Taxing the Second Nature*

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Abstract

This paper examines the tax competition for mobile capital between symmetric countries. All first-nature features are completely abstracted away in such a framework so agglomeration and dispersion forces come purely from the second nature features such as monopolistic competition, increasing returns to scale, and trade costs. A positive tax indicates dispersion while a negative tax implies agglomeration. We find that the income effect and the pro-competitive effect together foster a strong dispersion force which is explored by the local government in the form of a positive tax rate.

Keywords: tax competition, second nature, symmetric countries, finite choke prices


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1 Introduction

In the real world, some economic phenomena are determined by local natural advantages, while others come from the economic linkages. They are called first nature and second nature, respectively, as documented in the literature (Cronon, 1991; Krugman, 1993). While traditional trade theory focuses on trade problems when countries have some first-nature features such as production technologies, geographical advantages, and resource abundance, new trade theory explores the implications of second nature—firms’ economic activities are related to the presence of and interaction with other firms.

The tax competition literature shows that the second nature can be taxed. The core-periphery models (e.g., Baldwin and Krugman, 2004; Borck and Pfüger, 2006; Kind et al., 2000) reveal the agglomeration rent—a higher location rent in a large country or region can be taxed away by the local government. The frameworks used in this research assume asymmetry of countries—countries with differences in some first-nature features such as population size. Thus, the sources of the first and the second natures are mixed.

This paper reports a new fact of dispersion rent. We consider the tax competition for mobile capital between symmetric countries, in which all first-nature features have been completely taken away. Thus, all results originate from second-nature features, such as the technology of increasing returns to scale (IRS), the market of monopolistic competition, and trade costs. The footloose capital (FC) model of Martin and Rogers (1995) is a useful workhorse for investigating the role of mobile capital using a general equilibrium with these second natures. There are two production factors in FC: mobile capital and immobile labor. Capital mobility fully captures the mobility of firms in choosing their locations. This basic framework is applied to examine various issues of firm location and trade patterns in Baldwin et al. (2003).\footnote{The symmetric FC model is particularly examined in their Chapters 3 and 5. However, since it does not feature endogenous asymmetry and agglomeration, this symmetric case is not sufficiently explored. The authors show more interests in “almost symmetric regions” and “near-catastrophic agglomeration” (pp. 82-83).}

Following Persson and Tabellini (1992), the tax competition in our model does not involve a specific public good. Tax revenue in a country is assumed to be equally redistributed among the residents as one source of income. Because of this symmetry, all countries have the same firm share, and identical wage rates in equilibrium. Trade in goods is also balanced in each country. Thus, we cannot observe any kind of home mar-
ket effects (HMEs) that demonstrate the advantages of large countries. Nevertheless, the second-nature is in operation, making it possible to examine the dispersion force in agglomeration economics. We consider dispersion not spontaneous if the tax rate is negative. Dispersion is spontaneous and exploited if the tax rate is positive. Thus, a subsidy indicates the existence of a strong agglomeration force, while a tax indicates a strong dispersion force.

This paper assumes a non-CES additively separable utility—the CARA (Constant Absolute Risk Aversion, Behrens and Murata, 2007) function. This setup is featured with variable elasticity of substitution (VES). It allows us to capture both the pro-competitive effect\(^2\) and the income effect\(^3\). Our result shows that the tax rate is positive when trade costs are high and negative when trade costs are low. This indicates a dispersion rent when trade costs are high. The dispersion force is so strong that firms do not move out even if the tax rate is positive. This result starkly contrasts with the agglomeration rent known in the literature of new economic geography that a positive tax rate results from the priority of the core region in which all mobile skilled workers locate (Baldwin and Krugman, 2004). Such an agglomeration rent is observable only when the regions are asymmetric. However, we provide a more direct way to see how the second nature features interact with each other, resulting in the dispersion rent. This allows us to study agglomeration economics without agglomeration.

We show that both the income effect and the pro-competitive effect are crucial to derive this dispersion rent. In fact, on the one hand, Ottaviano and van Ypersele (2005) consider this tax competition problem by a quadratic quasi-linear utility function proposed by Ottaviano \textit{et al.} (2002). Their framework captures the pro-competitive effect but loses the income effect. The authors find that, as long as the transport costs are low enough to allow trade, the equilibrium tax rate is always negative.\(^4\) On the other hand, to disclose the necessity of pro-competitive effect, we examine two CES models, with and without a homogeneous good. We again obtain negative tax rates only.

The dispersion rent is illustrated in a space of two symmetric countries. For simplicity, we consider neither “harmful tax competition” (OECD, 1998; Huizinga and Nielsen, 2008) nor “tax havens” (Dharmapala and Hines, 2009; Slemrod and Wilson, 2009). The

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\(^{2}\)This refers to the phenomenon that markup in a market decreases as the mass of firm there increases.

\(^{3}\)Namely, a higher income induces a larger expenditure in the manufacturing sector.

\(^{4}\)The negative tax rate is given by the first equation in their Section 5.3. Note that there is an error in part (i) of Proposition 3, which is pointed out by Haufler and Wooton (2010, footnote 25).
symmetric assumption rules out discussion of the advantages of being small or big (Bu-
covetsky, 1991; Wilson, 1991; Haufler and Wooton, 1999). Nevertheless, our setup could
be used as a building block for exploring more meaningful policy implications, in addi-
tion to a lot of existing models associated with first-nature features, such as comparative
advantages and country size (see, for example, Wilson (1991, 1999), Keen and Konrad
(2013)).

There are a few studies on tax competition related to ours. Section 15.5 of Baldwin
et al. (2003) considers taxes with symmetric countries in a footloose entrepreneur model
and concludes that “lower trade costs would exacerbate the race to the bottom when
regions are similar” (p. 380). By comparing four models in this paper, we find that
such a relationship is not necessarily monotone. In contrast to the symmetric countries
in our paper, Baldwin and Krugman (2004) and Borck and Pflüger (2006) focus on the
asymmetry across regions. Since a Nash equilibrium may not exist in their models, the
authors utilize a Stackelberg strategy instead. This strategy is also applied in a later
paper of Borck et al. (2012).

Another interesting strand of tax competition with increasing returns to scale assumes
oligopolistic competition. The paper of Haufler and Wooton (2010) finds that an equilib-
rium tax rate for symmetric countries might become positive in an oligopolistic market.
Our results of a positive tax rate are different in the sense that free entry is assumed in
monopolistic competition setups.

The remainder of this paper is organized as follows. Section 2 presents the tax com-
petition game based on a VES model. The main result of dispersion rent is obtained. In
Section 3, we provide two alternative models to show that the pro-competitive effect is
necessary to observe the dispersion rent. We show that the income effect is also necessary
to drive the dispersion rent in Section 4. Finally, Section 5 concludes.

