2009. According to another point of view, following the "lean" approach, central banks should conduct monetary policy that leans against asset price bubbles and leads to increasing interest rates above the necessary reaction for the stabilization of inflation and employment. After the Global Financial Crisis, the "lean" approach has begun to seem more preferable. Nowadays, the focus of macroeconomic discussion has changed, from the question of whether central banks should respond on asset price bubbles or not, to the question of how central banks should respond to asset price bubbles, in which cases should they actively respond, and which strategy is best to use.[1]

Although the optimal response of monetary policy on asset price bubbles has been the subject of hot debate in mainstream macroeconomics for decades and has been studied by many authors (e.g. Bernanke and Gertler (2000), Cecchetti (2000), Bordo and Jeanne (2002), Bean (2004), Gruen et al. (2005)), surprisingly, there is a lack of research on cases when monetary policy serves to prick asset price bubbles, although this topic has been widely discussed in the literature (e.g. in Roubini (2006) and Posen (2006)). In the case of the identifying the need for pricking, the central bank has already missed time when the bubble grew and now it has to decide which strategy for pricking the bubble would be best, or whether it would be better not to do anything against the bubble. Almost all of the papers in the literature do not concentrate on such cases, when the central bank is already in an unfavorable position. But analysis of such cases is important, because of problems related to the identification of bubbles at the early stage. At this stage, misalignment is not yet large and monetary policy usually can start affecting the bubble only after it has grown to a substantial size. The lack of research on the pricking of asset price bubbles is caused by the lack in development of potentially appropriate methods for analysis. The framework constructed in our paper fills this gap.

In this paper, we build a joint model consisting of a New Keynesian model with a financial accelerator, à la Bernanke et al. (1999), which sets the real sector and an agent-based model, which sets the financial market populated by bounded rational traders. The market price of assets in this joint model, which is determined through trading (the interaction of traders in the financial market), can sometimes significantly deviate from the fundamental price of assets. In such cases, bubble cases, traders may start selling their assets, which leads to the bursting of the bubble and, perhaps, to a crisis in the economy. The monetary policy in the joint model, in addition to the change of the interest rule from the Taylor rule, can raise the interest rate in order to prick the bubble or can influence the expectations of traders much earlier than the bubble would burst by itself; for example, it can announce the existence of a bubble in the media. The influence of

[1] For more detailed discussion of the "clean" versus "lean" debate see, for example, Mishkin (2011) or Brunnermeier and Schnabel (2015).

the central bank on traders' expectations in the model is the central bank's information policy, which can be more or less effective, if traders believe or ignore the announcements of the central bank, respectively.

There are only two papers in this field that study the necessary response of monetary policy on asset price bubbles, Filardo (2004) and Fouejieu et al. (2014), in which the authors consider pricking bubbles. In both papers, the authors use simple New Keynesian models, in which the interest rate is determined through the Taylor rule and affects the probability of the bursting of asset price bubbles, so the bubble endogenously depends on the interest rate. In comparison to these papers, we construct a framework that includes not only basic equations for New Keynesian models, but also the production function, capital and labor markets, credit market frictions, and the amplification mechanism of the financial sector. This allows us to analyze the influence of pricking asset price bubbles on social welfare and other macroeconomic variables. Moreover, in our model, in addition to the endogenous relationship between the interest rate and the probability of bursting the asset bubble, there are two extra possible effects of monetary policy on asset price bubbles. First, the central bank can raise the interest rate above the necessary reaction from the Taylor rule at certain times, when the deviation of the market asset price from the fundamental asset price becomes too large. Second, the central bank can influence traders' expectations about the future development of asset bubbles through its information policy, for example, through verbal interventions.

We calculate social welfare losses and the volatility of output and inflation in various cases, which differ by the size of the response of monetary policy on asset price bubbles, by the efficiency of the central bank's information policy, or by the existence of the liquidity flow from the real sector to the financial sector that allows us to mimic situations in which bubbles are partially boosted by credit expansion, such as the Global Financial Crisis of 2008-2009. Our results demonstrate that, in some cases, pricking asset price bubbles by the central bank can reduce social welfare losses from asset price bubbles, as well as the volatility of output and the volatility of inflation. This effect is larger, in cases when asset price bubbles are caused by credit expansion and when the central bank conducts effective information policy; in other words, it can effectively influence traders' expectations. We also argue that pricking asset price bubbles only by raising the interest rate, with a lack of effective information policy, leads to negative consequences for social welfare and financial stability.

Our paper also concerns another stream of research, in which authors integrated financial market models with, or at least following the logic of, agent-based models, which may generate speculative phenomena in financial markets in traditional macroeconomic models of the real economy. The financial market, in the models from this field, is usually constructed following the logic of the chartists/ fundamentalists agent-based model of the financial market, because this model allows researchers to simply include the main stylized facts of financial markets in the analysis. In the early papers in the field Kontonikas and Ioannidis (2005) and Kontonikas and Montagnoli (2006), authors add rules for the behavior of a financial market to a simple New Keynesian model with rational expectations. A large part of more recent papers in the field (Scheffknecht and Geiger (2011), Ascari et al. (2013), Lengnick and Wohltmann (2013), Pecora and Spelta (2013), and Lengnick and Wohltmann (2016)) continue the development of this approach, but focus on using bounded-rational expectations in the real sector of economy. Some authors (Bask and Madeira (2011), Bask (2012) and Gwilym (2013)) still work with rational expectations, integrating financial market models into existing, more complex traditional frameworks (Westerhoff (2012), Naimzada and Pireddu (2014) and Naimzada and Pireddu (2015)) using traditional Keynesian income-expenditure models for the real sector, in order to pay more attention to the analysis of interactions between the real sector and the financial market, than on the complexity of the joint model. In our paper, we use a more complex agent-based model of the financial market than in previous literature, integrating it into a New Keynesian model with a financial accelerator, à la Bernanke et al. (1999), with rational expectations. Exactly this combination allows us to make the largest contribution of our paper: to study the optimal strategy for central banks in pricking asset price bubbles for the maximization of social welfare and the preservation of financial stability.

The paper is organized as follows. Section 2 includes the description of the model, where Sections 2.1 and 2.2 describe the New Keynesian part and the agent-based part of the model correspondingly, and Section 2.3 shows how we connect two parts in the joint model. The calibration of the model is discussed in Section 3. Section 4 presents the first simulations of our model and the discussion of their robustness, while Section 5 contains the analysis of the optimal strategy of monetary policy in pricking asset price bubbles. Finally, Section 6 concludes the paper.

2 Model

Our joint model consists of two parts: the real sector, which is similar to the financial accelerator framework with a bubble from Bernanke and Gertler (2000) and the financial sector, which is set by the agent-based model. The model includes 7 types of agents, 6 of them: households, entrepreneurs, retailers, capital producers, the central bank, and the government, as related to the New Keynesian part of the joint model. The agent-based financial market is populated by bounded rational traders, who trade futures contracts on capital from the real sector using behavior rules and, thereby, set the market price of capital in the real sector. We construct the agent-based financial market using the logic of the model from Harras and Sornette (2011), but change the essence of the behavior rules for traders, add the liquidity flow from the real sector to the financial market, and calibrate the model to the real data. The agent-based financial market is calibrated in order to reproduce possible bubbles.

In comparison to the original paper of Bernanke and Gertler (2000), we exclude money and exogenous shocks from the model, for the simplicity of the analysis. In the model, the central bank sets the interest rate according to the Taylor rule but can also additionally raise the interest rate to try to prick bubbles in the market price of capital. This may also affect traders' expectations or their opinions; we name the influence of the additional increase in the interest rate on the traders' opinions "the information policy of the central bank." The change of market price in the agent-based financial market is transmitted to the real sector through the market change impulse. The market price impulse is not a random shock, but is a variable with a complex dynamic that is set by the agent-based financial market. The deviation of the market price of capital from the fundamental price in the real sector, first of all, influences the costs of resources for entrepreneurs through the financial accelerator mechanism.

To sum up, the real sector and the financial market are connected through 4 transmission mechanisms: the dynamics of the financial market determines the market price of capital in the real sector; the real sector affects traders' opinions about the fundamental price of capital; the central bank can also affect traders' opinions by the information policy; and there are liquidity flows from the real sector to the financial market. It is worth noting, that the periods of operation in two parts of the joint model are different: in the real sector, the period corresponds to one quarter, in the financial market it corresponds to one week. Section 2.1 further presents the description of the New Keynesian part of the joint model that sets the operation of the real sector. Section 2.2 provides the specification of the agent-based part of the model - the financial market, while the interaction of parts with each other is discussed in Section 2.3.

2.1 Real Sector

2.1.1 Households

The model includes a continuum of households, normalized to 1. Each of them solves the following standard utility maximization problem:

$$maxE_0 \sum_{t=0}^{\infty} \beta^t U(C_t,L_t) = maxE_0 \sum_{t=0}^{\infty} \beta^t \{\log C_t - \frac{L_t^{1+\sigma_l}}{1+\sigma_l}\}, \tag{2.1}$$

which depends on the current consumption C_t and the labor supply L_t. $0 < \beta < 1$ denotes the discount factor and σ_l is the inverse elasticity of labor supply.

The budget constraint of a household has the following form:

$$C_t + B_t = W_t L_t + \frac{R_{t-1} B_{t-1}}{\pi_t} + \Pi_t - T_t, \tag{2.2}$$

which includes credits for entrepreneurs at time t - B_t, the interest on the same credits adjusted for inflation, but at time $(t-1)$ - $\frac{R_{t-1}B_{t-1}}{\pi_t}$, where $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate at time t. The interest rate R_{t-1} is set by the central bank. The household gets from entrepreneurs the wage W_t in exchange for its labor L_t, she also pays lump sum taxes T_t and owns retail firms, obtaining their profit - Π_t.

