The small open circular economy

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January 11, 2018

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[†] This document belongs to the Integrated economic modelling of material flows (IECOMAT) project, sponsored by the Belgian Science Policy (BELSPO). The authors would like to thank IECOMAT researchers of VITO and CORE for their valuable comments and suggestions. Likewise, we thank Theo Moons, Balazs Zelity and Lila Bayet for their willingness to review and comment on this article, Vernon Smith for literature suggestions and to the academic community that attended the CEDON seminars at KU Leuven, and the ORBEL 2017, BEED 2017, EAERE 2017 and 2017 FSR Climate conferences, in which this work has been presented.

Abstract

In this paper we present a fully integrated dynamic general equilibrium model of a small open economy taking into account material balance, irreversible loss of materials and population dynamics. For general formulations of production processes we derive conditions under which recycling and remanufacturing are socially optimal. These conditions enable us to identify conditions that drive a small open economy towards a more linear or more circular economy. These conditions are region specific and consist mainly of production efficiency and environmental sensitivity considerations. Finally we determine optimal prices that make coincide a competitive market outcome with the social optimum circularity of the economy.

Keywords: Recycling, re-manufacturing, circular economy
JEL classification: Q30, Q53, Q56, Q58

1 Introduction

Significant improvements in social welfare have recently been linked to large scale recycling programs across the world, projects that have been termed as *circular economies.* Moreau et al. (2017) cites Ellen Macarthur foundation's (EMF) definition of the circular economy: "A circular economy is restorative and regenerative by design, and aims to keep products, components, and materials at their highest utility and value at all times. The concept (....) is a continuous positive development cycle that preserves and enhances natural capital, optimizes resource yields, and minimizes system risks by managing finite stocks and renewable flows. It works effectively at every scale". Among the gains claimed to be associated with material recycling and re-manufacturing, the most appealing dividends that call to the attention of local and regional authorities - especially in the European Union, China and Japan - include job creation, economic growth, increased competitiveness, reduced dependence on foreign raw material, pollution control and environmental sustainability (e.g. European Commission 2015; State Council of the People's Republic of China, 2013). From different perspectives, the discussion on circular economies and expected impacts has been relatively rich and controversial, ranging from concepts and definitions (e.g. EMF 2015a, 2015b; Hollander et al., 2017; Blomsma and Brennan, 2017), metrics (e.g. EMF, 2015c.; Liender et. al., 2017; Tisserant et al., 2017; Haupt et al., 2017; Lebre et al., 2017) to more critical postures (e.g. Fellner et al., 2017; Zinc and Geyer, 2017; Richa and Gaustad, 2017). Despite this momentum, the economic literature, and more precisely the environmental and resource economics, has shed limited light on the topic, a formal input that is expected to play an important role in the comprehension of key circular mechanisms strongly determined by market forces (Liender et al., 2017).

A particular question that has recently gained attention in the circular economy literature is the condition under which extensive recycling can always be prescribed as a desirable system, and with this the guarantee that the economic possibilities lost in abandoning the extract-consume-dump linear scheme are to be more than compensated by the gains offered by looping factors of production over time without major refuel. This question, despite casting doubt on the desirability of extensive recycling, has proven difficult to address for obvious and semi-obvious reasons. Obvious reasons are related to the fact that circular economies are integrated and dynamic in nature, and therefore several dimensions (economic, temporal, environmental, social, geographical, demographic, chemical, among many others) should play a role in the final determination of net advantages offered by the circular system. Semi-obvious difficulties relate to the fact that, in order to pound the circular economy's advantages, a consistent *value* system is required; powerful enough to simultaneously operate over all the dimensions previously cited. In spite of these difficulties identification of potential gains in the circular economy is still important. Since circular economy projects are currently subject of debate in several European and Asian legislations, a better understanding of their advantages is by all means beneficial.

Shedding light on this debate, we devote this document to the analysis of the optimal activation of massive recycling from a purely economic perspective. We derive the conditions on fundamentals that explain *why* small open economies without direct ownership of raw material rationally activate or deactivate the main recycling loops of the circular economy, that is recycling and re-manufacturing. Throughout this document we understand long term as the situation in which the economy has reached its maximum (expected) dematerialization possibilities and *small open* as the condition in which the local open economy can affect neither the global consumption (depletion rate) of raw material nor the relative international price at which it is traded and where spatial heterogeneity plays no tangible role in the characterization of major externalities. We focus on the particular class of economies that experience an inevitable shortage of landfill capacity, toxic and potentially lethal pollution released from economic activity and population dynamics. In order to connect our investigation with the ongoing research, we assume that the economy is constrained by material balance restrictions, the impossibility of dematerialization of final goods and machinery, thermodynamic efficiency and inefficient recycling. Our model provides an explanation as to why small open economies behave in a linear (no input coming from recycling), fully circular (all material inputs come from recycled material) or mixed linear-circular manner. As an interesting result, we show that the circular economy is typically not consistent with the so called hierarchy of waste management, in line with insights provided by authors as Richa and Gaustad (2017). Finally, we optimally price the circular economy identifying crucial factors that can drive a small open country towards a more linear or a more circular economy model than socially desired.

Our model fits in the general equilibrium dynamic recycling literature, whose seminal works were provided by Plourde (1972) and Smith (1972). Plourde (1972) introduced the problem of control of pollution by transforming hazardous waste into a nonhazardous component. Smith (1972), building on Plourde (1972), allowed for the reintroduction of waste into the consumption chain under material balance restrictions, introducing a more familiar form of material reuse into the modelling. A rich exposition of mathematical methods for the classic study of the economics of dynamic recycling is exposed in Smith (1977). In the same spirit, Lusky (1975, 1975b, 1976) and Hoel (1978) study the introduction of material scarcity and its role in optimal recycling and re-manufacturing, emphasizing the effect of material depletion costs and environmental services on consumer's welfare. Stylized recycling dynamic models have been enriched with land-filling costs (Dinan, 1993; Huntala 1999) and renewable resources (Huntala, 1999). Capital accumulation with dynamic recycling is explored by DiVita (2001), Pittel et al. (2010), Akao and Managi (2007) and Fagnart and Germain (2011).