2 Dispersion rent

We consider an economic space of two symmetric countries. The population in each coun-
try is \( L/2 \). The preferences of consumers are described by the following utility function

\[
U_i = \int_0^{N_i} u(c_{ii}(\omega))d\omega + \int_0^{N_j} u(c_{ji}(\omega))d\omega, \quad i \neq j \in \{1, 2\},
\]

where \( u(\cdot) \) is thrice continuously differentiable, strictly increasing, and strictly concave on
\((0, +\infty)\). When \( u(x) = x^{\frac{1}{\sigma}} \), then (1) degenerates to the CES utility widely used in the
literature.

We follow Zhelobodko et al. (2012) and introduce the relative love for variety: \( r_u(c) = -\frac{\text{c}u''(c)}{u'(c)} \). As in Zhelobodko et al. (2012), we impose the following restrictions on \( u(\cdot) \):

\[
\begin{align*}
    u(0) &= 0, \quad u'(\cdot) > 0, \quad u''(\cdot) < 0, \quad r_u(\cdot) < 1, \quad r'_u(\cdot) > 0, \quad r''_u(\cdot) < 2.
\end{align*}
\]

Thus, a representative individual’s budget constraint is determined by:

\[
\int_0^{N_i} p_{ii}(\omega)c_{ii}(\omega)d\omega + \int_0^{N_j} p_{ji}(\omega)c_{ji}(\omega)d\omega = y_i,
\]

where \( y_i \) stands for his/her nominal income. From the utility maximization, we have the following first-order condition (FOC):

\[
u'(c_{ji}) = \lambda_ip_{ji},
\]

where \( \lambda_i \) is the shadow price of an individual who lives in country \( i \). Taking the derivative of (4) with respect to \( p_{ji} \), we have:

\[
\frac{\partial c_{ji}}{\partial p_{ji}} = \frac{\lambda_i}{u''(c_{ji})}.
\]

Combining (4) and (5) yields:

\[
\frac{\partial c_{ji}}{\partial p_{ji}} \frac{p_{ji}}{c_{ji}} = -\frac{1}{r_u(c_{ji})},
\]

which reveals the relationship between the elasticity of demand and the relative love for variety.

The demand \( c_{ji} \) can be zero when \( p_{ji} \) is high enough. Consumers have a positive demand \( c_{ji} \) if and only if \( p_{ji} \leq p^c_i \), where \( p^c_i \) is the choke price given by \( u'(0)/\lambda_i \). It is noteworthy that in the case of a CES utility, the choke price is infinity so that \( c_{ji} \) is always positive.

There are two production factors, mobile capital and immobile labor. Each resident holds one unit of capital so the total amount of capital is also \( L \). Tax is imposed on the mobile capital in every country, with a rate of \( t_i \) in country \( i \). Choose the capital return as the numéraire so that the post-tax rent is 1. The capital will not be invested if the pre-tax capital rent is negative. Thus, throughout this paper, we assume that

\[
1 + t_i > 0, \quad \text{for} \quad i = 1, 2.
\]

We assume the iceberg transport cost \( \tau \). The operating profit after taxing a firm in country \( i \) is given as follows:

\[
\pi_i(x) = (p_{ii} - w_i)\frac{L}{2}c_{ii} + (p_{ij} - \tau w_i)\frac{L}{2}c_{ij} - t_i.
\]
Note that the demand $c_{ij}$ of an individual in country $j$ depends on its delivered price $p_{ij}$, which is described by (5). Therefore, profit maximization yields:

$$p_{ij} = \frac{\tau w_i}{1 - \tau_u(c_{ij})}. \quad (7)$$

Free entry implies that each firm earns zero profit. Thus, capital rent (measured by the net profit after tax) in country $i$ is determined by $r_i = \pi_i$. Since all workers are employed, each country accommodates some firms. We have only interior equilibrium. The free mobility of capital yields:

$$(p_{ii} - w_i)\frac{L}{2}c_{ii} + (p_{ij} - \tau w_i)\frac{L}{2}c_{ij} - t_i = (p_{ji} - \tau w_j)\frac{L}{2}c_{ji} + (p_{jj} - w_j)\frac{L}{2}c_{jj} - t_j. \quad (8)$$

The total income of a representative individual is determined by:

$$y_i = w_i + 1 + 2k_i t_i,$$

where $k_i$ is the equilibrium share of capital employed in country $i$ so that the mass of firms in country $i$ is $n_i = k_iL$. The market-clearing condition for labor in country $i$ is given by:

$$1 = k_i(Lc_{ii} + L\tau c_{ij}). \quad (9)$$

To gain more tractability, we now specify the subutility as the CARA utility $u(x) = 1 - e^{-\beta x}$ with parameter $\beta > 0$. Behrens and Murata (2007) give some interesting properties of this utility function. It is noteworthy that the choke price is finite because $u'(0) = \beta$ is finite.

For convenience, let $k = k_1$ and $k_2 = 1 - k$. Under CARA, Equations (3), (4), (8), and (9) are specified as follows.

$$e^{-\beta(c_{11} - c_{21})} = \frac{p_{11}}{p_{21}}, \quad (10)$$
$$e^{-\beta(c_{22} - c_{12})} = \frac{p_{22}}{p_{12}}, \quad (11)$$
$$\frac{L}{2}c_{11}(p_{11} - w_1) + \frac{L}{2}c_{12}(p_{12} - \tau w_1) - t_1 = \frac{L}{2}c_{21}(p_{21} - \tau w_2) + \frac{L}{2}c_{22}(p_{22} - w_2) - t_2, \quad (12)$$
$$kLp_{11}c_{11} + (1 - k)Lp_{21}c_{21} = w_1 + 1 + 2t_1 k, \quad (13)$$
$$kLp_{12}c_{12} + (1 - k)Lp_{22}c_{22} = w_2 + 1 + 2t_2 (1 - k), \quad (14)$$
$$kL(\tau c_{11} + c_{21}) = 1, \quad (15)$$
$$(1 - k)L(\tau c_{21} + c_{22}) = 1, \quad (16)$$
where $p_{ij}$ is given by (7) as follows:

\[
\begin{align*}
  p_{11} &= \frac{w_1}{1 - \beta c_{11}}, \\
p_{12} &= \frac{\tau w_1}{1 - \beta c_{12}}, \\
p_{21} &= \frac{\tau w_2}{1 - \beta c_{21}}, \\
p_{22} &= \frac{w_2}{1 - \beta c_{22}}.
\end{align*}
\]

Now we investigate the tax competition game in which two governments simultaneously choose their tax rates to maximize their welfare levels. This is a two-stage game because firms and consumers make their economic choices based upon the tax rates as given. The solution will be a subgame-perfect Nash equilibrium.

