Risk and Return in a Behavioural Framework. An application To Russian Financial Markets.

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April 10-13, 2018

Representation of Risk Tolerance

The standard approach

Compare the investor's expected utility of with the mathematical expectation of the payoff.

If a security promises a payoff X_t in t, given the investor's utility $u_t()$, their *expected utility* is:

$$U(X_t) = \mathbb{E}u_t(X_t)$$

For discrete payoffs $\{x_{t1}, \ldots, x_{tn}\}$:

$$U(X_t) = \sum_i u_t(x_{ti})p_{ti}$$

$$\text{where } p_{ti} = \mathbb{P}[X_t = x_{ti}]$$

Representation of Risk Tolerance

Given the mathematical expectation of X_t, the standard definition of risk aversion, states an investor is:

$$\begin{aligned}
\textit{Risk Neutral} \text{ if } \quad & U(X_t) = u_t(\mathbb{E}X_t) \\
\textit{Risk Seeker} \text{ if } \quad & U(X_t) > u_t(\mathbb{E}X_t) \\
\textit{Risk Averse} \text{ if } \quad & U(X_t) < u_t(\mathbb{E}X_t)
\end{aligned}$$

Can investors be contemporaneously risk averse and risk tolerant?

Risk tolerance: Gains vs. Losses

Choose between gains:

Prospect G1	
5	with certainty

Prospect G2	
0	with probability .5
10	with probability .5

Choose between losses:

Prospect L1	
-5	with certainty

Prospect L2	
0	with probability .5
-10	with probability .5

Typical Investors

While the degree of risk tolerance may vary from individuals, almost systematically individuals:

- prefer the sure amount of prospect G
- prefer the risky payoff as regards prospect L.

Different Alternatives?

Both prospects propose alternatives with
the same expected payoff

- Prospect G:

$$\mathbb{E}(G_1) = 5\ ;$$
$$\mathbb{E}(G_2) = .5 \times 0 + .5 \times 10 = 5\ ;$$

- Prospect L:

$$\mathbb{E}(L_1) = -5\ ;$$
$$\mathbb{E}(L_2) = .5 \times 0 + .5 \times -10 = -5$$

Why there is a difference then?

An Anomalous Result

More formally, for the typical investor:

$$U(G_1) > U(G_2) \; ;$$
$$U(L_1) < U(L_2) \; .$$

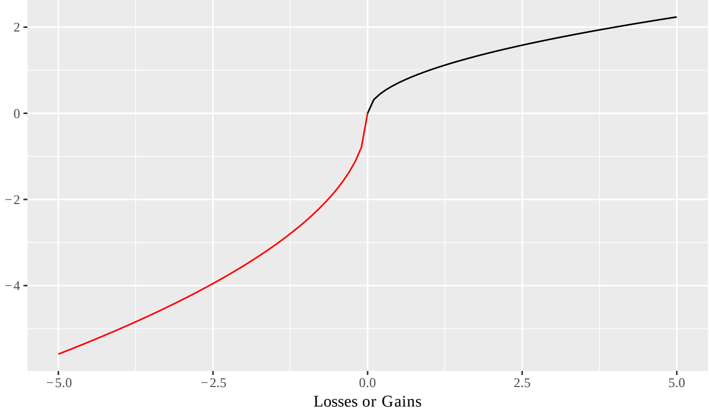
Rewriting, we get:

$$u(G_1) = u(\mathbb{E}\, G_2) > U(G_2) \; ;$$
$$u(L_1) = u(\mathbb{E} L_2) < U(L_2) \; .$$

We deduce an (apparently) absurd conclusion:
Investors are *contemporaneously risk adverse and risk seeker*.

S-shaped Value Function

Red part (convex) implies preference for risk, rather than sure loss.
Black part (concave) implies preference for the sure gain.