We contribute to the economics of recycling by formally introducing the study of small open circular economies in a dynamic setup. We develop a fully integrated model capable of analyzing circularities in the presence of environmental, technological, demographic and material constraints. We specifically consider the case where material and energy have strong substitution limitations. We show how, under absence of asymptotic de-materialization, Leontief technologies offer a natural, flexible and tractable way of studying circular phenomena on long run scenarios, overcoming the lack of steady state induced by inevitable loss of material.

This paper is organized as follows. Section 2 introduces the basic model and properties of the technologies we adopt. Section 3 derives the social optimum and in section 4 we derive optimal prices that make coincide a competitive market equilibrium with the optimum. Section 5 discusses the results and section 6 concludes.

2 The model

As far as we are concerned with the recycling of physical goods, and our objective is to deviate as little as possible from realistic assumptions, we restrict our analysis to a world constrained by two important properties, named minimal material balance and thermodynamic efficiency. The first property requires a minimum of material per unit of output, that is, excludes excessive dematerialization as a possibility in the economy. The second property imposes restrictions on the ability that a fixed quantity of energy has to transform arbitrary big stocks of physical material into goods in a specific period of time. This is to say, we prevent the economy to excessively materialize goods from immaterial energy sources. These considerations are formally presented in Assumption 1.

Assumption 1 *Anderson's limits. Let* α, A *and* $B \in \mathbb{R}_{++}$. *Any technology* $F(m,e) : \mathbb{R}^2_+ \to \mathbb{R}_+ \in \mathcal{C}^2$ *transforming physical input* (m) *and energy* (e) *into physical output satisfies:*

a. $\forall\, \alpha > 0 \ \lim_{e \to +\infty | F(m,e)=\alpha} m = A\alpha > 0$
b. $\forall\, \alpha > 0 \ \lim_{m \to +\infty | F(m,e)=\alpha} e = B\alpha > 0$
c. $F(m,e)$ *is quasi-concave in* (m,e)

Assumption 1.a rules out the possibility of asymptotic dematerialization. Assumption 1.b is Anderson's thermodynamic efficiency, and introduces restrictions on the capacity of transformation of arbitrary big stocks of raw material by limited quantities of energy. Assumption 1.c requires upper contours to be convex sets. Together with Assumption 1.a and Assumption 1.b, it prevents from

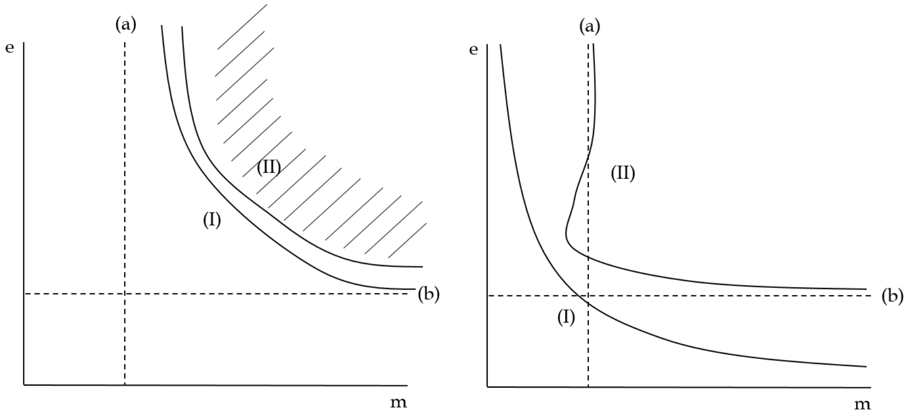

Figure 1: Assumption 1. On the left: Level curves satisfy assumptions 1.a, 1.b and 1.c. In this case technology (I) is more efficient than technology (II). On the right: Two technologies that violate Assumption 1. Technology (I) is quasi-concave but both asymptotically and non asymptotically excessively substitutes material and energy. Technology (II) satisfies assumptions 1.a and 1.b, but does not satisfy quasi-concavity. Non-asymptotic violations of minimal material content are evident.

non-asymptotic violations of excessive dematerialization and thermodynamic efficiency. This illustrated in Figure 1.[1]

If we understand the long term as the situation in which dematerialization has reached its maximum (expected) possibilities, it is possible to exploit Assumption 1 as follows. We say that a certain technology $F(e,m)$ is strictly more efficient than a counterpart $F'(e,m)$ if the upper-contour sets $S_\alpha : \{(m,e)|F(m,e) \geq \alpha \in \mathbb{R}_{++}\}$ and $S^{'}_\alpha : \{(m,e)|F(m,e) \geq \alpha \in \mathbb{R}_{++}\}$ satisfy $S' \subset S$ and $S' \cap S \neq \emptyset$. In these terms, it comes naturally that a technology $\frac{1}{A} \min\{m, e'\}$ with $e' = \frac{e}{B}$ satisfying Assumption 1 will report the biggest possible efficiency. Therefore, Leontief technologies can be seen as long term processes, either as representations of maximum efficiency in terms of dematerialization or as long term references to short term processes satisfying Assumption 1, as for instance, the CES function with elasticity of substitution $\sigma \in (0,1)$.[2] [3] According to these arguments, we consider the Leontief technology as our choice of long term transformation process.[4]

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[1] This key property is missing in Anderson's testing of CES properties (Anderson, 1987). However, the class of CES functions considered in his study satisfies the requisite of quasi-concavity.

[2] See Dasgupta and Heal (1974), Anderson (1987) and Fagnart and Germain (2010) for a rich discussion on CES technologies and their role in the modeling of physical transformation of goods.

[3] This interpretation can be naturally extended to monetary models considering the production of physical output and assuming constant marginal costs.

