Risk and Return in a Behavioural Framework.
An application To Russian Financial Markets.

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Representation of Risk Tolerance

The standard approach

*Compare the investor’s expected utility of with the mathematical expectation of the payoff.*

If a security promises a payoff $X_t$ in $t$, given the investor’s utility $u_t()$, their *expected utility* is:

$$U(X_t) = \mathbb{E}u_t(X_t)$$

For discrete payoffs $\{x_{t1}, \ldots, x_{tn}\}$:

$$U(X_t) = \sum_{i} u_t(x_{ti})p_{ti}$$

where $p_{ti} = \mathbb{P}[X_t = x_{ti}]$
Given the mathematical expectation of $X_t$, the standard definition of risk aversion, states an investor is:

- **Risk Neutral** if \( U(X_t) = u_t(\mathbb{E}X_t) \)
- **Risk Seeker** if \( U(X_t) > u_t(\mathbb{E}X_t) \)
- **Risk Averse** if \( U(X_t) < u_t(\mathbb{E}X_t) \)
Can investors be contemporaneously risk averse and risk tolerant?
### Risk tolerance: Gains vs. Losses

<table>
<thead>
<tr>
<th>Choose between gains:</th>
<th>Choose between losses:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prospect G1</strong></td>
<td><strong>Prospect L1</strong></td>
</tr>
<tr>
<td>5 with certainty</td>
<td>−5 with certainty</td>
</tr>
<tr>
<td><strong>Prospect G2</strong></td>
<td><strong>Prospect L2</strong></td>
</tr>
<tr>
<td>0 with probability .5</td>
<td>0 with probability .5</td>
</tr>
<tr>
<td>10 with probability .5</td>
<td>−10 with probability .5</td>
</tr>
</tbody>
</table>
While the degree of risk tolerance may vary from individuals, almost systematically individuals:

- prefer the sure amount of prospect $G$
- prefer the risky payoff as regards prospect $L$. 
Different Alternatives?

Both prospects propose alternatives with the same expected payoff

- **Prospect G:**
  
  \[ E(G_1) = 5 ; \]
  \[ E(G_2) = 0.5 \times 0 + 0.5 \times 10 = 5 ; \]

- **Prospect L:**
  
  \[ E(L_1) = -5 ; \]
  \[ E(L_2) = 0.5 \times 0 + 0.5 \times -10 = -5 \]

Why there is a difference then?
An Anomalous Result

More formally, for the typical investor:

\[ U(G_1) > U(G_2) ; \]
\[ U(L_1) < U(L_2) . \]

Rewriting, we get:

\[ u(G_1) = u(\mathbb{E}G_2) > U(G_2) ; \]
\[ u(L_1) = u(\mathbb{E}L_2) < U(L_2) . \]

We deduce an (apparently) absurd conclusion:
Investors are \textit{contemporaneously risk adverse and risk seeker}. 
S-shaped Value Function

Red part (convex) implies preference for risk, rather than sure loss.
Black part (concave) implies preference for the sure gain.
The "flat" probabilities are now replaced a weighting function $\pi()$. An explicit formulation for $v(x)$ is the piecewise power function:

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\beta & \text{if } x < 0. \end{cases}$$
The Decision Weights: $\pi()$

Inverse S-shaped weighting function (black)
We analyse MICEX, using the Cumulative Prospect Theory by Tversky and Kahneman (1992).

CPT advantages:

- Decision weights are based on the cumulative probability distribution of the risky payoff.
- Weighting function can be differentiated for gains and losses.
- The cumulative functional and can be applied in the presence of uncertainty (not only for riskiness.)
Preference Function

\[ V_c(X) = \sum_{i<0} (w^-(F_i^-) - w^-(F_{i-1}^-))v(x_i) \]
\[ + \sum_{i>0} (w^+(F_i^+) - w^+(F_{i+1}^+))v(x_i) \]