The first order conditions for the problem (2.1)-(2.2) are standard and have the following form:

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} E_t(\frac{R_t}{\pi_{t+1}}) \tag{2.3}$$

$$\frac{W_t}{C_t} = L_t^{\sigma_l}, \tag{2.4}$$

where (2.3) and (2.4) are the Euler equation and the labor-supply condition correspondingly.

2.1.2 Entrepreneurs

Entrepreneurs manage perfectly competitive firms that produce intermediate goods Y_t at time t and borrow from households, in order to finance the purchasing of the capital K_{t+1} for the production process at time $t+1$. In the production process at time t they also use the labor of households L_t; the production function of a representative entrepreneur is assumed to be of the Cobb-Douglas type:

$$Y_t = AK_t^{\alpha} L_t^{(1-\alpha)\Omega}, \tag{2.5}$$

where the parameter A represents technology process, α and $(1-\alpha)$ are the shares of capital and labor in the intermediate product, respectively, while Ω denotes the share of households' labor in the total labor. The amount of entrepreneurs' labor is normalized to 1, and the share of entrepreneurs' labor is equal to $(1-\Omega)$.

With the probability $(1-v)$ an entrepreneur can become bankrupt in each period. Under this assumption entrepreneurs always use for the financing of the purchase of capital not only their net worth N_t, but also credits from households in the amount B_t:

$$B_t = Q_t K_{t+1} - N_t, \tag{2.6}$$

where Q_t denotes the fundamental price of capital at time t.

Bernanke and Gertler (2000) introduce the "financial accelerator" mechanism from Bernanke et al. (1999), in which the interest rate for external financing R_t^F is greater than the interest rate R_t, because of agency costs and asymmetric information, and depends on the ratio of the market value of capital to the net wort:

$$E_t R_{t+1}^F = \frac{R_t}{\pi_{t+1}} (\frac{F_t K_{t+1}}{N_t})^{\psi}, \tag{2.7}$$

where R_{t+1}^F denotes the expected rate of external financing, F_t is the market price of capital at time t, ψ denotes the parameter of the financial accelerator mechanism. $lev = \frac{F_t K_{t+1}}{N_t}$ - is the ratio of the market value of capital to the net worth of entrepreneurs or their financial leverage.

The net worth of entrepreneurs is updated according to the following equation, which is similar to the same equation from Christensen and Dib (2008):

$$N_t = \nu[R_t^F F_{t-1} K_t - E_{t-1} R_t^F (F_{t-1} K_t - N_{t-1})] + W_t^e, \tag{2.8}$$

where W_t^e is the labor income of entrepreneurs. Entrepreneurs, who become bankrupt at time t, consume the rest of the net worth in the amount C_t^e.

The interest rate of external financing in (2.7) and the dynamics of entrepreneurs' net worth in (2.8) depend on the market price of capital F_t, which is changed as follows:

$$\ln(F_t) - \ln(Q_t) = \ln(F_{t-1}) - \ln(Q_{t-1}) + \tau_t^F, \tag{2.9}$$

where Q_t is the fundamental price of capital at time t, while the variable τ_t^F represents the exogenous market change impulse at time t that is set by the interaction of traders in the financial market, who trade futures on capital. Calculations in the real sector take place at the end of the current quarter, while τ_t^F is calculated on the basis of the dynamics of the financial market over 13 weeks in the current quarter. The setting of τ_t^F will be described further in Section 2.3. Equation (2.9) determines the size of the deviation of the market price of capital from the fundamental price of capital, and we suppose that without the market change impulse τ_t^F this change will keep the same size over time. This assumption seems very reasonable, because the prediction of financial markets is a very complicated problem, if it is at all possible. Financial markets can go up or down, so without proper prediction, the most suitable way is to suppose that the deviation will be the same over time without exogenous shocks. Moreover, the joint model is calibrated in such a way that $E_{t-1}\tau_t^F = 0$.

The following condition is fulfilled under the optimal demand on capital:

$$R_t^F = \frac{(R_t^k + (1-\delta)F_t)}{F_{t-1}}, \tag{2.10}$$

where R_t^k is the marginal return on capital. The first order conditions for entrepreneurs are as follows:

$$R_t^k = \tfrac{\alpha Y_t}{K_t} MC_t \tag{2.11}$$

$$W_t = \tfrac{(1-\alpha)Y_t}{L_t} MC_t \tag{2.12}$$

$$W_t^e = (1-\alpha)(1-\Omega)Y_t MC_t, \tag{2.13}$$

where $\frac{1}{MC_t}$ - is the markup of retailers at time t, its description will be given further.

2.1.3 Capital Producers

The representative competitive capital producer purchases the amounts of final goods I_t at the price P_t from retailers at the beginning of each period, after which she transforms final goods in the same amount of new capital and sells new capital to entrepreneurs at the price P_t^K. We assume that the representative capital producer maximizes the following function:

$$\max_{I_t}[Q_t I_t - I_t - \frac{\chi}{2}(\frac{I_t}{K_t} - \delta)^2 K_t], \tag{2.14}$$

where $\frac{\chi}{2}(\frac{I_t}{K_t} - \delta)^2 K_t$ is quadratic adjustment costs, $Q_t = \frac{P_t^K}{P_t}$ denotes the relative fundamental price of capital at time t, δ and χ represent the depreciation rate and the parameter of adjustment costs correspondingly. The first order condition in this case is the standard Tobin's Q equation:

$$Q_t - 1 - \chi(\frac{I_t}{K_t} - \delta) = 0 \tag{2.15}$$

The aggregate capital stock evolves according to:

$$K_t = (1-\delta)K_{t-1} + I_t \tag{2.16}$$

2.1.4 Retailers

We introduce nominal price rigidity through the retail sector following Calvo (1983), which includes a continuum of monopolistic competitive retailers of mass 1, indexed by z. Retailers purchase at time t intermediate goods Y_t at the price P_t^w from entrepreneurs in a competitive market, differentiate them at no costs into $Y_t(z)$, and then sell to households and capital producers in the amount Y_t^f at the price $P_t(z)$ using a CES aggregation with the elasticity of substitution $\varepsilon_y > 0$:

$$Y_t^f = (\int_0^1 Y_t(z)^{\frac{\varepsilon_y - 1}{\varepsilon_y}} dz)^{\frac{\varepsilon_y}{\varepsilon_y - 1}} \tag{2.17}$$

Each retailer faces the following individual demand curve:

$$Y_t(z) = (\frac{P_t(z)}{P_t})^{-\varepsilon_y} Y_t^f, \tag{2.18}$$

where P_t denotes the aggregate price index, which is determined as follows:

$$P_t = (\int_0^1 P_t(z)^{1-\varepsilon_y} dz)^{\frac{1}{1-\varepsilon_y}} \tag{2.19}$$

In each period, the share of retailers $(1-\theta_p)$ can adjust their prices in order to set them optimally, maximizing the following profit function:

$$\Pi_t = \sum_{k=0}^{\infty} \theta_p^k E_{t-1}[\Lambda_{t,k} \frac{P_t^* - P_{t+k}^w}{P_{t+k}} Y_{t+k}^*], \tag{2.20}$$

where $\Lambda_{t,k} \equiv \beta \frac{C_t}{C_{t+k}}$ denotes the discount factor of retailers, which is equal to the stochastic discount factor of households, P_t^* and $Y_t^*(z) = (\frac{P_t^*(z)}{P_t})^{-\varepsilon_y} Y_t$ are the optimal price and optimal demand at time t.

The first order condition of retailers is as follows:

$$\sum_{k=0}^{\infty} \theta_p^k E_{t-1}[\Lambda_{t,k} (\frac{P_t^*}{P_{t+k}})^{-\varepsilon_y} Y_{t+k}^*(R)[\frac{P_t^*}{P_{t+k}} - (\frac{\varepsilon_y}{\varepsilon_y - 1})\frac{P_{t+k}^w}{P_{t+k}}]] = 0 \tag{2.21}$$

2.1.5 Central Bank

The Central Bank in the model sets the interest rate using the standard Taylor rule:

$$\ln(\frac{R_t}{\overline{R}}) = \rho_r \ln(\frac{R_{t-1}}{\overline{R}}) + (1-\rho_r)(\rho_\pi \ln(\frac{\pi_t}{\overline{\pi}}) + \rho_y \ln(\frac{Y_t}{\overline{Y}})) + \Delta r_t^{Bubble}, \tag{2.22}$$

where $\overline{R}, \overline{\pi}, \overline{Y}$ represent the steady state values of R_t, π_t, Y_t, respectively; ρ_r, ρ_π, ρ_y denote weights in the Taylor rule. The variable Δr_t^{Bubble} is an additional increase in the interest rate that is set by the central bank in response to bubbles in the market price of capital in order to prick bubbles. We will discuss how pricking bubbles works in more details further in Section 2.3.

2.1.6 Government Sector

Government expenditures are financed by lump-sum taxes:

$$G_t = T_t \tag{2.23}$$

2.2 Financial Market

The agent-based part of the model, that sets the operation of the financial market for futures on capital, includes S traders, who form by trading $p_{m,w}$ - the market price of capital in the financial market in week w. We assume that the volume of capital, which is used by traders in trading in the financial market, is a small part of investment in each period, and the change of the amount of capital that is used in trading does not affect the amount of investment in each period. The main part of new capital is purchased by entrepreneurs from capital producers directly, but the market price of capital is determined in the financial market, often under the influence of speculative forces.

The behavior of traders in the model is based on the certain number of rules, parameters of which vary from trader to trader and are drawn randomly from distributions. This approach allows for taking into account the heterogeneity of agents on the financial market. The agent-based part of the model is partially based on the model from Harras and Sornette (2011), which is significantly changed for our research. From the original model, we use the elements that relate to the formation of price in the financial market and the structure of the main mechanisms, but we change the rules, by which traders make decisions. We also include in the models liquidity flows from the real sector to the financial market and calibrate the model parameters to the stylized facts about stock returns.