[4] Note that if a technology departs from a *CES* with elasticity of substitution between 0 and 1, variations in the elasticity of substitution will imply gains in efficiency. However, in order to reach the long term efficient technology the elasticity must reverse again to zero. This results suggests that, properly managed, long term information can be deduced from input-output tables. A formal result describing the reversing is presented in the Appendix.

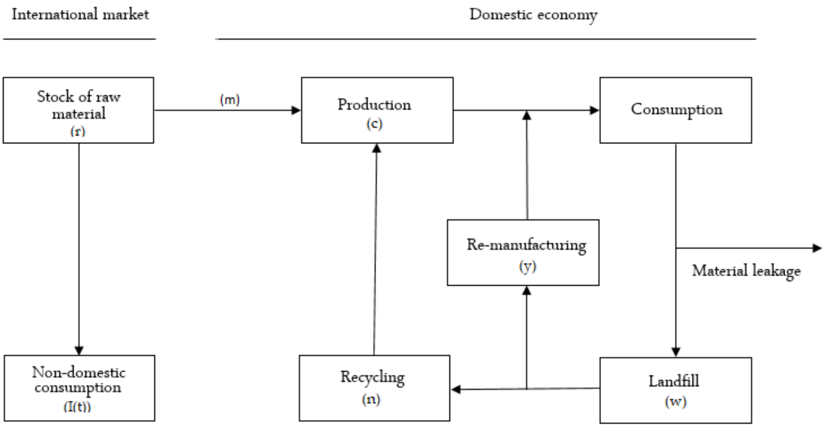

Figure 2: Material flow in the small open circular economy model

Now, we proceed to setup our circular economy model. The material flow in our model is illustrated in Figure 2. We consider an open economy consisting of L identical, infinitely lived consumers who derive utility over time from consumption of good $c \in \mathbb{R}_+$ (for instance, cellphones), an imperfect substitute re-manufactured good $y \in \mathbb{R}_+$ (repaired cellphones), and from environmental services. Environmental services are composed by available land space $w \in \mathbb{R}_+$ and perceived environmental quality $q \in \mathbb{R}_+$. We assume that any negative environmental quality level is lethal for the economy. Preferences are captured by a twice differentiable strictly concave instantaneous utility function $U(c, y, w, q)$ discounted over time by the factor $\delta \in \mathbb{R}_{++}$, satisfying $\frac{\partial U}{\partial c} > 0$, $\frac{\partial U}{\partial y} > 0$, $\frac{\partial U}{\partial w} > 0$, $\frac{\partial U}{\partial q} > 0$, $lim_{c \to 0} \frac{\partial U}{\partial c} = +\infty$ [5], $lim_{w \to 0} \frac{\partial U}{\partial w} = +\infty$, $lim_{q \to 0} \frac{\partial U}{\partial q} = +\infty$. Thus, utility increases with consumption, available land services and environmental quality. Goods c and y are physical in nature and must be disposed of right after consumption in a landfill with finite *remaining* capacity w.[6] We assume that at least one fraction of the perceived domestic pollution is inevitable and a proportional consequence of the local economic activity. Consumption good c is produced out of raw material $v \in \mathbb{R}_+$ (for instance gold) and labor $l_c \in \mathbb{R}_+$ according to the long term technology

$$Lc = \frac{1}{A^c} \min \{v, f(l_c)\}$$

for some $A^c > 0$, referring to minimal material content per unit of output required, and where $f(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing and concave function satisfying $f(0) = 0$. Note that in this case v and $f(l_c)$ play the role of m and e

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[5] We assume that at least one good needs to be consumed. Since initial consumption of c is needed in order to be able to consume re-manufactured good y, we assume that good c will always be consumed at least to some minor extent.

[6] Throughout this document, time subscripts are avoided for notational convenience, except for exogenous motions.

respectively. Technological gains given by A^c implies a reduction in the long term quantity of material and energy required to transform the good c. In turn, and with similar technical constraints, raw material v is composed of either imported material m or material recycled domestically n, according to the long term technology

$$n = \frac{1}{A^n} \min \{w_n, h(l_n)\}$$

$A^n \in (0, 1)$ capturing the long term economy's ability to minimize loss of material in the recycling sector. w_n denotes recyclable waste streams, $l_n \in \mathbb{R}_+$ stands for labor and $h(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing and concave function satisfying $h(0) = 0$.

In order to import virgin material (m), the domestic economy must trade a non-physical service $x \in \mathbb{R}_+$ with the rest of the world,[7] with real terms of exchange perceived by the local economy given by $\frac{m}{x} = B(t)$, where the exogeneity of $B(t) : \mathbb{R}_+ \to \mathbb{R}_{++}$ reflects the impossibility of the small open economy of altering its terms of exchange.[8] Service x is produced with the long term technology

$$x = s(l_x)$$

where $s(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly concave and increasing function satisfying $s(0) = 0$ and with $l_x \in \mathbb{R}_+$ denoting labor. Assuming perfect substitution between imported material and recycled material, the amount of material that is available to be used in the production process in the local economy is given by $v = n + m$. Further, recyclable waste streams $w_s \in \mathbb{R}_+$ and labor $l_y \in \mathbb{R}_+$ are required in order to produce a re-manufactured good y, according to the long term rule

$$Ly = \frac{1}{A^y} \min \{w_s, g(l_y)\}$$

for some $A^y > 0$ referring to the minimal material content per unit of re-manufactured output required and where Ly stands for the aggregate re manufactured good, $g(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing and concave function satisfying $y(0) = 0$. Total waste production is assumed to be a linear function of consumption and satisfies material linear balance conditions

$$w_c = \underbrace{\phi_c}_{\text{Material leakage}} \quad \overbrace{A^c}^{\text{Material converter}} \quad Lc \tag{1}$$

$$w_y = \phi_y A^y Ly \tag{2}$$

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[7] For simplicity but without loss of generality, we assume that the local consumer does not derive any utility from out of service goods.