The role of country $i$’s government with a utilitarian social welfare function is to choose an optimal tax rate in order to maximize the utility of a representative resident:

\[
\max_{t_i} v_i = k_i Lu(c_{ii}) + k_j Lu(c_{ji}).
\]

The FOC is written as

\[
\frac{dv_i}{dt_i} = 0, \quad i = 1, 2. \quad (17)
\]

The second-order (sufficient) condition for optimality is described by:

\[
\frac{\partial^2 v_i}{\partial t_i^2} \bigg|_{t_1 = t_1^*, t_2 = t_2^*} < 0.
\]

Taking $t_1$ and $t_2$ as given, the above seven equations, (10)-(16), endogenously determine seven unknowns: $c_{11}(t_1, t_2), c_{12}(t_1, t_2), c_{21}(t_1, t_2), c_{22}(t_1, t_2), w_1(t_1, t_2), w_2(t_1, t_2),$ and $k(t_1, t_2)$. By substituting these values into (17), we can derive the optimal tax rate of the two countries: $t_1^*$ and $t_2^*$.

Because the countries are symmetric, we have the following explicit expressions for the equilibrium variables.

**Lemma 1** As long as trade starts, the industry share, wages, demands, and tax rates are given by

\[
k = \frac{1}{2}, \quad w_1 = w_2 \equiv w_{\text{CARA}}, \quad c_{11} = c_{22} \equiv c_d, \quad c_{12} = c_{21} \equiv c_x, \quad \text{and} \quad t_1 = t_2 \equiv T_{\text{CARA}},
\]

respectively, where $c_d$ is implicitly given by

\[
e^{-\frac{2}{L}(1+\tau c_d-\frac{2}{L})} = \frac{L(\tau + \beta c_d) - 2\beta}{L\tau^2(1 - \beta c_d)},
\]

7
and
\[ c_x = \frac{2 - c_d L}{\tau L}, \quad w^{\text{CARA}} = \frac{2(1 - \beta c_d)(1 + T^{\text{CARA}})[(\tau L - \beta(2 - Lc_d)]}{\beta\{(\beta + L)c_d - 2\}^2 + (\tau L^2 - \beta^2)c_d^2}, \]

\[ T^{\text{CARA}} = \frac{A_4 c_d^4 + A_3 c_d^3 + A_2 c_d^2 + A_1 c_d + A_0}{B_4 c_d^4 + B_3 c_d^3 + B_2 c_d^2 + B_1 c_d + B_0}, \]

where \( A_i, B_i \ (i = 0, 1, 2, 3, 4) \) are given in Appendix A.

**Proof:** See Appendix A. \( \square \)

In autarky, \( c_x = 0 \) holds, so (18) implies \( c_d = 2/L \). Assumption (2) implies \( \tilde{\beta} \equiv \beta/L \in (0, 1/2) \). Equation (10) indicates that the threshold value of the trade cost for autarky is given by
\[ \tau^{\text{autarky}} \equiv \frac{e^{2\tilde{\beta}}}{1 - 2\tilde{\beta}} > 1, \]

Meanwhile, the wage rate in autarky is \( w^{\text{CARA}} = [1/(2\tilde{\beta}) - 1](1 + T^{\text{CARA}}) \), and the utility of a representative resident is given by \( L(1 - e^{-2\tilde{\beta}})/2 \), which is independent of \( T^{\text{CARA}} \).

Thus, the equilibrium tax rates of the two countries are arbitrary in autarky.

**Proposition 1** As long as trade occurs, the equilibrium tax rate \( T^{\text{CARA}} \) has a non-monotone relationship with trade cost \( \tau \). It is positive and decreasing when \( \tau \) falls from \( \tau^{\text{autarky}} \) but negative and increasing to zero when \( \tau \) is close to 1.

**Proof:** See Appendix B. \( \square \)

The left panel of Figure 1 plots the equilibrium tax rate when parameters are given by \( L = 1 \) and \( \beta = 0.1 \). In this numerical example, \( \tau^{\text{autarky}} \approx 1.52675 \). This figure displays the property that \( T^{\text{CARA}} \) first decreases and then increases when \( \tau \) falls.

We obtain a U-shaped curve, and the tax rate is positive for a large \( \tau \). The CARA utility implies a finite choke price. When trade costs are high, as opposed to the case of a CES utility, a foreign product may not be consumed. Thus, a firm aiming at the domestic market cannot choose a location in another country, even though the tax rate on capital in Home is positive. This can be alternatively explained by the pro-competitive effect. In a dispersion distribution, firms do no move to the other country even if a positive tax is imposed because the markups and profits will be reduced in a market with more firms.\(^5\)

\(^5\)Although the population sizes are the same, the masses of firms are not necessarily the same due to different tax rates.
The U-shape of the tax rate $T_{\text{CARA}}$ is related to the U-shape of the national welfare. The welfare curve has a U-shape in this CARA model, as shown in the right panel of Figure 1, with the same parameters as those of the left panel. Bykadorov et al. (2016) show that such a U-shape is typical for a preference with decreasing elasticity of substitution, including CARA. When trade costs fall from a high level, the positive tax rate of mobile capital slows down the decreasing speed of the welfare level.

3 Models without pro-competitive effect

3.1 The CES setup with a homogeneous good

We add tax competition in the FC of Ottaviano and Thisse (2004, Section 3.2.1) with two symmetric countries.$^6$ The symmetric countries are named $i = 1, 2$. The residents in country $i = 1, 2$ have a preference represented by utility function

$$U_i = M_i^\mu A^{1-\mu}, \quad \mu \in (0, 1),$$

$$M_i = \left( \int_0^{n_i} c_{ii}^{\sigma-1}(\omega)d\omega + \int_0^{n_j} c_{ji}^{\sigma-1}(\omega)d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $c_{ji}$ is the demand in country $i$ for a variety produced in country $j$, $^7$ $\sigma$ represents the elasticity of substitution between two varieties. To simplify the analysis, we further

$^6$Kind et al. (2000) examine the tax competition to attract internationally mobile capital based on the industrial linkage model of Krugman and Venables (1995). In their analysis of dispersion equilibrium, their setup results in analytically intractable characterization, and the tax competition is based on simulations only.

$^7$Because all firms are symmetric, we drop the firm index $\omega$ in the following development.
assume $\sigma \geq 2$, which is consistent with the empirical results in the literature.\footnote{For example, Redding and Venables (2004) and Hanson (2005) find that $\sigma$ is between 5 and 10.}

A homogeneous (agricultural) good, $A$, is assumed, which can be costlessly traded across countries. The agricultural market is perfectly competitive, and each worker can produce one unit of $A$ in both countries. Thus, the wage rates are equal everywhere. Let them be $w$.