2.2.1 Trading Strategies

In each week w trader i makes one of the three decisions: buy futures, sell futures or do not participate in trading. It is worth noting, that our model does not include the possibility of short positions. The trader's decision is based on her opinion about future price movements. The opinion of the trader i in week w - $\omega_{i,w}$ can be affected by 3 types of information sources: the fundamental information $Fundamental_w$ about the fundamental price of capital that is common for all traders on the market; the market information $Market_w$ that is also common for all traders on the market; and the private information $Private_{i,w}$ that is different for each trader. $\omega_{i,w}$ is determined by the following equation:

$$\omega_{i,w} = c_{1i} * Fundamental_w + c_{2i} * Market_w + c_{3i} * Private_{i,w}, \tag{2.24}$$

where c_{1i}, c_{2i} and c_{3i} represent individual for each trader coefficients that have a uniform distribution over the respective intervals $[0, C_1]$, $[0, C_2]$ and $[0, C_3]$, where C_1, C_2 and C_3 denote parameters in the model. The private information has a simple standard normal distribution $Private_{i,w} \sim N(0, 1)$.

The market information is market sentiments, including market trend and economic, political, and geopolitical news, and it is set as:

$$Market_w = \epsilon_w^{market} + \overline{LRtrend} + MRtrend_w, \tag{2.25}$$

where ϵ_w^{market} is a random global news shock in week w that has a standard normal distribution, $\overline{LRtrend}$ represents a model parameter - a fix long-run component in the market information, while $MRtrend_w$ denotes a changing medium-run component in the market information and is equal to the difference in week w between the two moving averages of the market price for the last 52 and 104 weeks multiplied by the parameter $trendpar$:

$$MRtrend_w = trendpar * (\sum_{j=w-52}^{w-1} p_{m,j} - \sum_{j=w-104}^{w-1} p_{m,j}) \tag{2.26}$$

The fundamental information is based on the deviation of the market price $p_{m,w}$ in week w in the financial market from the fundamental price of capital Q_{t-1} in the last quarter $t-1$ from the real sector of the model:

$$Deviation_w = \frac{p_{m,w} - Q_{t-1}}{Q_{t-1}} \tag{2.27}$$

When $Deviation_w$ becomes larger, more and more traders take it into consideration in the decision process. It may not lead to the bursting of the bubble because traders may believe that the bubble will exist further, but in some cases, when traders fear that the bubble will burst in the near future, the large $Deviation_w$ will force traders sell futures and will cause the bursting of the bubble. In order to include this phenomenon in the model we suppose that the fundamental information dependes also on the traders' fear in week w - $Fear_w \geq 0$ about the bursting of the bubble in the near future:

$$Fundamental_w = Fear_w * (Deviation_w + \epsilon_w^{fundam}), \tag{2.28}$$

where ϵ_w^{fundam} is a normally distributed fundamental information shock, which has zero mean and standard deviation σ^{fundam} - $\epsilon_w^{fundam} \sim N(0, \sigma^{fundam})$. The variable $Fear_w$ is updated according to the following rule:

$$Fear_w = (1 + |\max(0, Deviation_{w-1})| * |\min(0, \frac{p_{m,w-1} - p_{m,w-memory}}{p_{m,w-memory}})|^{f1} * f2) * \\ *(1 + IPparam * \Delta r_{t-1}^{Bubble}), \tag{2.29}$$

where the first factor on the right side of (2.29):

$$(1 + |\max(0, Deviation_{w-1})| * |\min(0, \frac{p_{m,w-1} - p_{m,w-memory}}{p_{m,w-memory}})|^{f1} * f2) \geq 1 \tag{2.30}$$

represents the influence of the deviation of the fundamental price from the market price in last week - $Deviation_{w-1}$ and the influence of the market return over several last weeks - $\frac{p_{m,w-1} - p_{m,w-memory}}{p_{m,w-memory}}$ on the fear. $memory$ denotes the number of weeks that is used in the calculation of the cumulative market return, while $f1$ and $f2$ are model parameters. The first factor is equal 1, if the fundamental price is greater than the market price, or if the market return over several last weeks is positive.

The second factor - $(1 + IPparam * \Delta r_{t-1}^{Bubble}) \geq 1$, shows the influence of the information policy of the central bank on the traders' fear, where Δr_{t-1}^{Bubble} - is the additional increase in the interest rate from the New Keynesian part of the model, using which the central bank tries to prick the bubble. $IPparam$ denotes the parameter of information policy, which determines the efficiency of the central bank's information policy and shows the influence of the additional increase in the interest rate on the fear of traders about the bursting of the bubble in the near future.

In the initial period the portfolio of the trader i consists of cash - $cash_{i,0}$ and some amount of futures - $futures_{i,0}$, these values have uniform distributions over the intervals $[0, \overline{cash}]$ and $[0, \overline{futures}]$ correspondingly, where $\overline{cash}$ and $\overline{futures}$ are the parameters of the model.

As in Harras and Sornette (2011) in order to introduce the differences in risk aversion for traders we suppose that in each week the trader i decides about her participation in trading on the basis of the parameter $\underline{\omega_i}$ that is randomly set for each trader over the interval $[0, \underline{\Omega}]$, where $\underline{\Omega}$ is the parameter of the differences in risk aversion. The value of $\underline{\omega_i}$ is compared with the value of $\omega_{i,w}$, and the trader i makes the decision on the basis of the following rules:

$$
\begin{aligned}
&if \quad \omega_{i,w} > \underline{\omega_i} : \\
&\qquad\qquad s_{i,w}^d = +1\ (buying),\ \ v_{i,w}^d = share * \frac{cash_{i,w-1}}{p_{m,w-1}} \\
&if \quad -\underline{\omega_i} \leq \omega_{i,w} \leq \underline{\omega_i} : \\
&\qquad\qquad s_{i,w}^d = 0\ (hold),\ \ v_{i,w}^d = 0 \\
&if \quad \omega_{i,w} < -\underline{\omega_i} : \\
&\qquad\qquad s_{i,w}^d = -1\ (selling),\ \ v_{i,w}^d = share * futures_{i,w-1},
\end{aligned}
\tag{2.31}
$$

where $v_{i,w}^d$ is the number of futures, which the trader i wants to buy or sell on the basis of her decision, $s_{i,w}^d$ denotes the indicator of the trading operation. When the trader decides to buy, she uses the share *share* of her cash, where *share* is a model parameter. At the same time, when she decides to sell, she also uses the share *share* of her futures. Following Harras and Sornette (2011) we use the value for *share*, which is much smaller than 1 in order to ensure time diversification.

2.2.2 Liquidity Flows

We assume that the portfolio of each trader may vary not only due to trading operations that are based on the trader's decisions, but also due to liquidity flows from the real sector to the financial market. Traders also buy (sell) futures in the case of the positive (negative) flow of the liquidity from the real sector to the financial market.

The variable $liquidity_w$ shows in how many times the value of the trader's portfolio should be changed due to the liquidity flow: if $liquidity_w > 0$ then the liquidity flow is positive, and if $liquidity_w < 0$ then the liquidity flow is negative. The trader i additionally buy or sell futures according to the following rules:

$$
\begin{aligned}
&if \quad liquidity_w > 0: \\
&\qquad\qquad s^l_{i,w} = +1,\ v^l_{i,w} = liquidity_w * futures_{i,w-1}, \\
&if \quad liquidity_w = 0: \\
&\qquad\qquad s^l_{i,w} = 0,\ v^l_{i,w} = 0 \\
&if \quad liquidity_w < 0: \\
&\qquad\qquad s^l_{i,w} = -1,\ v^l_{i,w} = liquidity_w * futures_{i,w-1},
\end{aligned} \tag{2.32}
$$

where $v^l_{i,w}$ is the number of futures, which the trader i wants to buy or sell due the liquidity flow, $s^l_{i,w}$ denotes the indicator of the trading operation.

2.2.3 Price Clearing Condition

After the time when all traders made decisions on the basis of opinions and liquidity flows, they send their orders without transaction costs to a market maker, who has an unlimited amount of cash and stocks. The market maker sets the price in week w according to the following market clearing rules:

$$
r_w = \frac{1}{\lambda * S} \sum_{i=1}^{S} (s^d_{i,w} v^d_{i,w} + s^l_{i,w} v^l_{i,w}) \tag{2.33}
$$

$$
\log[p_{m,w}] = \log[p_{m,w-1}] + r_w, \tag{2.34}
$$

where $r(week)$ is the market return and λ represents the market depth - the relative impact of the excess demand upon the price.

2.2.4 Cash and Futures Positions

The cash and futures positions of the trader i are updated as follows:

$$cash_{i,w} = liquidity_w * cash_{i,w-1} - (s^d_{i,w}v^d_{i,w} + s^l_{i,w}v^l_{i,w})p_{m,w} \tag{2.35}$$

$$futures_{i,w} = futures_{i,w-1} + s^d_{i,w}v^d_{i,w} + s^l_{i,w}v^l_{i,w} \tag{2.36}$$

2.3 The Interaction of the Real Sector and the Financial Market

As mentioned earlier, the period in the New Keynesian part of the model, which is one quarter, does not match with the period of the agent-based part, which is one week. In order to connect the two parts of the joint model we suppose that one quarter always consists of 13 weeks, so one year, which is four quarters, always includes 52 weeks in the model.

The interaction of the New Keynesian part and the agent-based part of the joint model is based on four transmission mechanisms. Firstly, the market price of capital in the financial market determines the market price of capital in the real sector. The deviation of the market price of capital F_t from the fundamental price of capital Q_t in the New Keynesian part of the model in the current quarter t is set through equation (2.9) using the market change impulse τ^F_t, which is calculated according to the following equation based on the average market price in financial market for 13 weeks $\frac{\sum_{week=1}^{13} p_{m,w}}{13}$:

$$\tau^F_t = sensitivity_1 * \left(\frac{\frac{\sum_{week=1}^{13} p_{m,w}}{13} - Q_{t-1}}{Q_{t-1}}\right) - (f_{t-1} - q_{t-1}), \tag{2.37}$$

where $sensitivity_1$ represents the model parameter, which is responsible for the sensitivity of changes in the real sector, due to changes on the financial market, while $f_{t-1} = \frac{F_{t-1}-\bar{F}}{\bar{F}}$ and $q_{t-1} = \frac{Q_{t-1}-\bar{Q}}{\bar{Q}}$ are the deviations of the market price of capital and the fundamental price of capital from their steady state values $\bar{F}$ and $\bar{Q}$, respectively. All calculations in the New Keynesian part of the model take place at the end of each quarter, when we know the dynamics of the agent-based model in this quarter.