[8] For instance, one could consider some $B(t)$ satisfying $\frac{\partial B(t)}{\partial t} > 0$, reflecting an increasing scarcity of global raw material.

for some $\phi_c \in (0,1)$ and $\phi_y \in (0,1)$ capturing inevitable loss of material due to the natural course of consumption. Therefore, we have made the impossibility of 100% recycling explicit (e.g. Ayres, 1999). We assume that the motion of *remaining landfill capacity* of the economy follows the linear rule

$$\dot{w} = -w_c - w_y + w_s + w_n \tag{3}$$

with initial condition $w(0) > 0$. The change in the available land capacity is thus determined by the amount of waste produced by consumption, the amount of repairable waste used in the re-manufacturing industry and the amount of recyclable waste transformed in the recycling industry. We allow recyclers to mine the landfill and therefore the landfill acts as a storage facility albeit one with negative environmental externalities. Equations (1), (2) and (3) restrict the model by a material balance principle that prevents spontaneous creation of material throughout the production process.

We consider the motion that captures environmental quality (q)

$$\dot{q} = Q(n, m, w(0) - w, Ly, q)$$

with initial condition $q(0) > 0$ and satisfying $\frac{\partial Q}{\partial n} < 0, \frac{\partial Q}{\partial m} < 0, \frac{\partial Q}{\partial w_0 - w} < 0, \frac{\partial Q}{\partial y} < 0, \frac{\partial Q}{\partial q} \geq 0$, where the last derivative reflects nature's absorption capacity. Loss in environmental quality is thus caused by the industrial transformation of recycled material as well as the impact on the local environment of imported material, by landfilling waste and by re-manufacturing activities. Abatement efforts to reduce this environmental impact are not taken into account. Following Zink and Geyer (2017), we allow for the possibility of more than one externality and, unlike most of past studies, we model the economy's impact on landfill and environmental quality in two different motions.

We assume that the stock of virgin material abroad is finite and that the same material is also consumed by the rest of the world, whose demand cannot be influenced by the small open economy. We denote the stock of international raw material as r and the share of consumption of the rest of the world as the motion $I(t) : \mathbb{R}_+ \to \mathbb{R}$. Therefore we assume

$$\dot{r} = -m - I(t)$$

For simplicity, we assume that the international stock of material is never fully depleted or $\int_0^\infty I(t)dt < r(0)$, where $r(0) > 0$ is the initial value of r, and it cannot be directly stockpiled by the small economy.[9] Further, we also assume that the local labor supply is equal to the population and completely shared among sectors, i.e. $L = l_c + l_y + l_n + l_x$. In turn, we assume that population dynamics are governed by the exponential motion $L = L(0)e^{\psi t}$, where $\psi < \delta$ is

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[9] In the case the stock of raw material is depleted, it suffices to assume that $B(t) = +\infty$, with similar consequences.

assumed to hold.[10] We normalize $L(0)$ to 1 so L can be immediately read as the growth rate of population from time zero to time t.[11]

3 Derivation of the socially optimal circularity

3.1 Welfare maximization problem

Letting $\Theta = \{c, y, x, n, m, w_c, w_y, w_s, w_k, l_c, l_y, l_k, l_x\}$ be the control set, we assume that the welfare functional problem capturing all inter-generational frictions in the long term economy takes the form

$$\max_{\Theta} \int_0^{\infty} U(c, y, w, q) e^{(\psi - \delta)t} dt \tag{4}$$

s.t.

$$\begin{aligned}
Lc &= \frac{1}{A^c} \min\{v, f(l_c)\} \\
Ly &= \frac{1}{A^y} \min\{w_s, g(l_y)\} \\
n &= \frac{1}{A^n} \min\{w_n, h(l_n)\} \\
L &= l_c + l_y + l_n + l_x \\
x &= s(l_x) \\
m &= xB(t) \\
\dot{r} &= -m - I(t) \\
\dot{w} &= -w_c - w_y + w_s + w_n \\
\dot{q} &= Q(n, m, w(0) - w, Ly, q) \\
w_c &= \phi_c A^c Lc \\
w_y &= \phi_y A^y Ly \\
v &= m + n
\end{aligned}$$

With $L = e^{\psi t}$. We refer to this program as the welfare optimality problem or problem (4). Anticipating that nested-Leontief functions satisfy $z(x^*, y^*) = \min\{x^*, f(y^*)\} = x^* = f(y^*)$,[12] the associated Hamiltonian of a convex-differentiable program equivalent to problem (4) can be written as

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[10] We rely on this assumption to ensure convergence in the inter-temporal utility function.

[11] We have assumed this particular population regime in order to simplify the problem, maintain the regular concavity in the Hamiltonian and ensure the correct characterization and existence of solutions. More general population regimes may be considered although for our purposes similar results are to be expected and stronger assumptions generally required. Yet a more flexible and realistic exogenous population dynamics of the form $L = A_l(t)$ may be assumed.

[12] Omitted variables at optimal levels are to be recovered accordingly.

$$
\begin{aligned}
H =& e^{\psi t} U\left(\frac{n+m}{e^{\psi t} A^c}, y, w, q\right) \\
& + \lambda_c (f(l_c) - m - n) \\
& + \lambda_y \left(\frac{1}{A^y} g(l_y) - y e^{\psi t}\right) \\
& + \lambda_m \left(s(l_x) B(t) + \frac{1}{A^n} h(l_n) - m - n\right) \\
& + \lambda_l (e^{\psi t} - l_c - l_y - l_n - l_x) \\
& + \rho_r (-m - I(t)) \\
& + \rho_w (-\phi_c (n+m) - \phi_y A^y y e^{\psi t} + A^y e^{\psi t} y + A^n n) \\
& + \rho_q (Q(n, m, w(0) - w, y e^{\psi t}, q))
\end{aligned}
$$

Since labor is assumed finite, the constraint set is closed, bounded and non-empty, the existence of a solution is guaranteed by Weierstrass theorem. Uniqueness of the solution comes from the strictly concavity of the utility function. A lack of steady state in the flow of materials results from the inclusion of material leakages ϕ_c and ϕ_y. Multipliers can be interpreted in the following way. λ_c is the intrinsic value of the consumption good c, λ_y is the intrinsic value of the re-manufactured good y, λ_m is the intrinsic value of raw material and λ_l is the intrinsic wage of the economy. On the other hand, the co-estate variables ρ_r, ρ_w and ρ_q are the intrinsic marginal values of the stock of global raw material, available landfill space and environmental quality.