The production technology in the manufacturing sector is the same in two countries. Specifically, one unit of capital is required as the fixed input, and one unit of labor is required as the marginal input for production. Thus, the total cost of producing $c$ units of a variety is $r_i + wc$.

The total incomes in two countries are

$$Y_1 = L \left( \frac{1+w}{2} + kt_1 \right) \quad \text{and} \quad Y_2 = L \left[ \frac{1+w}{2} + (1-k)t_2 \right],$$

where $k$ is the firm share in country 1. As in the previous section, we assume the iceberg transport cost $\tau$. Notation $\phi \equiv \tau^{1-\sigma}$ denotes trade freeness. According to some CES properties, the equilibrium price of a variety produced in country $i$ and consumed in country $j$ is

$$p_{ij} = \begin{cases} \frac{\sigma}{\sigma - 1} w & \text{if } i = j, \\ \frac{\mu}{\sigma - 1} w \tau & \text{if } i \neq j. \end{cases}$$

The price indices in two countries are

$$P_1 = \left[ Lk + L(1-k)\phi \right]^\frac{1}{1-\sigma} \frac{\sigma}{\sigma - 1} w \quad \text{and} \quad P_2 = \left[ Lk\phi + L(1-k) \right]^\frac{1}{1-\sigma} \frac{\sigma}{\sigma - 1} w.$$

The market clearing conditions for each variety in two countries are written as

$$\frac{Y_1}{L[k + (1-k)\phi]} + \phi \frac{Y_2}{L(1-k+k\phi)} = \frac{\sigma(1+t_1)}{\mu},$$

$$\frac{Y_2}{L(1-k+k\phi)} + \phi \frac{Y_1}{L[k + (1-k)\phi]} = \frac{\sigma(1+t_2)}{\mu}.$$

Solving these two equations, we obtain the endogenous wage rate $w$ and firm share $k$ as follows.

$$w = \frac{\sigma - \mu}{\mu} \left\{ 1 + \frac{t_1 + t_2}{2} + \frac{(1+\phi)[\phi(\sigma + \mu) + \sigma - \mu](t_1 - t_2)^2}{2(1-\phi)\Psi(t_1,t_2)} \right\},$$

$$k = \frac{1}{2} + \frac{(1+\phi)[\phi(\sigma + \mu) + \sigma - \mu]}{2(1-\phi)\Psi(t_1,t_2)}.$$
where \( \Psi(t_1, t_2) \equiv [\phi(\sigma + \mu) - (\sigma - \mu)](t_1 + t_2) - 2\sigma(1 - \phi) \).

The indirect utility of each resident in country 1 is
\[
V_1 = \frac{1 + \omega + 2kt_1}{\omega^{\frac{1}{\sigma}} - \mu} \left( \frac{\sigma - 1}{\sigma} \right)^\mu 1 + w + 2kt_1 \left[ k + (1 - k) \phi \right] \frac{\omega^{\frac{1}{\sigma}}}{\sigma^{\frac{1}{\sigma}} - 1}.
\]

By use of (20) and (21), we can derive \( dV_1/dt_1 \) to show how the welfare level depends on the tax rate \( t_1 \). A similar expression can be found for country 2. In a Nash equilibrium, both of them are zero. We then obtain the equilibrium tax rate:
\[
t^*_1 = t^*_2 = \frac{1 - \phi \mu + 2 - \sigma}{\phi \mu + 2 - \sigma} \equiv T_{CESagr}.
\]

**Proposition 2**

(i) There is a local Nash equilibrium in the tax competition game in the CES setup with a homogeneous good. (ii) The equilibrium tax rate (23) is zero for \( \phi = 1 \) and always negative for \( \phi \in (0, 1) \). (iii) When \( \phi \) increases, (23) has a U-shape.

**Proof:** See Appendix C.

Since \( \phi = \tau^{1-\sigma} \) is a decreasing function of \( \tau \), the equilibrium tax rate has a U-shape when \( \tau \) falls. Figure 2 plots this tax rate with parameters \( \sigma = 5 \) and \( \mu = 0.95 \). As given by the previous proposition, this tax rate firstly falls from a negative level \( T_{CESagr}|_{\tau \to \infty} = -(1 - \mu)/(\sigma - \mu) \) and then increases to zero when the economy gradually moves from autarky to free trade.

![Figure 2: The tax rate for the CES setup with a homogeneous good](image)

The negative optimal tax rate indicates that the dispersion force resulting from the second-nature features in this CES framework is dominated by the agglomeration force. In
this CES framework, the markups are constant. Firms do not worry about their markup loss in an agglomerated country. Thus, the agglomeration force in this model is larger than that of Section 2.

With tax rate (23), the welfare level of (22) becomes

\[
V_{CESagr} = \frac{\sigma}{\sigma - \mu} \left( \frac{\sigma - 1}{\sigma} \right)^\mu \left[ \frac{L(1 + \phi)}{2} \right]^{\frac{\mu}{\sigma - 1}},
\]

which increases with \( \phi \). In this setup, globalization raises the welfare in a monotonic way, which is contrastive with Figure 1.

### 3.2 The CES setup with endogenous wages

We reformulate the previous FC model by removing the homogeneous good as done in Takahashi et al. (2013). Again, there are two symmetric countries, \( i = 1, 2 \), and two factors, mobile capital and immobile labor. Each resident holds one unit of capital. The total population is \( L \). Tax is imposed on the mobile capital in every country, with a rate of \( t_i \) in country \( i \). The capital return is again chosen as the numéraire so that the post-tax rent is 1. The pre-tax rent \( r_i \) is positive by (6).

Without the homogeneous good, residents’ preferences can be simply represented by

\[
U_i = \left( \int_0^{n_i} c_i^{\frac{\sigma-1}{\sigma}}(\omega)d\omega + \int_0^{n_j} c_j^{\frac{\sigma-1}{\sigma}}(\omega)d\omega \right)^{\frac{\sigma}{\sigma - 1}},
\]

where \( c_{ji} \) is the demand in country \( i \) for a variety produced in country \( j \).

Because of the constant elasticity of demand, in a firm’s optimal production plan, the ratio of the variable costs to the fixed costs is always \( \sigma - 1 \). Given the fixed cost \( r_i \) in country \( i \), the total production cost is \( \sigma r_i \) and the labor cost of this firm becomes \( (\sigma - 1) r_i \).