The second transmission mechanism is liquidity flows from the real sector to the financial market. We supposethat liquidity flows are proportional to the change of the net worth of entrepreneurs and are set according to the following rule:

$$liquidity_w = sensitivity_2 * (n_{t-1} - n_{t-2})^{\frac{1}{13}}, \tag{2.38}$$

where $n_{t-1} = \frac{N_t-\bar{N}}{\bar{N}}$ - is the deviation of the net worth of entrepreneurs from its steady state value. $sensitivity_2$ denotes the model parameter that shows how the change of the deviation of the net worth affects liquidity flows to the financial market. The assumption about the relationship between the net worth and liquidity flows to the financial market in the joint model seems reasonable, because the growth of the net worth of firms in the economy in the reality means the increase in the amount of possible collateral for credits, which leads to the growth in available liquidity in the economy as well as to the increase in liquidity flows to financial markets. Moreover, with the higher value of the net worth firms or institutional investors can invest more, including investments in different funds, like mutual and hedge funds, which operate on financial markets.

The third transmission mechanism is the influence of the central bank's information policy on traders' fear about the bursting of the bubble in the future, which is defined by equation (2.29). As earlier discussed in the reality asset price bubbles can be hard for identification, so we suppose that the central bank starts to recognize the danger of a bubble only, when the market price has already significantly deviated from the fundamental price. After the time, when the central bank starts to recognize the danger of the bubble, it can raise the interest rate by Δr_t^{Bubble}. In order to simulate not an instant response of the central bank on bubbles, we introduce in the model the reaction parameter of the central bank $levCB$. This parameter means the value of the deviation of the market price from the fundamental price (equation (2.28)), after which the central bank starts raising the interest rate in each quarter until the deviation exceeds the value of $levCB$. The central bank uses the following rule in the setting of the additional increase of the interest rate Δr_t^{Bubble} in the Taylor rule:

$$
\begin{aligned}
&If \;\; Deviation_w \geq levCB: \\
&\qquad\qquad\qquad \Delta r_t^{Bubble} = \Delta r^{Bubble} \\
&Else: \\
&\qquad\qquad\qquad \Delta r_t^{Bubble} = 0,
\end{aligned}
\tag{2.39}
$$

where Δr^{Bubble} is a model parameter.

The fourth transmission mechanisms is the impact of the fundamental price of capital from the real sector on traders' opinions about the fundamental price, which is set by equation (2.27).

To sum up, the central bank in the joint model can prick bubbles on the capital market by three possible ways. First of all, the cumulative growth of the interest rate from the Taylor rule (2.22) increases the probability of the bursting of the bubble, because it slows down the economy, and so it leads to the decrease of the fundamental price of capital, which affects the traders' opinions about the true fundamental price in equation (2.28).

The second way is the additional increase of the interest rate in the Taylor rule (2.22); in this case, the central bank significantly raises the interest rate in the situation when the bubble has already had a quite large size, in order to suddenly diminish the fundamental price of capital and try to influence traders' opinions about the true fundamental price in equation (2.27). And, finally, the central bank can conduct information policy and affect traders' fear about the bursting of the bubble in the near future in equation (2.29).

In order to solve the joint model we firstly loglinearize the New Keynesian part, which sets the real sector, and obtain transition matrixes following typical steps in the solving of DSGE models. The loglinearized version of the New Keynesian part is presented in Appendix A. After it, we simulate the dynamics of the agent-based model during 13 weeks in the current quarter for the calculation of the market price change impulse τ_t^F from the real sector. At the end of the current quarter, using transition matrixes, we compute the values of the variables from the New Keynesian part. And then we again simulate the agent-based model during 13 weeks in the next quarter and so on.

3 Calibration

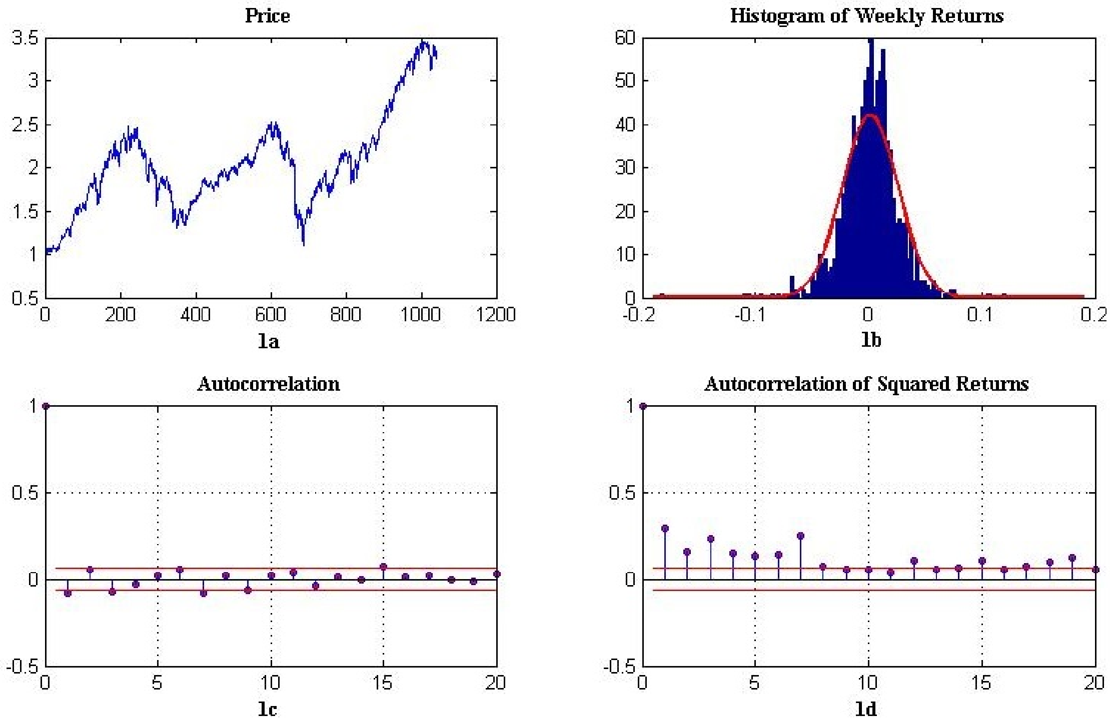
Our goal in the calibration of the model is to reproduce the dynamics of the economy in deviations from the steady state for the period of 20 years, mainly focusing on the realistic dynamics of the financial market with possible bubbles and crashes. As a benchmark for the financial market we use the S&P 500 stock market index. A summary of the parameter values used can be found in Appendix B.

We calibrate the agent-based model in order to reproduce the main stylized facts - faithful statistical characteristics of the realistic price dynamics of the S&P 500 index for the period of 1996-2016 years for the weekly data that is presented in Figure 1a. Over the sample period on the US stock market there were 2 large crashes of the stock market: the Dot-com bubble and the financial crisis of 2007-2009 years, so for each realization of random shocks our model should generate approximately from 1 to 4 crises. A greater number of crises seems unrealistic, because usually the life cycle of the bubble is approximately 5-6 years, for example, the growth and the burst of the Dot-com bubble took 6 years as well as in the case of the bubble that preceded the Asian financial crisis of 1997 year. The statistical characteristics of the market price of capital that is set by the agent-based model should comply with the following stylized facts:

- Weekly returns have small autocorrelation. Figure 1c shows that autocorrelation in weekly returns over the period 1996-2016 is insignificant for any lag.

Figure 1: Statistical Characteristics of the S&P 500 index.

The figure presents the following data for the period of 1996-2016: the weekly adjusted price of the S&P 500 index (Figure 1a), the histogram of weekly S&P 500 returns (Figure 1b) , the autocorrelation of weekly S&P 500 returns (Figure 1c), and the autocorrelation of squared weekly S&P 500 returns (Figure 1d). The red line in Figure 1b shows the probability density function of a normal distribution with the mean and standard deviation of weekly S&P 500 returns over the sample period.



- The distribution of weekly returns does not follow the normal distribution. Figure 1b also illustrates that the real distribution has fatter tails, is more peaked around zero (positive kurtosis) and is also negatively skewed. Moreover, it is not possible to reject the hypothesis about a zero mean return.

- The dynamics of market price can be divided on several volatility clusters; in some periods volatility will be high, while in others it will be low. A positive aurocorrelation in squared returns on Figure 1d represents this phenomenon.

- In periods with high volatility the market price is more likely to fall, while in periods with low volatility, it is more likely to grow, so there is a negative correlation between volatility and stock returns.

Our agent-based part of the joint model has many possible combinations of parameters that correspond to mentioned stylized facts (as usual for agent-based models), so in the description of the calibration of the parameters from the agent-based part of the model we mainly focus on the explanation how a parameter affects the statistical characteristics of the market price.