3.2 Optimality rules for the circular economy

Despite the highly integrated circular economy considered so far, to find the tension among fundamentals that explain the activation (deactivation) of recycling and re-manufacturing it suffices to focus on the first order conditions

$$
\begin{aligned}
H_n =& \frac{1}{A^c} U_c - \lambda_c - \lambda_m + \rho_w (A^n - \phi_c) + \rho_q Q_n && \leq 0 \qquad (5) \\
H_m =& \frac{1}{A^c} U_c - \lambda_c - \lambda_m + \rho_r + \rho_w (-\phi_c) + \rho_q Q_m && \leq 0 \qquad (6) \\
H_y =& e^{\psi t} (U_y - \lambda_y + \rho_w A^y (1 - \phi_y) + \rho_q Q_y) && \leq 0 \qquad (7)
\end{aligned}
$$

Equations (5) and (6) show that, since the substitution of imported and re-purified material is perfect, the relative amount of each that is contained in good c is irrelevant to the consumer. However, from an efficiency point of view the marginal gains from choosing imported or recycled material must be balanced by the associated marginal economic and environmental costs. In the case of the use of recycled material, the factor $(A^n - \phi_c)$ reveals that the recycled material has a positive effect on welfare by temporally alleviating landfill pressures. Equation (7), related to the re-manufactured sector, may be interpreted similarly.

From the optimality conditions we can identify the tension among fundamentals consistent with a fully circular economy in the primary sector (no imported material flows; $m = 0$) or to a fully linear economy (no domestic recycling; $n = 0$).

Similarly, for the re-manufacturing sector (inner loop or cascade) we can identify fundamental conditions that optimally activate the optimal production of good y. These results are formally summarized in the following rules.

Rule 1 *Recycling. Consider problem (4). In equilibrium it follows that* $\forall t$*:*

Case a - Mixed economy. $n^* > 0, m^* > 0$ *if* $-A^n \rho_w = \rho_r + \rho_q(Q_m - Q_n)$

Case b - Circular economy. $n^* > 0, m^* = 0$ *if* $-A^n \rho_w < \rho_r + \rho_q(Q_m - Q_n)$

Case c - Linear economy. $n^* = 0, m^* > 0$ *if* $-A^n \rho_w > \rho_r + \rho_q(Q_m - Q_n)$

Proof. Cases (a)-(c) come from the fact that, since $\lim_{c \to 0} U_c = +\infty$ holds, $n^* = 0$ and $m^* = 0$ cannot occur simultaneously. Then, K.T. conditions imply that if $n^* > 0$ and $m^* > 0$, then (5)=(6). Similarly, when $n^* > 0$ and $m^* = 0$, (5)<(6). In case $n^* > 0$ and $m^* = 0$, (5)>(6) ♣

Rule 2 *Re-manufacturing. Consider problem (4). It follows that* $\forall t$ $y^* > 0$ *iff*

$$U_y = \lambda_y - A^y \rho_w (1 - \phi_y) - \rho_p P_y$$

Proof. This result follows from the sufficiency of K.T. conditions ♣

Rule 1 (a)-(c) has an intuitive interpretation. The main condition compares the marginal net social cost of importing raw material with the marginal net social costs of recycling material. This comparison thus crucially determines the choice between a linear or circular path in a small open economy. Firstly, when the tension holds with equality a *mix of a linear and circular economy* of the type $n^* > 0$ and $m^* > 0$ is optimally expected (case a). This happens when the social cost of landfilling ($-\rho_w$) scaled by the green design capacity A^c is exactly equal to the sum of social cost of exporting services (λ_m), the social cost of dependence on international raw material (ρ_r), the net cost of the difference between the marginal value of environmental externality caused by imported raw material (Q_m) and the marginal value of environmental externality caused from recycling material. Similarly, if the condition holds as an '<'-inequality, the economy behaves as a *circular-economy*, that is $n^* > 0$ and $m^* = 0$ (case b). Finally, if the condition holds as an '>'-inequality, a *linear economy* solution of the type $n^* = 0$ and $m^* > 0$ is optimally expected (case c).

Rule 2 characterizes the optimal condition required to activate the remanufacturing sector. If the tension holds with equality, that is if the average marginal utility equals the social cost of the remanufacturing activity, the sector is optimally active. This cost includes the intrinsic cost of production (λ_y), the marginal cost associated with pollution ($\rho_q Q_y$) and the effect of the proportion of material lost each cycle $(1-\phi_y)\rho_w$. Given our assumptions, if follows that the average marginal utility U_y will tend to be higher when the population increases[13], making positive

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[13] Recall that the utility function is concave and the aggregate level of consumption must be divided by the size of population.

re-manufacturing more likely to be beneficial.[14] The reader can easily notice that when $y^* = 0$ a positive re-manufacturing $y' > 0$ may be feasible but not optimal, contradicting the hierarchy of waste management.

3.3 Discussion

Rules 1 and 2 reveal four key factors in the optimal behavior of small circular economies. First, the absence of marginal utility terms in Rule 1 suggests that the activation (deactivation) of recycling is directly related to the relative impact on environmental quality. This is expected since the circular economy is capable of perfect substitution among materials, making the consumers indifferent, in terms of consumption, to the source of inputs. Second, the absence of the motion of population reveals that this rule is expected to hold for a very rich family of population regimes. Third, our derivation shows that, in line with Geyer et al. (2016), when considering optimal decisions on recycling or linear-behaving, heterogeneity across countries matters. This is to say that homogeneous environmental policies aimed at rising (decreasing) recycling behavior among different countries as a part of hierarchy of waste management schemes (including those suggested by Stahel (2010) and Hollander (2017)) are likely to be suboptimal in stimulating a transition towards a circular economy. Optimal circular policies are dependent on local circumstances and are thus quite likely to differ between countries. Fourth, Rules 1 and 2 suggest that corrective policies must be integrated in the sense that environmental quality, material dependency and economic efficiency must be simultaneously taken into account.