The mass of firms in country \( i \) is calculated as

\[
n_i = \frac{Lw_i}{2(\sigma - 1)r_i}.
\]

The tax income in a country is evenly redistributed to all residents. Thus, the nominal income of a resident in country \( i \) is

\[
y_i = w_i + 1 + \frac{1}{\sigma - 1} \frac{w_it_i}{1 + t_i}.
\]

Because of the constant markup of CES frameworks, the equilibrium domestic price of a variety produced in country \( i \) is \( p_{ii} = [\sigma/(\sigma - 1)]w_i \), and the delivered price in the
other country $j$ is $p_{ij} = p_{ii} \tau$. The price index in country $i$ is written as $P_i = (n_ip_{ii}^{1-\sigma} + \phi n_j p_{jj}^{1-\sigma})^{1/(1-\sigma)}$, and the total demand for this variety is

$$y_i L \frac{P_i^{-\sigma}}{2P_1^{1-\sigma}} + \phi y_j L \frac{P_i^{-\sigma}}{2P_j^{1-\sigma}}.$$  

On the other hand, for a firm in country $i$, the total variable cost is $(\sigma - 1)r_i$ while the marginal input for each unit is one unit of labor. Accordingly, the supply of its product is $(\sigma - 1)r_i/w_i$. By use of $p_{ii} = w_i/(\sigma - 1)$, the market clearance condition implies

$$p_i^{\sigma-1} r_1 = \frac{y_1 L}{2P_1^{1-\sigma}} + \frac{\phi y_2 L}{2P_2^{1-\sigma}} \quad \text{and} \quad p_2^{\sigma-1} r_2 = \frac{y_2 L}{2P_2^{1-\sigma}} + \frac{\phi y_1 L}{2P_1^{1-\sigma}}.$$  

The above two expressions indicate that

$$\frac{\sigma}{\sigma - 1} (p_i^{\sigma-1} r_1 - \phi p_2^{\sigma-1} r_2) = (1 - \phi^2) \frac{y_1 L}{2(\sigma - 1)P_1^{1-\sigma}} = (1 - \phi^2) \frac{w_1}{r_1} p_1^{1-\sigma} + \frac{\phi w_2}{r_2} p_2^{1-\sigma}. \quad (25)$$

For convenience, let

$$w_i^t \equiv \frac{w_i}{(\sigma - 1)(1 + t_i)},$$

which is the relative price of labor to the post-tax capital rent in country $i$.

Meanwhile, the capital clearance condition is

$$\frac{Lw_1}{2(\sigma - 1)} \frac{1}{1 + t_1} + \frac{Lw_2}{2(\sigma - 1)} \frac{1}{1 + t_2} = L,$$

which can be written as

$$w_1^t + w_2^t = 2. \quad (26)$$

Since wage rates are positive, (6) and (26) imply that

$$w_1^t < 2, \quad w_2^t < 2.$$  

Furthermore, we have

$$w_2^t = 2 - w_1^t, \quad \frac{dw_2^t}{dt_i} = -\frac{dw_1^t}{dt_i}, \quad i = 1, 2. \quad (27)$$

By using (27), Equation (25) can be rewritten as

$$\mathcal{F}(w_1^t, t_1, t_2, \phi) \equiv Q_2(w_1^t, t_1, t_2)\phi^2 + Q_1(w_1^t, t_1, t_2)\phi + Q_0(w_1^t, t_1, t_2) = 0, \quad (28)$$

where

$$Q_2(w_1^t, t_1, t_2) = 1 + w_1^t(\sigma - 1 + \sigma t_1) - \sigma(2 - w_1^t)(1 + t_2),$$
\[ Q_1(w^t_1, t_1, t_2) = \sigma(1 + t_1)w^t_1 \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma-1} \left( \frac{w^t_1}{2 - w^t_1} \right)^{\sigma-2} \left[ 1 - \left( \frac{1 + t_1}{1 + t_2} \right)^{1-2\sigma} \left( \frac{w^t_1}{2 - w^t_1} \right)^{3-2\sigma} \right], \]

\[ Q_0(w^t_1, t_1, t_2) = w^t_1 - 1. \]

We call (28) the wage equation, which determines wage rate \( w^t_1 \) implicitly when trade freeness and tax rates are given.

If \( t_1 \leq t_2 \), we have

\[ F(1, t_1, t_2, \phi) = (1 + t_1) \left[ \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma-1} - \left( \frac{1 + t_2}{1 + t_1} \right)^{\sigma} \right] \sigma \phi + (t_1 - t_2)\sigma\phi^2 \leq 0 \]

from (6). On the other hand, when \( w^t_1 \to 2 \), we have

\[ \lim_{w^t_1 \to 2} Q_2(w^t_1, t_1, t_2) > 0, \]

\[ \lim_{w^t_1 \to 2} Q_1(w^t_1, t_1, t_2) = \begin{cases} +\infty & \text{if } \sigma > 2, \\ \sigma(1 + t_1) \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma-1} > 0 & \text{if } \sigma = 2. \end{cases} \]

\[ \lim_{w^t_1 \to 2} Q_0(w^t_1, t_1, t_2) = 1 > 0. \]

Thus, the wage equation has a solution \( w^t_1 \in [1, 2) \). Similarly, we know that the wage equation has a solution in \((0, 1)\) if \( t_1 > t_2 \). By use of \( \sigma \geq 2 \), both \( Q_2\phi^2 + Q_0 \) and \( Q_1\phi \) are increasing functions of \( w^t_1 \). Thus,

\[ \frac{\partial F}{\partial w^t_1} > 0 \] (29)

holds and the wage equation has a unique solution. Let \( w^t_1(t_1, t_2, \phi) \) be the solution of the wage equation, which depends on \( t_1, t_2, \) and \( \phi \). It is noteworthy that \( w^t_1(t_1, t_2, \phi) = 1 \) if and only if\(^9\) either \( \phi = 0 \) or \( t_1 = t_2 \).

Now we investigate the tax competition game in which two governments simultaneously choose their tax rates to maximize their welfare levels.