The number of traders in the model is set at a relatively large value $S = 10000$. If only a small part of all traders in some weeks decides to participate in trading, in any case it will be a sufficient number of traders for the deriving of statistical characteristics. The distribution parameters of the amount of cash and futures in the initial week $\overline{cash} = 1$ and $\overline{futures} = 1$ are taken from Harras and Sornette (2011), as well as the share of traders' cash or stocks that they trade per action $share = 0.02$. In the reality the world economy has positive average long-term growth rates over the last several decades after the World War II, so we suppose that the fix long-run component in the market information - $\overline{LRtrend}$ is positive and equal 0.6. In order to create a growing dynamics of the financial market with $\overline{LRtrend} = 0.6$, we find that the parameter of a changing medium-run component in the market information, $trendpar$, the distribution parameter of fundamental information, C_1, and the distribution parameter of market information, C_2, should be approximately equal to the following values: $trendpar = 1.2$, $C_1 = 1$, and $C_2 = 20$. At the same time for the possibility of bursting of bubbles the parameters of traders' fear $f1$ and $f2$, the memory parameter $memory$, and the standard deviation of the fundamental expectation shock σ^{fundam} may have the following values: $f1 = 3$, $f2 = 750$, $memory = 12$, and $\sigma^{fundam} = 2$. The parameters of the differences in risk aversion, $\underline{\Omega}$, and the market depth, λ, specify the form and the scale of the distribution of returns correspondingly. In order to match the form and the scale of the distribution of returns from the agent-based part with the same distribution from Figure 1b, we calibrate these parameters as $\underline{\Omega} = 40$ and $\lambda = 0.05$. The distribution parameter of private information, C_3, allows us to simultaneously adjust the autocorrelation of returns and the autocorrelation of squared returns. We find that with $C_3 = 15$ the market price in the agent-based part has realistic levels of the autocorrelation of returns and the autocorrelation of squared returns, which are similar to levels in Figures 1c and Figure 1d. A smaller value of C_3 leads to a higher value of autocorrelations and visa-versa.

In the case of the sensitivity parameter $sensitivity_1$, which transforms the differences between the market price $p_{m,w}$ in the financial market and the fundamental price in the last quarter Q_{t-1} in the real sector into the market change impulse τ_t^F in the real sector, we take the value that leads to realistic fluctuations of output over the 20 years - $sensitivity_1 = 0.06$. In the case of another sensitivity parameter $sensitivity_2$, we take the maximum value that does not distort the stylized facts of the financial market: $sensitivity_2 = 0.075$.

In the New Keynesian part of the model we use the majority of parameters' values from Bernanke et al. (1999), except the inverse elasticity of labor supply σ_l, the parameters in the Taylor rule ρ_r, ρ_π, ρ_y, and the parameter of financial accelerator mechanism ψ. In Bernanke et al. (1999) the authors do not use the inverse elasticity of labor supply in the utility function, so we take $\sigma_l = 1$, because this value is frequently used in the literature. The market price in the New Keynesian part is almost all time greater than the fundamental price; it can be smaller than the fundamental price only at the time of market crash. The positive difference between the market price and the fundamental price leads to a situation, when inflation is almost always greater than its steady state value, in more details we will discuss this phenomenon further in Section 4.3. This situation is not standard for New Keynesian models. We use values for the parameters in the Taylor rule and the parameter of financial accelerator mechanism that differs from the values in Bernanke et al. (1999), in order to keep welfare properties of the financial accelerator model. We use $\rho_r = 0.7$, $\rho_\pi = 1.1$, and $\rho_y = 0.2$ for the values of the weights in the Taylor rule. Although these values differ from the similar values in Bernanke et al. (1999), they also are frequently used in the literature. The value of the additional increase in the interest rate for pricking asset price bubbles Δr^{Bubble} is set to $\Delta r^{Bubble} = 0.25\%$, because it is the minimum value that is typically used by the Federal Reserve System. In the case of the parameter of the financial accelerator mechanism, we take the value $\psi = 0.02$, this value decreases the financial accelerator effect, but it completely keeps the causal relationships in the model.

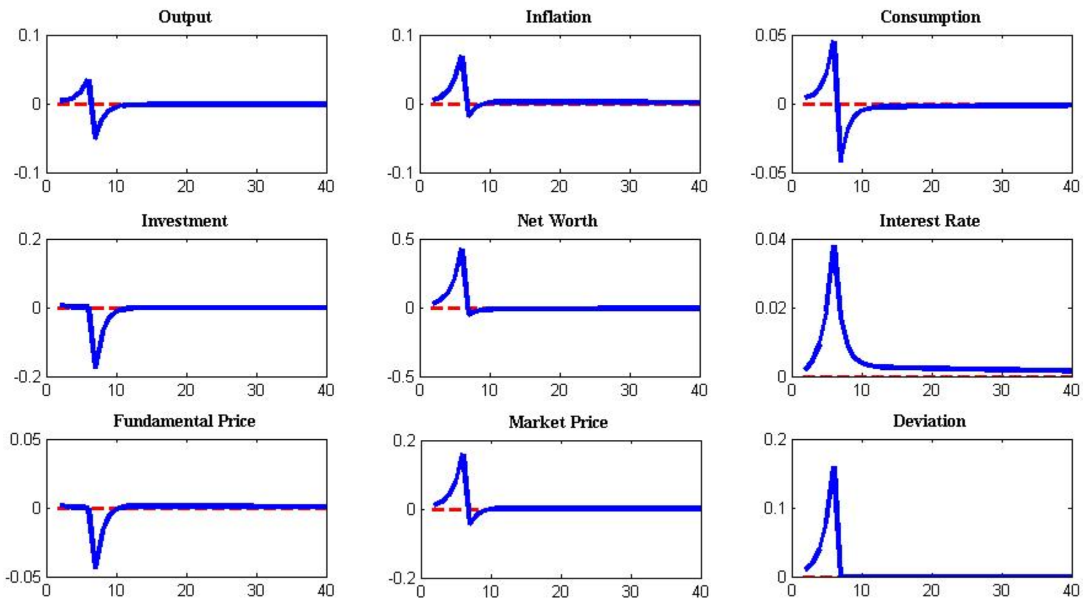
4 The Dynamics of the Model

In this section we analyze the dynamics of the model for the period of 1040 weeks. In the model we suppose that each quarter consists of 13 weeks, so the analyzed period is equal to 20 years or 80 quarters. In Section 4.1 we firstly show the dynamics of the New Keynesian part of the joint model in response to an exogenous bubble as in Bernanke and Gertler (2000). After this, in Section 4.2 we consider the dynamics of the agent-based financial market in the case, when it operates without any connections with the New Keynesian part of the model. Finally, in Section 4.3 we analyze the dynamics of the joint model.

It is worth noting, that we present the dynamics of the agent-based model in Section 4.2 and the dynamics of the joint model in Section 4.3 for some realization of random shocks. For other realizations the dynamics will be different, but will also generate bubbles and will correspond to the stylized facts that are set by calibration.

The goal of the joint model is the explanation of the behavior of the economy in deviations from the long-run trend, so the steady state in the model should be considered as the long-run trend.

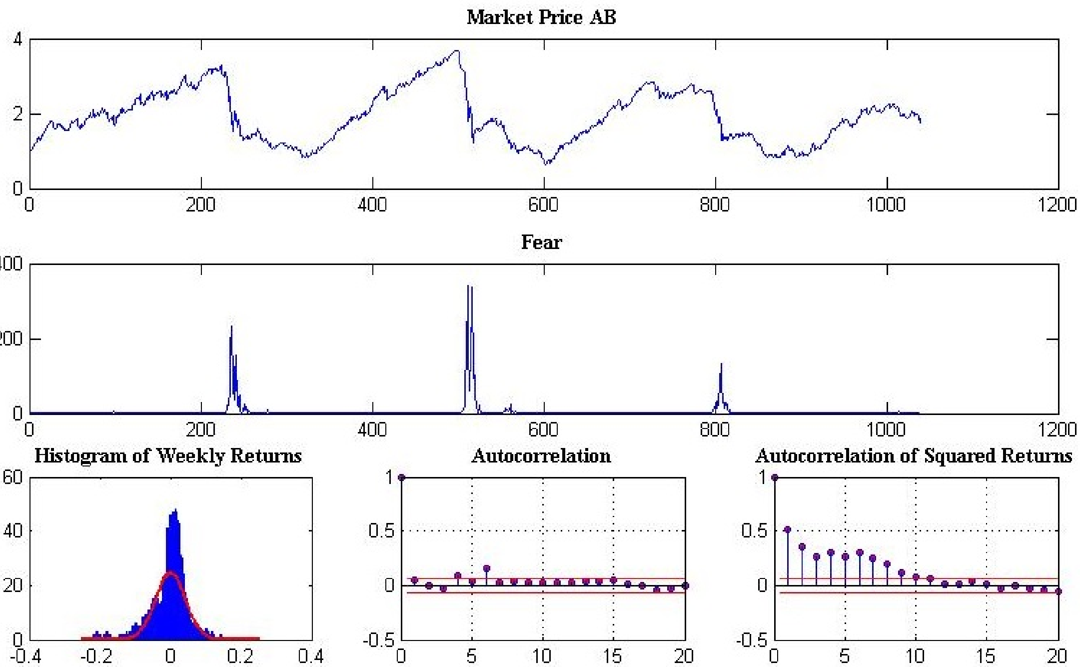
Figure 2: Impulse Responses of the New Keynesian Part of the Joint Model to an Exogenous Bubble.



4.1 The Dynamics of the New Keynesian Part in the Case of an Exogenous Bubble

As in Bernanke and Gertler (2000) we consider an exogenous bubble - a 1% market price change impulse, which grows twice in each quarter and bursts, when the market price is 16% percent higher than the fundamental price. The impulse responses to the bubble are presented in Figure 2. The creation of the bubble leads to the rapid growth in the market price of capital, which causes the increase in the net worth of entrepreneurs and the acceleration of inflation. The increase in the net worth means that entrepreneurs start borrowing more funds from households, purchase more capital, hire more household labor and produce more output, but the acceleration of inflation and the growth of output force the central bank to raise the interest rate. For the values of the parameters, which are used in our paper, the negative effect from the increase of the interest rate on capital and investment during the creation phase of the bubble is approximately the same as the positive effect from the increase in the net worth. The growth of output leads to the growth of consumption before the bursting of the bubble. After the bursting of the bubble almost all key variables in the economy, including consumption, sharply fall, so the welfare of households also decreases.