3.3.1 Extension: recycling and physical accumulation of machines

In recycling models labor and machines are usually abstracted as the same factor of production. A natural justification for this proceeding may be that both factors are energy providers and not energy-material complements. Although this argument is in general sound for static models, however, as noted by Di Vita (2001), Akao and Managi (2007) and Pittel et al. (2010), durable machinery can be recycled and built using recycled material, while the labor force cannot. An interesting question is whether dismantling and re-looping its material into the production process is optimal for the open circular economy in the long run. A second question targets the conditions that ensure a circular economy is expected to optimally fuel its machinery stock with recycled refuse. We address these questions in this subsection. The revised material flow in our model is illustrated in Figure 3.

We start assuming that new machines (k^{new}) are built employing labor (l_k) and material (i) according to the rule

$$k^{new} = \frac{1}{A^z} min\left\{i, z(l_k)\right\}$$

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[14] Notice that this result is driven by material scarcity. As population becomes a pressure in the system per-capita consumption is compromised. In this case, re-using appears a natural solution to increase society's welfare.

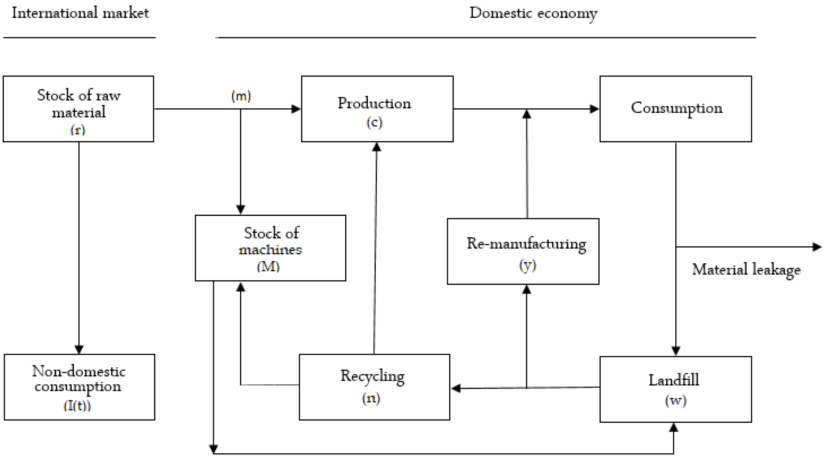

Figure 3: Material flow in the small open circular economy model with durable machines.

For some $A^z > 0$ referring to the minimal material content required to build one machine and $z(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing and concave function satisfying $z(0) = 0$.

Without loss of generality, we assume that only final good c is produced with energy provided for both labor and physical machines. Letting M be the raw material contained in the stock of machines of the economy, we can write

$$Lc = \frac{1}{A^c} min\left\{ m + n - i, f\left(l_c, \frac{M}{A^z}\right)\right\} \tag{8}$$

where $f(.,.) : {\mathbb{R}_+}^2 \to \mathbb{R}_+$ is a strictly increasing and concave function satisfying $f(0,.) = f(.,0) = 0$. We consider the motion that governs the accumulation of raw material in the stock of machines

$$\dot{M} = i - \epsilon M - D(l^d)$$

Where $\epsilon \in (0,1)$ is the depreciation rate and $D(l^d)$ stands for a clean technology that dismantles machines into waste[15], with $l^d \in \mathbb{R}_+$ denoting labor, and $D(.)$ is a strictly increasing and concave function satisfying $D(0) = 0$. We denote by ρ_M the new co-estate variable that governs material accumulation in machines and λ_k the social cost of transforming material and labor into new machines. Taking into account that new waste inflows are given by $-\phi_M D(l^d) - \epsilon\phi_\epsilon M$ with cyclical costs $\phi_k, \phi_\epsilon \in (0,1)$, the tension that activates dismantling can be deduced from optimality conditions. The new Hamiltonian is

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[15] We assume that the dismantling technology does not cause pollution. However, assuming a dirty technology, the main insights would remain untouched.

$$\begin{aligned}
H =& e^{\psi t} U\left(\frac{n+m-i}{e^{\psi t} A^c}, y, w, q\right) \\
&+ \lambda_c \left(f\left(l_c, \frac{M}{A^z}\right) - m - n\right) \\
&+ \lambda_y \left(\frac{1}{A^y} g(l_y) - y e^{\psi t}\right) \\
&+ \lambda_m \left(s(l_x)B(t) + \frac{1}{A^n} h(l_n) - m - n\right) \\
&+ \lambda_l (e^{\psi t} - l_c - l_y - l_n - l_x - l_k - l_d) \\
&+ \lambda_k (z(l_k) - i) \\
&+ \rho_r (-m - I(t)) \\
&+ \rho_w (-\phi_c(n+m) + A^y y e^{\psi t}(1 - \phi_y) + A^n n - \phi_M D(l^d) - \epsilon \phi_\epsilon M) \\
&+ \rho_q Q(n, m, w(0) - w, y e^{\psi t}, q) \\
&+ \rho_M (i - \epsilon M - D(l^d)))
\end{aligned}$$

Rule 3 *Durable loop. Consider problem (4) with all constraints implied by (8). Moreover, assume that the economy is regular in the sense that* $\rho_M > 0$*. In equilibrium it follows that* $D^* = 0$*.*

Proof. This result follow since K.T. implies that $\lambda_l = D_{l_d}(-\rho_M - \phi_M)$ cannot occur in equilibrium. ♣

Rule 3 suggests that in a regular long term open circular economy where the social value of the stock of machinery is positive, dismantling is always suboptimal. Therefore, besides leakage caused by depreciation, material embedded in the machines remains there forever.