We have already examined the behavior of firms and consumers. Now, we focus on the behavior of governments in the first stage. According to the implicit function theorem and (27), we have

\[ \frac{dw^t_1}{dt_1} = -\frac{\partial F / \partial t_1}{\partial F / \partial w^t_1} \bigg|_{w^t_1 = w^t_1(t_1, t_2, \phi)}, \quad \frac{dw^t_2}{dt_2} = \frac{\partial F / \partial t_2}{\partial F / \partial w^t_1}. \]

\(^9\)This is because \( F(1, t_1, t_2, \phi) = \sigma\phi\mathcal{G} \), where \( \mathcal{G} \equiv (1 + t_2) \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma} - (1 + t_1) \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma} + (t_1 - t_2)\phi \) is a monotonic function of \( t_1 \).
The partial derivatives of $F$ are
\[
\frac{\partial F}{\partial t_1} = \sigma \left( \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma - 1} \left( \frac{w_1^t}{2 - w_1^t} \right)^{\sigma - 1} (2 - w_1^t)\sigma \right.
+ \left. \frac{w_1^t}{1 + t_1} \left( \frac{2 - w_1^t}{w_1^t} \right)^{\sigma - 1} (\sigma - 1)\phi + w_1^t\phi^2 \right)
\]
\[> 0,\]
\[
\frac{\partial F}{\partial w_1^t} = -\sigma \left\{ - (1 + t_2) \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma} \left( \frac{w_1^t}{2 - w_1^t} \right)^{\sigma - 2} \frac{\phi}{w_1^t - 2}
+ (1 + t_2) \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma} \left( \frac{2 - w_1^t}{w_1^t} \right)^{\sigma - 1} \phi \left( 1 + \frac{2(\sigma - 1)}{w_1^t - 2} \right)
- \phi^2 \left( 2 - \frac{1}{\sigma} + t_1 + t_2 \right) - \frac{1}{\sigma} \right\},
\]
\[
\frac{\partial F}{\partial t_2} = -\sigma \left\{ \left( \frac{1 + t_2}{1 + t_1} \right)^{\sigma - 1} \left[ \frac{2 - w_1^t}{w_1^t} \right]^{\sigma - 1} w_1^t \phi
+ (2 - w_1^t) \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma} \left[ \frac{w_1^t}{2 - w_1^t} \right]^{\sigma - 1} (\sigma - 1)\phi + (2 - w_1^t)\phi^2 \right\}
< 0.
\]

By using (29), we know that $w_1^t(t_1, t_2, \phi)$ decreases with $t_1$ and increases with $t_2$, while $w_2^t$ increases with $t_1$ and decreases with $t_2$.

Now the welfare levels in country 1 can be written as
\[
V_1 = \frac{y_1}{P_1} = \frac{1}{\sigma} \left( \frac{L}{2} \right)^{\sigma - 1} \left[ 1 + (\sigma - 1)w_1^t + \sigma t_1 w_1^t \right]
\]
\[
\left[ (1 + t_1)^{1 - \sigma}(w_1^t)^{-2 - \sigma} + (1 + t_2)^{1 - \sigma}(2 - w_1^t)^{-2 - \sigma} \phi \right]^{\frac{1}{1 + \sigma}}.
\]

The total differential of $V_1$ with respect to $t_1$ is given by
\[
\frac{1}{V_1} \frac{dV_1}{dt_1} = \frac{w_1^t}{1 + t_1 w_1^t + (w_1^t - 1)\frac{2 - 1}{\sigma} (1 + t_1) + (1 + t_2) \left( \frac{1 + t_1}{1 + t_2} \right)^\sigma \left( \frac{2 - w_1^t}{w_1^t} \right)^{-2 - \sigma} \phi}
+ \left\{ \frac{(2 - \sigma)\left[ 1 - \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma - 1} \left( \frac{w_1^t}{2 - w_1^t} \right)^{\sigma - 1} \phi \right]}{(\sigma - 1)\left[ w_1^t + \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma - 1} \left( \frac{w_1^t}{2 - w_1^t} \right)^{\sigma - 1} (2 - w_1^t)\phi \right]}
+ \frac{t_1 + \frac{2 - 1}{\sigma} \left[ w_1^t + \left( \frac{1 + t_1}{1 + t_2} \right)^{\sigma - 1} \left( \frac{w_1^t}{2 - w_1^t} \right)^{\sigma - 1} (2 - w_1^t)\phi \right]}{1 + t_1 w_1^t + (w_1^t - 1)\frac{2 - 1}{\sigma}} \right\} \frac{\partial w_1^t}{\partial t_1}.
\]

We have a similar expression for $dV_2/dt_2$. The Nash equilibrium tax rates $(t_1^*, t_2^*)$ satisfy the conditions $dV_1/dt_1 = 0$ and $dV_2/dt_2 = 0$.  

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Proposition 3  

(i) A symmetric Nash equilibrium exists in the tax competition game, in which the tax rates and wages are given by

\[ t^*_1 = t^*_2 = T_{CES\text{noA}} \equiv -\frac{1 - \phi}{2\sigma - 1 - \phi}, \quad \text{for all} \quad \phi \in (0, 1], \tag{30} \]

\[ w^*_1 = w^*_2 = 1. \tag{31} \]

(ii) Tax rate (30) decreases with \( \tau \).

Proof: (i) If two countries adopt the same tax rate \( t^* \), then Equation (28) gives (31), and both \( dV_1/dt_1 = 0 \) and \( dV_2/dt_2 = 0 \) have a unique solution of (30). The following calculation shows that the second-order condition is satisfied at this equilibrium:

\[
\left. \frac{d^2V_1}{dt_1^2} \right|_{t_1 = t_2 = t^*} = -\frac{(2\sigma - 1 - \phi)^2}{\sigma - 1} \frac{(6\sigma - 3 - \phi)\phi}{(2\sigma - 1 + \phi)[(1 - \phi)^2 \frac{1}{\sigma} + 4(\frac{\sigma - 1}{\sigma})^2 \phi]} \left[ \frac{L(1 + \phi)}{2^{2\sigma - 1}(\sigma - 1)} \right]^{\frac{1}{\tau - 1}} (\sigma - 1)^{\frac{2 - \sigma}{\tau - 1}} < 0.
\]

(ii) By the definition of \( \phi = \tau^{1-\sigma} \), the monotone property is given by the following inequality.

\[
\frac{\partial T_{CES\text{noA}}}{\partial \tau} = -\frac{\sigma - 1}{\tau} \phi \frac{\partial T_{CES\text{noA}}}{\partial \phi} = -\frac{2(\sigma - 1)^2 \phi}{\tau(2\sigma - 1 - \phi)^2} < 0.
\]

\( \square \)

Figure 3 plots a numerical example of tax rate (30) with \( \sigma = 5 \).

![Figure 3: The tax rate for the CES setup without a homogeneous good](image-url)
As illustrated in Figure 3, the tax rate of (30) is monotone and always negative. Globalization reduces tax competition, and the tax rate eventually becomes zero in the case of free trade. Intuitively, the advantage of local production disappears when trade is free enough, so there is no incentive to attract capital. This property is observed because the wage rate is now endogenously determined. In fact, the tax curves of Figures 2 and 4 are not monotone but U-shaped.

Although there is no homogeneous good in this model, the utility of residents is

\[ V_{\text{CESnoA}} = \left[ \frac{L(1 + \phi)}{2} \right]^{\frac{1}{1-\sigma}} = \left[ \frac{L(1 + \tau^{1-\sigma})}{2} \right]^{\frac{1}{1-\sigma}}, \]

which is monotonically increasing in \( \phi \) and decreasing in \( \tau \), as we have observed in the CES model with a homogeneous good.