Discrete payoffs \( x_j \) are ranked so that if \( j < 0 \), \( x_j \) is a loss, if \( j > 0 \), \( x_j \) is a gain, \( x_0 = 0 \) and \( x_j < x_{j+1} \).
Preference Function

where:

\[ F_{i<\text{m}} = F_{i>n} = 0 \]  \hspace{1cm}  (1)

\[ F_i^- = \sum_{j=-\text{m}}^{i} p_j \]  \hspace{1cm}  (2)

\[ F_i^+ = \sum_{j=i}^{n} p_j \]  \hspace{1cm}  (3)

\[ w^-(F) = \frac{F^\delta}{(F^\delta + (1 - F)^\delta)^\frac{1}{\delta}} \]  \hspace{1cm}  (4)

\[ w^+(F) = \frac{F^\gamma}{(F^\gamma + (1 - F)^\gamma)^\frac{1}{\gamma}} \]  \hspace{1cm}  (5)
The weighting functional is applied to probabilities related to histograms bins and the value function to their average return.
## Results: Details

<table>
<thead>
<tr>
<th>Return</th>
<th>Prb.</th>
<th>Value</th>
<th>$F^-$</th>
<th>$F^u$</th>
<th>$w^-$</th>
<th>$w^+$</th>
<th>$DW^-$</th>
<th>$DW^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.05,-0.04]</td>
<td>0.01</td>
<td>-3689.86</td>
<td>0.01</td>
<td>0.03</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.04,-0.03]</td>
<td>0.00</td>
<td>-2957.76</td>
<td>0.01</td>
<td>0.03</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.03,-0.02]</td>
<td>0.01</td>
<td>-2199.73</td>
<td>0.02</td>
<td>0.06</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.02,-0.01]</td>
<td>0.07</td>
<td>-1403.28</td>
<td>0.09</td>
<td>0.16</td>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.01,0]</td>
<td>0.39</td>
<td>-533.67</td>
<td>0.48</td>
<td>0.44</td>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,0.01]</td>
<td>0.38</td>
<td>237.19</td>
<td>0.52</td>
<td>0.43</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.01,0.02]</td>
<td>0.12</td>
<td>623.68</td>
<td>0.50</td>
<td>0.42</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.02,0.03]</td>
<td>0.02</td>
<td>977.66</td>
<td>0.38</td>
<td>0.36</td>
<td>0.36</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Results: Net Values

<table>
<thead>
<tr>
<th>Cumulative Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative loss value</td>
<td>-465.87</td>
</tr>
<tr>
<td>Cumulative gain value</td>
<td>392.07</td>
</tr>
<tr>
<td>Net value</td>
<td>-73.81</td>
</tr>
</tbody>
</table>
Portfolio approach

- We group by level of correlation MICEX shares.
- For each group we identify a representative shares (which maximises correlation in the group).

We identify the five shares:

- Ros Agro
- MFON MegaFon
- Sberbank Rossii
- Rossiyskiye Seti
- GMK Norilskiy Nikel

We assume the investor wants to beat the MICEX.
For each MICEX bin we show the optimal portfolio combination.

<table>
<thead>
<tr>
<th>Return</th>
<th>Optimal frequency</th>
<th>w1</th>
<th>w2</th>
<th>w3</th>
<th>w4</th>
<th>w5</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0</td>
<td>0.60</td>
<td>0.10</td>
<td>0.08</td>
<td>0.50</td>
<td>0.18</td>
<td>0.13</td>
</tr>
<tr>
<td>&gt; 0.01</td>
<td>0.51</td>
<td>-1.19</td>
<td>-0.40</td>
<td>1.99</td>
<td>0.60</td>
<td>-0.00</td>
</tr>
<tr>
<td>&gt; 0.02</td>
<td>0.35</td>
<td>0.32</td>
<td>0.35</td>
<td>0.83</td>
<td>-2.31</td>
<td>1.81</td>
</tr>
<tr>
<td>&gt; 0.03</td>
<td>0.36</td>
<td>0.02</td>
<td>0.11</td>
<td>-0.85</td>
<td>4.09</td>
<td>-2.37</td>
</tr>
</tbody>
</table>