Rule 4 *New machines. Consider problem (4) with all constraints implied by (8). In equilibrium it follows that* $i^* > 0$ *iff*

$$\rho_M = \frac{1}{A^c} U_c + \lambda_k$$

Proof. This result follows from the sufficiency of K.T. conditions ♣

Rule 4 reveals that, since raw material is fueling both consumption and machinery, it is optimal to allocate part of it in new machines if the marginal value of the raw material contained in machines equals the average marginal utility of consumption of the economy plus the social marginal cost of transforming material into machines. When Rule 4 and Rule 1 a. or Rule 1 b. hold simultaneously, it may be inferred that part of the waste that is flowing from the landfill is being directed to the stock of machinery to be returned only as depreciation.

4 Optimal pricing and restorative policy

In this section we address the problem of basic optimal pricing for a small open circular economy for the case in which the outcome of the economy's activity is a laissez-faire equilibrium. We show that, in general, policy intervention, and therefore complex insight on the dynamics of fundamentals is required. We assume a simple institutional arrangement where consumers take prices and environmental services as given, and sell labor to firms at a competitive wage. Landfills are assumed to be public and free to use. For simplicity, we abstract from physical capital and the presence of illegal dumping. Firms are representative for each sector and behave competitively. Each dynasty of consumers born in time t' owns an equal share in all firms and solves the program

$$\max_{c,y} \int_{t'}^{\infty} U(c, y, w, q) e^{-\delta t} dt$$

s.t.

$$(P_c + \tau_c)c + (P_y + \tau_y)y = P_w l^c + (\pi_c + \pi_y + \pi_n + \pi_x)L^{-1} + \Pi$$

Where P_c and P_y are the market prices of goods c and y. τ_c and τ_y represent consumer taxes on c and y respectively. P_w is the monetary wage, l^c is the amount of labor supplied by the consumer, and $(\pi_c + \pi_y + \pi_k + \pi_x)L^{-1}$ denotes total gains from firm's ownership. $\Pi \in \mathbb{R}$ denotes any other monetary transfer. The system has the following optimal conditions:

$$U_c = \lambda^m (P_c + \tau_c)$$
$$U_y \leq \lambda^m (P_y + \tau_y)$$

Where λ^m stands for the Lagrangian multiplier associated to the consumer's problem. These conditions reveal that at the optimum marginal utility of consumption, equals its respective marginal cost. Inequalities leading to corner solutions (of this and further programs) must be interpreted as in the controlled solution case.

The producer of good c buys labor and material v, facing market prices P_c, P_w and P_m :

$$\max_{l_c, v} \pi_c = \int_0^{\infty} \left(P_c \frac{1}{A^c} \min\{v, f(l_c)\} - P_w l_c - P_m v \right) e^{-\delta t} dt$$

The re-manufacturing sector faces market prices P_y and P_w. In order to simplify the problem, we assume that the price of waste for all sectors is zero. The program can be expressed as

$$\max_{l_y, w_s} \pi_y \int_0^\infty \left(P_y \frac{1}{A^y} \min\{w_s, g(l_y)\} - P_w l_y \right) e^{-\delta t} dt$$

The recycling sector which provides a perfect substitute for virgin material to the economy faces prices P_m and P_w. The program can be expressed as

$$\max_{l_n, w_n} \pi_n \int_0^\infty \left(P_m \frac{1}{A^n} \min\{w_n, h(l_n)\} - P_w l_n \right) e^{-\delta t} dt$$

Finally we consider the import-export sector that we assume is composed by a representative firm that can trade services for raw material on the international market, according to real terms $B(t)$, facing the local price of raw material P_m.[16] Therefore, the problem of the firm can be expressed as:

$$\max_{l_x} \pi_x \int_0^\infty (P_m B(t) s(l_x) - P_w l_x) e^{-\delta t} dt$$

In addition to equilibrium and clearing conditions implied by problem (4), some τ_c, τ_y and Π complete the market problem of the economy. Given our institutional arrangement laissez-faire prices are given by $P_c = A^c \frac{\lambda_c + \lambda_m}{\lambda^m}$, $P_y = \frac{\lambda_y}{\lambda^m}$, $P_m = \frac{\lambda_m}{\lambda^m}$ and $P_w = \frac{\lambda_l}{\lambda^m}$. In general this situation implies that a laissez-faire circular economy is characterized by the presence of several externalities that are unaccounted for. As a result of the impossibility of market forces to fully internalize externalities, material and energy are likely to be allocated in a suboptimal way, provoking a wrong level of circularity including undesirable rebounds. An example of one such rebound effect is the excessive level of pollution caused by too stringent recycling requirements. Thus, corrective signaling is required. Basic Pigouvian corrections are given by

$$\begin{aligned}
\tau_c|_{m^*=0} &= -\ A^c(\rho_w(A^n - \phi_c) + \rho_q Q_n) \\
\tau_c|_{n^*=0} &= -\ A^c(\rho_r + \rho_w(-\phi_c) + \rho_q Q_m) \\
\tau_y &= -\ A^y(\rho_w(1 - \phi_y) + \rho_q Q_y)
\end{aligned}$$

These corrections reveal that externalities are linked to pollution, raw material depletion, landfill capacity and the pressure of population dynamics in social welfare. Note that the efficacy of corrective measurements will depend directly on the quality of information gathered by authorities, which in turn must possess a decent understanding of technological and environmental interaction. This information, unfortunately, in its required quality is currently missing in public databases. When policy makers possess accurate information on externalities,

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[16] We may consider the profit functional $\int_0^\infty \beta(P_s s(l_x) - P_a r) + (P_m + \tau^m)r - P_w l_x)e^{\delta t} dt$ where P_s and P_a are prices expressed in foreign currency and β stands for the nominal exchange rate. In equilibrium $s(l_x) = \frac{P_a}{P_s} r$.

nothing prevents a regulated circular economy to allocate energy and physical material in a way such that a maximum utility over time is achieved. In reality, however, an adequate understanding of environmental damages is far from perfect, which makes the design of an optimal policy unlikely in practice.