### 4 Income effect

Ottaviano and van Ypersele (2005) examine the tax competition game by the assumption of a quasi-linear utility function proposed by Ottaviano et al. (2002). This framework successfully capture the pro-competitive effect but loses the income effect.

They derive the following equilibrium tax rate\(^\text{10}\)

\[
t_1^* = t_2^* = T_{\text{OTT}} \equiv \frac{(b + cL) L T}{8(2b + cL)^2} \left[ (2b^2 + 4bcL + c^2L^2)\tau - 4a(b + cL) \right],
\]

where \( a, b, \) and \( c \) are positive parameters, \( \tau \) is the trade cost satisfying\(^\text{11}\)

\[
\tau \leq \tau_{\text{trade}} \equiv \frac{4a}{4b + cL}.
\]

It is immediately verified that \( T_{\text{OTT}} \) is always negative when \( \tau \in (0, \tau_{\text{trade}}] \). Thus, this model is unable to observe the dispersion rent.

We conduct a simulation for the equilibrium tax rate by the quasi-linear utility (OTT) setup obtained in Ottaviano and van Ypersele (2005). The tax curve of Figure 4 is plotted with parameters \( a = b = c = 1, L = 1 \).

The curve of Figure 4 is U-shaped, which is similar to Figure 2. Both models have a homogeneous good so that wage rates are fixed.

\(^{10}\)This expression is a special case of their equilibrium tax rate on P.40 when countries are symmetric. The statement in their Proposition 3 (i) is incorrect. See our footnote 4.

\(^{11}\)Their threshold value of \( \tau_{\text{trade}} \) for trade to start is extended from \( 2a/(2b + cL) \) to \( 4a/(2b + cL) \) because the two countries are symmetric in size and the firm share is 1/2 in equilibrium.
5 Conclusion

This paper examines capital tax competition between symmetric countries. All first-nature features are removed and we are able to explore the second nature only.

A positive tax rate indicates a strong dispersion force because the firm dispersion of equilibrium is spontaneous and exploited. Meanwhile, a negative tax rate implies a strong agglomeration force because the dispersion is sustainable by the subsidy. Since the agglomeration force is strong when trade costs are sufficiently low, we always observe negative tax rates when trade costs are low in various models. However, our result shows that the income effect and the pro-competitive effect foster dispersion rent when trade costs are high. Namely, in the Nash equilibrium, each country impose a positive tax when trade costs are high.

Three alternative models are applied to show that both the income effect and the pro-competitive effect are necessary to derive the dispersion rent. The following table summarizes how these models derive different tax competition results.
Table 1: Different effects in different models

<table>
<thead>
<tr>
<th></th>
<th>CES with $A$</th>
<th>CES without $A$</th>
<th>OTT</th>
<th>CARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>income effect</td>
<td>$t \uparrow$</td>
<td>$t \uparrow$</td>
<td>$\times$</td>
<td>$t \uparrow$</td>
</tr>
<tr>
<td>marg. prod. cost</td>
<td>$\times$</td>
<td>$t \downarrow$</td>
<td>$\times$</td>
<td>$t \downarrow$</td>
</tr>
<tr>
<td>choke price</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$t \uparrow$</td>
<td>$t \uparrow$</td>
</tr>
<tr>
<td>pro-comp. effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tax curve shape</td>
<td>U-shape</td>
<td>monotone</td>
<td>U-shape</td>
<td>U-shape</td>
</tr>
<tr>
<td>dispersion rent</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

In Table 1, three important effects (three rows) are compared for four models given in the columns. Notation $t \uparrow$ indicates that the effect in the row encourages the local government to levy a high tax, notation $t \downarrow$ implies the opposite, and notation $\times$ means that the effect does not appear in the model. In the last row, a “+” indicates the existence of a positive tax rate when trade cost $\tau$ is high while a “−” indicates a negative rate.

**Appendix A: The proof of Lemma 1**

In the symmetric equilibrium, $k = 1/2$, $w_1 = w_2 \equiv w_{CARA}$, $c_{11} = c_{22} \equiv c_d$, $c_{12} = c_{21} \equiv c_x$, and $t_1 = t_2 \equiv T_{CARA}$ hold. Equations (10)–(16) immediately imply that $c_d$ satisfies $\mathcal{H}(c) = 0$, while $c_x$, $w_{CARA}$ are given by (18), where

$$\mathcal{H}(c) = e^{-\hat{\beta}(1+\tau)} + \frac{2\hat{\beta}}{\tau} - \frac{\tau - 2\hat{\beta} + \hat{\beta}c}{\tau^2(1-\beta c)}.$$

In the above equation, $\hat{\beta} \equiv \beta/L \in (0,1/2)$ is defined in the context after Lemma 1. It is readily verified that the following inequalities hold:

$$\mathcal{H}'(c) = -\frac{\beta}{\tau^2} \left[ \tau(1+\tau)e^{-\hat{\beta}(1+\tau)} + \frac{2\hat{\beta}}{\tau} + \frac{\tau}{(1-\beta c)^2} + \frac{1-2\hat{\beta}}{(1-\beta c)^2} \right] < 0,$$

$$\lim_{c \to 0} \mathcal{H}(c) = e^{2\tau} - \frac{\tau - 2\hat{\beta}}{\tau^2} > \frac{\tau(\tau-1) + 2\hat{\beta}}{\tau^2} > 0,$$

$$\lim_{c \to \frac{1}{\hat{\beta}}} \mathcal{H}(c) = -\infty.$$

Thus, Equation $\mathcal{H}(c) = 0$ has a unique root $c_d \in (0, \frac{1}{\hat{\beta}})$. 