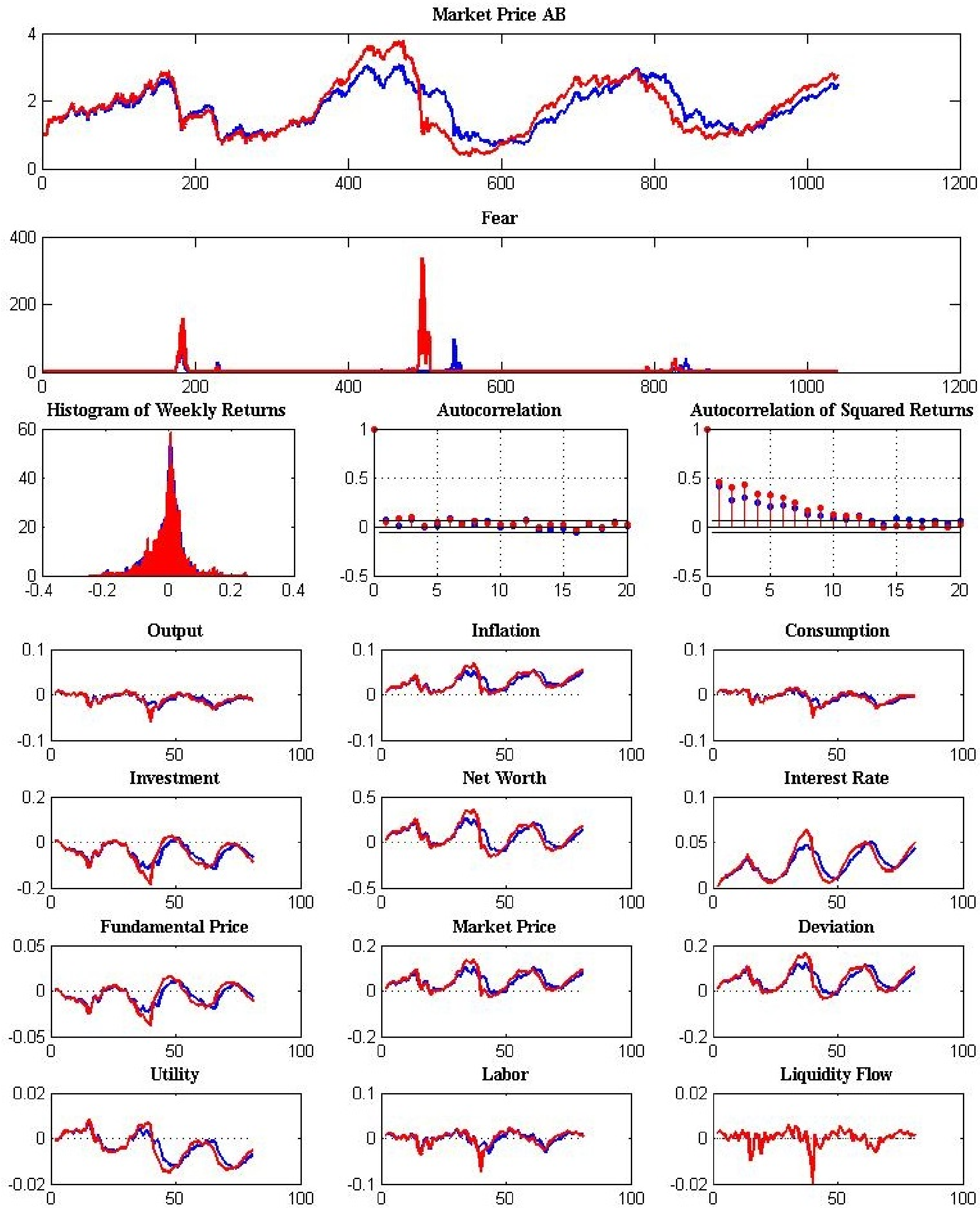
Figure 3: The Dynamics of the Agent-Based Financial Market.



4.2 The Dynamics of the Agent-Based Financial Market

As already mentioned, the dynamics of the market price in the agent-based part of the model depends on the realization of random shocks, and there are an infinite number of possible realizations. A typical realization of the agent-based financial market looks like Figure 3. The dynamics of the market price, the distribution of market returns, and the autocorrelation of market returns and squared market returns are quite similar to the real data in Figure 1. Over 1040 weeks in the agent-based financial market, there were two large crashes of the financial market and one smaller correction. The first two episodes are very similar to bubbles, where the creation of a bubble takes approximately four years. As in real data weekly market returns on the agent-based financial market have small autocorrelation, but there is a significant autocorrelation in squared market returns. The distribution of weekly market returns on the agent-based financial market also has fat tails, is more peaked around zero than normal distribution and is negatively skewed. We also present in Figure 3 the dynamics of the variable $Fear_w$ that shows traders' fear in week w about the bursting of the bubble. We can see that a rapid growth of $Fear_w$ precedes a sharp fall in the market price.

Figure 4: The Dynamics of the Joint Model.



4.3 The Dynamics of the Joint Model

Figure 4 presents the dynamic of New Keynesian and agent-based parts of the joint model in the case, when both parts operate simultaneously with endogenous relationships between two parts. The blue lines on both graphs show the dynamics of the joint model without liquidity flows from the real sector to the financial market, while the red lines show the dynamics of the joint model, but with liquidity flows from the real sector to the financial market. From the graphs we can see that the growth of the market price in the agent-based part leads to the increase in output, consumption, the net worth of entrepreneurs, and the utility of households. In addition to the growth of output, inflation accelerates that leads to the increase in the interest rate. There is also the liquidity flow from the real sector to the financial market, because the net worth of entrepreneurs grows that leads to the appearance of free liquidity in the economy.

In the case of the sharp fall of the market price, which is similar to the bursting of the bubble or to the market crash, the dynamic becomes opposite. Moreover, the bursting of the bubble leads to a larger change in absolute value of output, consumption, and the utility of households than during the time, when the market price increases. Usually the market crash in the model occurs quickly, but the recovery of output requires long time; this dynamics is very similar to the reality. From Figure 4 we can see that the including of liquidity flows increases the amplitude of variables in the joint model.

4.4 The Robustness of the Dynamics

In order to check the robustness of the dynamics of the model we simulate the joint model, changing values of each parameter from the agent-based part, which can affect statistical characteristics of the market price of capital, by 10%. So we check the robustness for the following parameters: $\underline{\Omega}, C_1, C_2, C_3, trend, trendpar, f1, f2, \lambda, memory$. The statistical characteristics of the market price of capital remain approximately the same for each 10% change of a parameter, when other parameters are fixed.

The parameters $IPparam$ and $levelCB$ are used for the analysis of the optimal policy of the central bank in pricking asset price bubbles, which is presented further in Section 5, where we also analyze how the changes of these parameters affect our results. The parameter $sensitivity_1$ is set, in order to correspond to reasonable fluctuations of the real sector, its change leads only to the change of the scale of these fluctuations and does not affect the key statistical characteristics of the market price of capital. The parameter $sensitivity_2$ has the maximum value that keeps the key statistical characteristics of the market price of capital, so only smaller values of $sensitivity_2$ may make sense for our analysis. We check that smaller values of $sensitivity_2$ do not change the statistical characteristics of the market price of capital. We do not check the robustness of other parameters from the New Keynesian part of the model, because the values of these parameters are frequently used in the literature.

5 Should monetary policy prick the bubble?

In the analysis of the response of monetary policy on market bubbles we calculate the welfare of households and the volatility of output and inflation in both cases: when monetary policy follows the Taylor rule from equation (2.22) without the additional response to asset price bubbles, and when it suddenly raises the interest rate by Δr^{Bubble} trying to prick asset price bubbles.

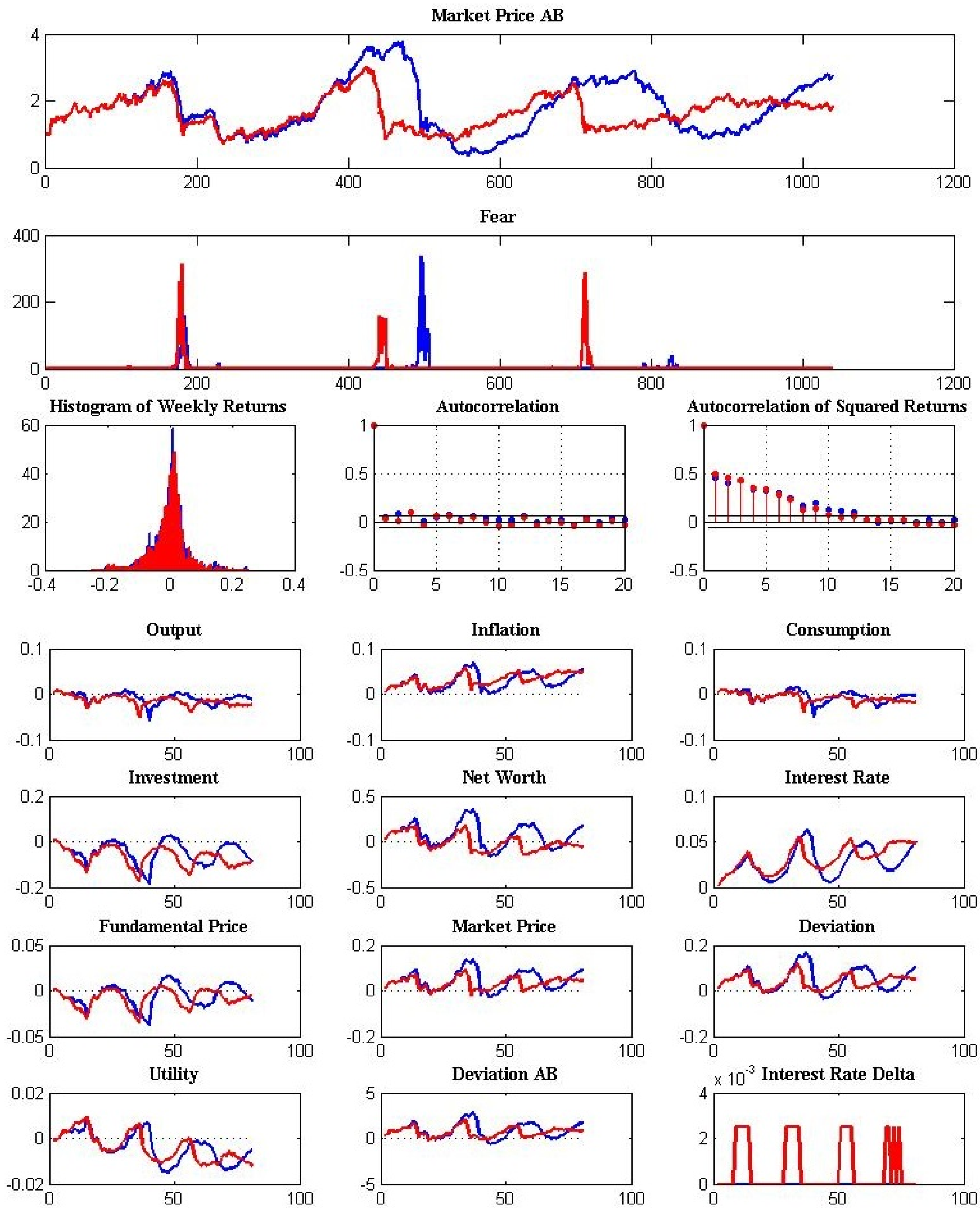
We use several possible levels for the reaction parameter of the central bank: $levCB \in [1; 1.2; 1.4; 1.6; 1.8; 2]$; with the larger values of $levCB$ the central bank in some realizations does not operate, so in such cases the larger values of $levCB$ are equivalent to the case, when the central bank does not prick asset price bubbles.