5 Discussion

Circular economy transition projects have been welcomed in different spheres as a solution to evident disadvantages that are imposed upon society by the extract-use-dump linear model. Despite apparent advantages of massive recycling schemes, the nature of the conditions under which the circular economy can always be prescribed as a desired system is still an open question in the literature. This legitimate concern has been proved difficult to address, and contradictory conclusions coming from different real life examples show themselves sound and justified. Some lessons, however, can be taken from our study.

From a purely economic perspective, we have found that the reasoning behind optimal activation of recycling operations heavily depends on the fundamentals of the economy under consideration, and more specifically, on its (in)ability to trade the necessary material from the rest of the world so as to fuel local industries without excessively damaging the local environment. In these lines, it can be expected that economies with highly efficient service exporting sectors will delay any transition to circular economies and serious circular initiatives will therefore be rather unpopular among industrials and policy makers in such economies.[17] This result may be reversed in time if the local recycling sector evolves and becomes efficient and clean enough to be considered as a non-harming and even improving option to social welfare. Scientific research is a natural example that comes to mind when we consider ways to stimulate recycling efficiency.

On the other hand, our results suggest that in regular economies physical machines are not likely to be a big source of circularities besides those associated to depreciation. With respect to material looping through re-manufacturing, we have found that a big barrier faced by the repairing industry is its ability to effectively provide goods with a non-trivial impact on consumer's welfare, compensating any real or imaginary advantage provided by brand new purchases. When population pressures make per-capita consumption more expensive and resources become more scarce, this loop however is more likely to play a visible role in everyday consumer's life.

Another interesting result refers to the ability a society has to manage waste streams in a harmless way. According to our derivations, as far as waste is not causing considerable negative externalities to the population, limited landfill capacity does not constitute in itself a trigger of recycling. These conclusions may be somehow reversed in extreme cases, although material looping is predicted to have a limited power as credible remedy of a waste crisis. In those cases a general reduction of aggregate consumption, incineration or exporting of waste to the rest of the world might be considered as more reasonable actions.

Policy development concerning circular economies is also worth mentioning. Although we have proven that restorative taxation is a theoretical possibility, its real

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[17] This may explain why the recycling theory in economics is concentrated in the 70's.

implementation is unlikely due to imperfect information. Already Baumol and Oates (1971) commented on this lack of information and on its consequences for environmental policy: *"We simply do not, in general, have the information needed to determine the appropriate set of Pigouvian taxes and subsidies.* (Baumol and Oates, 1971, p51). We therefore emphasize that the required data describing material flows and waste streams in useful detail is currently missing from regulatory databases, in part because these have been designed, within the so-called hierarchy of waste management, to serve linear rather than circular purposes. Without qualitative information it is virtually impossible to transfor theoretical insights into useful real life exercises -including transition policy-, a common complaint heard in empirical work related to large scale recycling. So, more powerful legislation on minimal information to be reported to environmental authorities regarding components and technical procedures (or impossibilities) to recover embedded material would be a great help in alleviating this issue.

6 Conclusion

In this paper we have studied the behavior of an integrated circular economy in which several factors such as trade, scarcity, waste, recycling and population dynamics play a simultaneous role in determining allocations of scarce material and energy. Departing from Leontief specifications we have managed to derive, following a relatively smooth procedure, four general rules on the expected optimal behavior of the small open circular economy. We have characterized the activation (deactivation) of its main loops in a highly integrated setup. Results, however, are to be interpreted as long run references to a rich class of transformation processes, named quasi-concave technologies restricted by Anderson's limits. We have derived conditions on the fundamentals that explain why and when material loops as those prescribed by the circular economy are always preferred than the extract-use-dump practice. We have included in our analysis recycling, re-manufacturing and dismantling of durable capital as main possibilities of reuse. Our derivations suggest that, in general, circular economies and the so-called waste hierarchy are not consistent. Finally, although we have concentrated our attention on the first best and laissez-faire allocation of resources in a small open economy, different market structures and numerical exercises including non-competitive firms, waste markets, and expected policy corrections are a source of future research.

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Appendix

Proposition 1 *Consider the technology* $Q(m,e;\sigma,a,A) = A(am^{\frac{\sigma-1}{\sigma}} + (1-a)e^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ *where* $A \in \mathbb{R}_{++}$*,* $a \in (0,1)$ *and* $\sigma \in [0,1)$*. Then it follows that positive variations in the elasticity of substitution* σ *imply (short term) gains in efficiency for any* $\sigma \in [0,1)$*.*

Proof. Normalizing without loss of generality $A = 1$, first notice that since $a \in (0,1)$, $lim_{m\to+\infty} e|Q = \alpha \in \mathbb{R}_{++} = \frac{\alpha}{(1-a)^{\frac{\sigma}{\sigma-1}}}$ and $lim_{e\to+\infty} m|Q = \alpha \in \mathbb{R}_{++} = \frac{\alpha}{a^{\frac{\sigma}{\sigma-1}}}$, therefore $\frac{\partial lim_{m\to+\infty} e|Q=\alpha\in\mathbb{R}_{++}}{\partial\sigma} < 0$ and $\frac{\partial lim_{e\to+\infty} m|Q=\alpha\in\mathbb{R}_{++}}{\partial\sigma} < 0$. Second, we can rewrite the level curve $Q(m,e;\sigma) = \alpha$ as $m(e;\sigma) = ((\alpha^{\frac{\sigma-1}{\sigma}} - ae^{\frac{\sigma-1}{\sigma}})(1-a)^{-1})^{\frac{\sigma}{\sigma-1}}$. Finally, notice that $\frac{\partial m(e;\sigma)}{\partial\sigma} \leq 0$ which completes the proof ♣

Since the long term takes a Leontief shape, Proposition 1 implies a reverse in the elasticity of substitution to zero.