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We then take derivatives of Equations (10)–(16) with respect to \( t_1 \). Evaluate the partial derivatives of \( c_{11}, c_{12}, c_{22}, c_{21}, w_1, w_2 \) and \( k \) at the above symmetric equilibrium and solve them out by those equations. This allows us to evaluate \( dv_1/dt_1 \) at the symmetric equilibrium, which becomes an equation of \( T_{CARA} \). Solving for this variable we can obtain (19), where

\[
A_4 \equiv - \tau \tilde{\beta}^4 L (\tau - 1) [2 \tilde{\beta} + 1 + \tau] < 0,
\]

\[
A_3 \equiv \tilde{\beta}^3 [8 \tilde{\beta}^2 \tau (\tau - 1) + 2 \tilde{\beta} \tau (\tau^2 + 5 \tau - 4) - (\tau^4 - 4 \tau^3 - 2 \tau^2 + 1)],
\]

\[
A_2 \equiv - \frac{\tilde{\beta}^2}{L} [8 \tilde{\beta}^2 \tau (\tau - 1) + 4 \tau \tilde{\beta}^2 (8 \tau - 5) - 2 \tilde{\beta} (\tau^4 - 3 \tau^3 - 7 \tau^2 + 3)
- \tau (5 \tau^3 - 3 \tau^2 + \tau - 3)],
\]

\[
A_1 \equiv \frac{2 \tilde{\beta}}{L^2} [4 \tilde{\beta}^3 \tau (3 \tau - 2) + 2 \tilde{\beta}^2 (7 \tau^2 - 3) - \tilde{\beta} (3 \tau^3 - 3 \tau^2 + 2 \tau - 6)
- \tau^2 (3 \tau^2 - \tau + 2)],
\]

\[
A_0 \equiv \frac{2}{L^3} [2 \tilde{\beta}^2 \tau (\tau - 3) - 4 \tilde{\beta}^3 (2 \tau^2 - 1) + 2 \tilde{\beta} \tau^2 (\tau^2 - \tau + 2) + \tau^3 (\tau - 1)],
\]

\[
B_4 \equiv \tilde{\beta}^4 L (2 \tilde{\beta} + \tau + 1) (\tau^2 - 1) > 0,
\]

\[
B_3 \equiv \tilde{\beta}^3 [-4 \tilde{\beta}^2 (2 \tau^2 - 3) - 2 \tilde{\beta} (\tau^3 + 5 \tau^2 - 4) + (\tau^4 - 3 \tau^3 - 4 \tau^2 - 3 \tau + 1)],
\]

\[
B_2 \equiv \frac{\tilde{\beta}^2}{L} [8 \tilde{\beta}^3 (\tau^2 - 3) + 4 \tilde{\beta}^2 (7 \tau^2 + 3 \tau - 6) - 2 \tilde{\beta} (\tau^4 - 3 \tau^3 - 6 \tau^2 - 9 \tau + 3)
- \tau (5 \tau^3 - \tau^2 + 3 \tau - 3)],
\]

\[
B_1 \equiv \frac{2 \tilde{\beta}}{L^2} [8 \tilde{\beta}^4 - 4 \tilde{\beta}^3 (3 \tau^2 + 2 \tau - 4) - 2 \tilde{\beta}^2 (4 \tau^2 + 9 \tau - 3) + 3 \tilde{\beta} \tau (\tau^3 - \tau^2 + 2 \tau - 2)
+ \tau^2 (3 \tau^2 - \tau + 2)],
\]

\[
B_0 \equiv - \frac{2}{L^3} [8 \tilde{\beta}^4 - 4 \tilde{\beta}^3 (2 \tau^2 + 3 \tau - 1) + 6 \tilde{\beta}^2 \tau (\tau - 1) + 2 \tilde{\beta} \tau^2 (\tau^2 - \tau + 2) + \tau^3 (\tau - 1)].
\]

The second-order condition is too complicated to be written down. The following figure plots \( \partial^2 v / \partial t_1^2 \) in a numerical example with parameters \( L = 1, \beta = 0.1 \), which is negative.
Appendix B: The proof of Proposition 1

We use notation $T_{\text{CARA}}(\tau)$ to emphasize that $T_{\text{CARA}}$ depends on $\tau$. According to Lemma 1, we have

$$T_{\text{CARA}}(1) = 0, \quad T_{\text{CARA}}(\tau_{\text{autarky}}) = \frac{2\tilde{\beta}[e^{2\tilde{\beta}} - 1 - 2\tilde{\beta} - \frac{1}{2}(2\tilde{\beta})^2] + [1 + 2\tilde{\beta} + (2\tilde{\beta})^2 - e^{2\tilde{\beta}}]}{(1 - 2\tilde{\beta})(e^{2\tilde{\beta}} - 1 - 2\tilde{\beta})} > 0,$$

where the inequality holds because of $1 + x + \frac{1}{2}x^2 < e^x < 1 + x + x^2$ for $x \in (0, 1)$.

Furthermore, we have

$$T'_{\text{CARA}}(1) = -\frac{(1 + 3\tilde{\beta})(1 - \tilde{\beta}) + \tilde{\beta}^3}{2\beta(2 - \beta)(1 - \beta)} < 0,$$

$$T'_{\text{CARA}}(\tau_{\text{autarky}}) = \frac{le^{-l}}{4(e^l - 1 - l)^2[(2 - l)e^l + 2(1 - l)^2]}\tilde{l} > 0,$$

where $l \equiv 2\tilde{\beta} \in (0, 1)$ and

$$\tilde{l} = le^l\{2l(1 - l)(1 - l + l^2) + (1 + l + l^2 - e^l)[1 + (1 - l)^2]\} + 2l(1 - l)^2(1 + l)(e^l - 1) + 4e^l(e^l - 1 - l)[1 + (1 - l)^2] > 0.$$

Appendix C: The proof of Proposition 2

(i) In the above text, we have shown that a tax rate of (23) satisfies the first-order conditions for the optimal welfare levels in two countries. The second-order condition is
also satisfied as follows:

\[
\frac{\partial^2 V(t_1, t_2)}{\partial t_1^2} \bigg|_{t_1=t_2=T_{CESagr}} = -\frac{\sigma \mu}{4(\sigma - \mu)(\sigma - 1)^2} \left( \frac{\sigma - 1}{\sigma} \right)^\mu \left( \frac{1 + \phi}{2} \right)^{\frac{\mu}{\sigma - 1}} \\
\times \left[ \mu(1 - \phi)^2 + 4\phi + \sigma(\phi^2 - 1 - 4\phi) \right] \\
\times \left( \frac{3 - \phi)[1 - \mu + (1 + \mu)\phi] + (\sigma - 1)(3 + 10\phi - \phi^2)}{(1 + \phi)^2(\mu + \sigma - 1)[\sigma - \mu + (\mu + \sigma)\phi]} \right) \\
< 0.
\]

(ii) This property is evident from (23).

(iii) At first, we have

\[
\frac{dT_{CESagr}(1)}{d\phi} = \frac{\sigma}{2(\sigma - 1)} > 0,
\]

\[
\frac{dT_{CESagr}(0)}{d\phi} = -\frac{2(\sigma - 1)[\sigma - 2(1 - \mu)]}{(\sigma - \mu)^2} < 0,
\]

where the inequalities are from the assumption of \( \sigma \geq 2 \). Our conclusion follows from the fact that the tax rate curve crosses any horizontal line no more than twice because equation \( T_{CESagr}(\phi) = \text{constant} \) has at most two roots for any “constant.”

References