Another important parameter in our model for the analysis of pricking asset price bubbles is the parameter of the efficiency of the central bank's information policy $IPparam \geq 0$. From the calibration, we find that this parameter should be $IPparam \leq 1000$. If it is larger 1000, then the model may generate unrealistic value of $Fear_w$. In order to understand the influence of $IPparam$ on the results of pricking asset price bubbles we consider several different possible values for $IPparam \in [0; 200; 400; 600; 800; 1000]$.

Figure 5 shows the possible effect from pricking asset price bubbles for $levCB = 1$ and $IPparam = 500$. The blue lines show the dynamics of different variables in the joint model for the case without pricking asset price bubbles, while the red lines show it for the case with pricking asset price bubbles. From Figure 5 we can see that the dynamics of the market price in the case, when the central bank pricks asset price bubbles, substantially differs from the case, when the central bank does not prick bubbles. The highest values of the market price are lower in the case of pricking bubbles, so the deviations of the market price from the fundamental price in this case are smaller. It leads to the differences in the dynamics of the variables in the real sector, and the falls of output, consumption and the utility of households in the times of market crashes are also smaller.

Figure 5: The Dynamics of the Joint Model in the Case of Pricking Asset Price Bubbles.

The blue lines show the dynamics of variables in the case without pricking asset price bubbles, while the red lines show the same information in the case, when the central bank pricks asset price bubbles with the values of the reaction parameter of the central bank $levCB = 1$ and the parameter of the efficiency of the central bank's information policy $IPparam = 500$.



For each combination of the parameters $levCB$ and $IPparam$ we calculate welfare losses from bubbles on the futures market during the considered period as the discounted differences between the utility of households in each period and the utility of households in the steady state divided by the consumption of households in the steady state:

$$W = \sum_{t=0}^{T} \beta^t \left(\frac{U_t - \overline{U}}{\overline{C}}\right), \tag{5.1}$$

where U_t, is the utility of households at time t, $\overline{U}$ denotes the value of the steady state utility of households. Schmitt-Grohé and Uribe (2004) show that for the welfare analysis, it is necessary to use the second-order approximation of the welfare function:

$$\begin{aligned} U_t &= \overline{U} + \frac{1}{\overline{C}}(C_t - \overline{C}) - \overline{L}^{\sigma_l}(L_t - \overline{L}) - \frac{1}{\overline{C}^2}\frac{(C_t - \overline{C})^2}{2} - \sigma_l \overline{L}^{\sigma_l - 1}\frac{(L_t - \overline{L})^2}{2} = \\ &= \overline{U} + c_t - \overline{L}^{\sigma_l + 1} l_t - \frac{c_t^2}{2} - \sigma_l \overline{L}^{\sigma_l + 1}\frac{l_t^2}{2}, \end{aligned} \tag{5.2}$$

where $c_t = \frac{C_t - \overline{C}}{\overline{C}}$ and $l_t = \frac{L_t - \overline{L}}{\overline{L}}$ are the deviations of consumption and labor from the steady state values $\overline{C}$ and $\overline{L}$ at time t correspondingly.

Using (5.1) and (5.2) we get:

$$W = \sum_{t=0}^{T} \beta^t \left(\frac{1}{\overline{C}} c_t - \frac{\overline{L}^{\sigma_l + 1}}{\overline{C}} l_t - \frac{1}{2\overline{C}} c_t^2 - \frac{\sigma_l \overline{L}^{\sigma_l + 1}}{2\overline{C}} l_t^2\right) \tag{5.3}$$

As already mentioned, the dynamics of the model depends on the realization of random shocks for the considered period of 1040 weeks, and it is different for different realizations of random shocks, although each realization corresponds to the stylized facts that have been discussed in Section 3. In order to compare the values of welfare losses, the volatility of output, and the volatility of inflation for the different values of the parameters $levCB$ and $IPparam$, we calculate the average differences of welfare losses $\Delta W_{average}^{loss}$, the volatility of output $\Delta Var_{average}(y)$, and the volatility of inflation $\Delta Var_{average}(\pi)$ between the case, when the central bank does not prick bubbles, and other cases for 200 realizations. For example, for $levCB = 1$ these values are calculated as:

$$\begin{aligned} \Delta W_{average,levCB=1} &= \frac{\sum_{j=1}^{200} \Delta W_{j,levCB=1}}{200} = \frac{\sum_{j=1}^{200} (W_{j,levCB=1} - W_{j,wp})}{200} && (5.4) \\ \Delta Var_{average,levCB=1}(y) &= \frac{\sum_{j=1}^{200} \Delta Var_{j,levCB=1}(y)}{200} = \frac{\sum_{j=1}^{200} \frac{Var_{j,levCB=1}(y) - Var_{j,wp}(y)}{Var_{j,wp}(y)}}{200} && (5.5) \\ \Delta Var_{average,levCB=1}(\pi) &= \frac{\sum_{j=1}^{200} \Delta Var_{j,levCB=1}(\pi)}{200} = \frac{\sum_{j=1}^{200} \frac{Var_{j,levCB=1}(\pi) - Var_{j,wp}(\pi)}{Var_{j,wp}(\pi)}}{200}, && (5.6) \end{aligned}$$

where j is the number of realization, $Var_{j,levCB=1}(y)$ and $Var_{j,levCB=1}(\pi)$ are the values of the volatilities of output and inflation correspondingly in the realization j, when $levCB =$

1, while the values for the case without pricking bubbles are denotes by the subscript "wp".

Table 1 reports the results for different combinations of the parameters $levCB$ and $IPparam$ for the joint model without liquidity flows from the real sector to the financial market (in this configuration of the joint model $sensitivity_2 = 0$), while Table 2 reports the same information for the joint model with liquidity flows (in this configuration of the joint model $sensitivity_2 = 0.075$).

Table 1: The Main Results for the Case Without Liquidity Flows from the Real Sector to the Financial Market.

***, ** and * indicate signficance at the 1%, 5% and 10% level, respectively.

$sensitivity_2 = 0$						
	$\Delta W_{average}, \%$					
$levCB$	1	1.2	1.4	1.6	1.8	2
$IPparam = 0$	-6.09^{***}	-5.07^{***}	-4.10^{***}	-3.10^{***}	-2.22^{***}	-1.62^{***}
$IPparam = 200$	-3.68^{***}	-2.79^{***}	-2.08^{***}	-1.46^{***}	-1.07^{***}	-0.69^{***}
$IPparam = 400$	-1.96^{***}	-1.15^{***}	-0.62^{***}	-0.27^{**}	-0.11	0.06
$IPparam = 600$	-0.73^{***}	-0.01	0.42^{***}	0.62^{***}	0.61^{***}	0.66^{***}
$IPparam = 800$	0.27	0.86^{***}	1.14^{***}	1.30^{***}	1.11^{***}	1.15^{***}
$IPparam = 1000$	1.21^{***}	1.63^{***}	1.78^{***}	1.76^{***}	1.61^{***}	1.59^{***}
	$\Delta Var_{average}(y), \%$					
$levCB$	1	1.2	1.4	1.6	1.8	2
$IPparam = 0$	96.67^{***}	74.68^{***}	53.01^{***}	35.86^{***}	23.05^{***}	15.24^{***}
$IPparam = 200$	64.99^{***}	45.87^{***}	28.93^{***}	18.50^{***}	11.13^{***}	6.89^{***}
$IPparam = 400$	42.44^{***}	25.17^{***}	13.51^{***}	6.66^{***}	2.89^{***}	1.68^{**}
$IPparam = 600$	24.11^{***}	10.30^{***}	1.83	-1.65	-2.99^{***}	-2.27^{***}
$IPparam = 800$	9.27^{***}	-1.33	-6.29^{***}	-7.69^{***}	-6.94^{***}	-4.76^{***}
$IPparam = 1000$	-2.47	-9.99^{***}	-12.82^{***}	-12.44^{***}	-9.36^{***}	-6.33^{***}
	$\Delta Var_{average}(\pi), \%$					
$levCB$	1	1.2	1.4	1.6	1.8	2
$IPparam = 0$	80.24^{***}	53.91^{***}	31.55^{***}	16.64^{***}	8.03^{***}	3.44^{***}
$IPparam = 200$	46.16^{***}	24.47^{***}	8.68^{***}	0.72	-1.96^{***}	-2.88^{***}
$IPparam = 400$	23.36^{***}	4.80^{***}	-5.69^{***}	-9.91^{***}	-9.51^{***}	-7.41^{***}
$IPparam = 600$	5.39^{***}	-9.12^{***}	-15.83^{***}	-17.24^{***}	-13.64^{***}	-10.45^{***}
$IPparam = 800$	-8.41^{***}	-19.57^{***}	-23.25^{***}	-20.96^{***}	-16.68^{***}	-12.46^{***}
$IPparam = 1000$	-18.67^{***}	-27.21^{***}	-28.91^{***}	-25.30^{***}	-18.97^{***}	-13.94^{***}

From Table 1 we can see that pricking bubbles in the joint model without liquidity flows and without the effective information policy is completely useless for the central bank. It usually leads to additional social welfare losses and to the growth of the volatility of output and the volatility of inflation. In this case, the central bank only raises the interest rate on Δr^{Bubble} beyond the Taylor rule in each quarter until the bubble bursts, without affecting traders' opinions. The gains from pricking bubbles are the possible decreasing of the size of the output drop after financial market crashes, but in this case, these gains are lower than the losses from the raising of the interest rate, which slows down the economy. Moreover, in some realizations of random shocks the raising of the interest rate may not achieve pricking bubbles with the reasonable increase in the interest rate, in other words, the interest rate can rise, but the bubble can continue to exist. These effects are widely discussed in the literature as a part of the " clean" versus " lean" debate.

With the growth in the effectiveness of the information policy the welfare losses start decreasing for all levels of $levCB$. When $IPparam = 1000$ pricking bubbles has highest social welfare gains for all values of $levCB$, with the maximum 1.78% of the steady state consumption level in the case $levCB = 1.4$. The similar dynamics are observed in the decreasing of the volatility of output and inflation; the maximum reductions also occur in the case $levCB = 1.4$ and have the following values: -12.82% for the volatility of output and -28.91% for the volatility of inflation.

With respect to Table 2, when the joint model includes endogenous liquidity flows, it is observed that the main results are very similar. In comparison with Table 1, the maximum value of social welfare gains of 4.04% of the steady state consumption with $IPparam = 1000$ is achieved when $levCB = 1.6$, but the maximum value of the decreasing in the volatilities of output and inflation are obtained when $levCB = 1.2$.

It is worth noting, that the maximum social welfare gains from pricking bubbles in the case of endogenous liquidity flows are approximately 2.3 times larger than the same number in the case without endogenous liquidity flows, but the maximum decreasing of the volatility of output and inflation are approximately 3 and 1.9 times larger, respectively. This effect is caused by the fact that the volatility of output and inflation are higher in the case of endogenous liquidity flows.

Table 2: The Main Results for the Case With Liquidity Flows from the Real Sector to the Financial Market.

***, ** and * indicate signficance at the 1%, 5% and 10% level, respectively.

$sensitivity_2 = 0.075$						
	$\Delta W_{average}, \%$					
$levCB$	1	1.2	1.4	1.6	1.8	2
$IPparam = 0$	-5.51^{***}	-4.80^{***}	-4.18^{***}	-3.53^{***}	-2.98^{***}	-2.42^{***}
$IPparam = 200$	-2.48^{***}	-1.84^{***}	-1.22^{***}	-0.80^{***}	-0.51^{***}	-0.36^{**}
$IPparam = 400$	-0.31	0.43^{**}	0.90^{***}	1.27^{***}	1.39^{***}	1.31^{***}
$IPparam = 600$	1.14^{***}	1.79^{***}	2.22^{***}	2.43^{***}	2.49^{***}	2.34^{***}
$IPparam = 800$	2.30^{***}	2.83^{***}	3.13^{***}	3.26^{***}	3.42^{***}	3.08^{***}
$IPparam = 1000$	3.23^{***}	3.63^{***}	3.74^{***}	4.04^{***}	3.89^{***}	3.74^{***}
	$\Delta Var_{average}(y), \%$					
$levCB$	1	1.2	1.4	1.6	1.8	2
$IPparam = 0$	33.74^{***}	27.31^{***}	22.58^{***}	18.77^{***}	15.05^{***}	12.15^{***}
$IPparam = 200$	10.40^{***}	5.53^{***}	1.29	-0.56	-1.73	-1.16
$IPparam = 400$	-6.39^{***}	-10.53^{***}	-12.23^{***}	-12.90^{***}	-12.44^{***}	-11.71^{***}
$IPparam = 600$	-19.48^{***}	-23.09^{***}	-22.95^{***}	-22.23^{***}	-19.42^{***}	-17.75^{***}
$IPparam = 800$	-29.84^{***}	-32.01^{***}	-30.60^{***}	-28.45^{***}	-25.87^{***}	-21.74^{***}
$IPparam = 1000$	-37.08^{***}	-38.93^{***}	-37.37^{***}	-33.89^{***}	-30.39^{***}	-25.47^{***}
	$\Delta Var_{average}(\pi), \%$					
$levCB$	1	1.2	1.4	1.6	1.8	2
$IPparam = 0$	13.54^{***}	5.79^{***}	0.98	-2.17^{***}	-4.05^{***}	-4.88^{***}
$IPparam = 200$	-10.32^{***}	-15.86^{***}	-19.27^{***}	-19.73^{***}	-19.03^{***}	-17.00^{***}
$IPparam = 400$	-25.92^{***}	-30.73^{***}	-31.94^{***}	-30.16^{***}	-27.98^{***}	-25.08^{***}
$IPparam = 600$	-37.08^{***}	-41.09^{***}	-40.65^{***}	-37.99^{***}	-34.12^{***}	-30.10^{***}
$IPparam = 800$	-45.08^{***}	-47.92^{***}	-47.01^{***}	-43.42^{***}	-38.83^{***}	-33.69^{***}
$IPparam = 1000$	-51.55^{***}	-54.04^{***}	-52.39^{***}	-47.71^{***}	-42.45^{***}	-36.79^{***}

6 Conclusion

In the paper, we develop the approach based on the synthesis of New Keynesian macroeconomics and agent-based models, which allows for the incorporation of behavioral and speculative factors in macroeconomic models. For this purpose, we construct a joint model that consists of a New Keynesian model with a financial accelerator, à la Bernanke et al. (1999), which sets the real sector, and the agent-based model of the financial market, on which traders determine the market price of assets by selling and buying assets from each other. The model allows for the existence of bubbles on the financial market, and considers the cases when the central bank may try to prick asset price bubbles by raising the interest rate above the reaction from the Taylor rule, or by conducting its information policy, for example, by verbal interventions.

Using the model, we, for the first time in the literature, study the optimal strategy of the central bank in pricking asset price bubbles. The results show that in some cases, pricking asset price bubbles by the central bank can reduce the social welfare losses from asset price bubbles, as well as the volatility of output and the volatility of inflation. This effect is larger, especially in the cases when asset price bubbles are caused by credit expansion, or when the central bank conducts an effective information policy. Our results also demonstrate that pricking asset price bubbles only by raising the interest rate with the lack of the effectiveness of information policy leads to negative consequences for social welfare and financial stability.

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A The Log-linearized New Keynesian Part of the Model

$$\lambda_t = -c_t \tag{A.1}$$

$$\lambda_t + \pi_{t+1} = \lambda_{t+1} + r_t \tag{A.2}$$

$$y_t = a + \alpha k_{t-1} + (1-\alpha)\Omega l_t \tag{A.3}$$

$$w_t = y_t + mc_t - l_t \tag{A.4}$$

$$r_t^k = y_t + mc_t - k_{t-1} \tag{A.5}$$

$$q_{t+1} = \chi(i_t - k_{t-1}) \tag{A.6}$$

$$\beta\pi_{t+1} = \pi_t - \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p} mc_t \tag{A.7}$$

$$c_t^e = f_t + k_t \tag{A.8}$$

$$\frac{n_t}{v\overline{R^F}} = r_t^F + n_{t-1} \tag{A.9}$$

$$b_t = \frac{\bar{K}}{\bar{B}}(q_t + k_t) - \frac{\bar{N}}{\bar{B}} n_t \tag{A.10}$$

$$y_t = c_t^e \frac{\overline{C_e}}{\overline{Y}} + c_t \frac{\overline{C}}{\overline{Y}} + i_t \frac{\bar{I}}{\overline{Y}} + g_t \frac{\overline{G}}{\overline{Y}} \tag{A.11}$$

$$r_t^F = r_t + \psi(f_t + k_t - n_t) - \pi_{t+1} \tag{A.12}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\rho_\pi \pi_t + \rho_y y_t) + \Delta r_t^{Bubble} \tag{A.13}$$

$$f_t - q_t = f_{t-1} - q_{t-1} + \tau_t^F \tag{A.14}$$

B Model Parameters

Table 3: Calibrated Model Parameters

The New Keynesian part of the model		The agent-based part of the model	
Parameters	Values	Parameters	Values
β	0.99	S	10000
σ_l	1	$\overline{cash}$	1
α	0.35	$\overline{futures}$	1
Ω	0.99	$\underline{\Omega}$	40
A	1	C_1	1
v	0.9728	C_2	20
lev	2	C_3	15
δ	0.025	$\overline{LRtrend}$	0.6
χ	0.25	$trendpar$	1.2
θ_p	0.75	σ^{fundam}	2
ε_y	6	$f1$	3
$\overline{R^F} - \overline{R}$	0.02	$f2$	750
ρ_r	0.7	$memory$	12
ρ_π	1.1	$share$	0.02
ρ_y	0.2	λ	0.05
Δr^{Bubble}	0.0025	$sensitivity_1$	0.06
ψ	0.02	$sensitivity_2$	0.075