Analytical Formulation of Preference Function

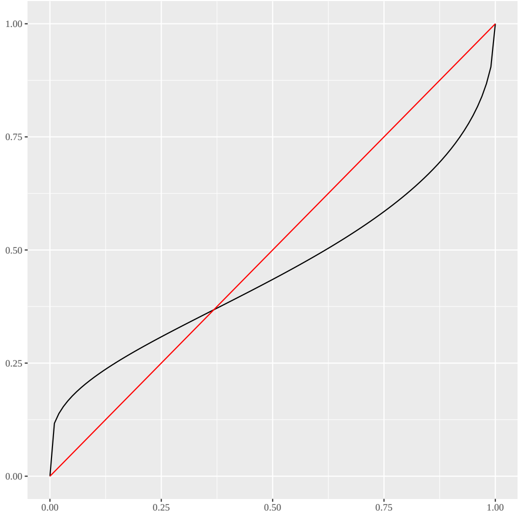
$$V(X) = \sum_i \pi(p_i) v(x_i)$$

The "flat" probabilities are now replaced a weighting function $\pi()$.
An explicit formulation for $v(x)$ is the piecewise power function:

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geqslant 0 \\ -\lambda(-x)^\beta & \text{if } x < 0. \end{cases}$$

The Decision Weights: $\pi()$

Inverse S-shaped weighting function (black)



An Analysis of MICEX

We analyse MICEX, using the Cumulative Prospect Theory by Tversky and Kahneman (1992).
CPT advantages:

- Decision weights are based on the cumulative probability distribution of the risky payoff.
- Weighting function can be differentiated for gains and losses.
- The cumulative functional and can be applied in the presence of uncertainty (not only for riskiness.)

Preference Function

$$V_c(X) = \sum_{i<0} (w^-(F_i^-) - w^-(F_{i-1}^-))v(x_i) + \sum_{i>0} (w^+(F_i^+) - w^+(F_{i+1}^+))v(x_i)$$

Discrete payoffs x_j are ranked so that if $j < 0$, x_j is a loss, if $j > 0$, x_j is a gain, $x_0 = 0$ and $x_j < x_{j+1}$.

Preference Function

where:

$$F^{-}_{i<-m} = F^{+}_{i>n} = 0 \tag{1}$$

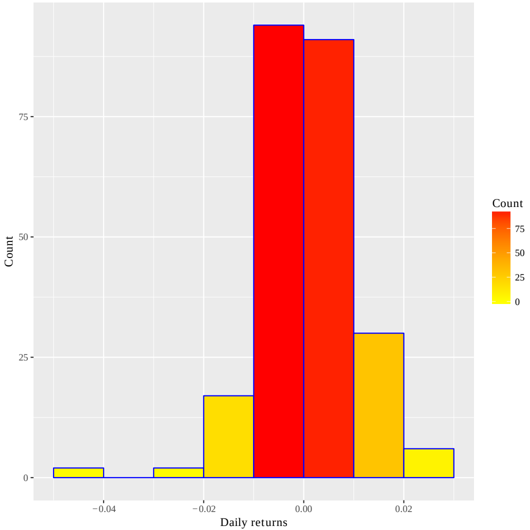
$$F^{-}_{i} = \sum_{j=-m}^{i} p_j \tag{2}$$

$$F^{+}_{i} = \sum_{j=i}^{n} p_j \tag{3}$$

$$w^{-}(F) = \frac{F^{\delta}}{(F^{\delta} + (1-F)^{\delta})^{\frac{1}{\delta}}} \tag{4}$$

$$w^{+}(F) = \frac{F^{\gamma}}{(F^{\gamma} + (1-F)^{\gamma})^{\frac{1}{\gamma}}} \tag{5}$$

Discretization



The weighting functional is applied to probabilities related to histograms bins and the value function to their average return.

Results: Details

Return	Prb.	Value	F^{-}	F^{u}	w^{-}	w^{+}	DW^{-}	DW^{+}
(-0.05,-0.04]	0.01	-3689.86	0.01		0.03		0.03	
(-0.04,-0.03]	0.00	-2957.76	0.01		0.03		0.00	
(-0.03,-0.02]	0.01	-2199.73	0.02		0.06		0.02	
(-0.02,-0.01]	0.07	-1403.28	0.09		0.16		0.10	
(-0.01,0]	0.39	-533.67	0.48		0.44		0.28	
(0,0.01]	0.38	237.19		0.52		0.43		0.01
(0.01,0.02]	0.12	623.68		0.50		0.42		0.06
(0.02,0.03]	0.02	977.66		0.38		0.36		0.36

Results: Net Values

Cumulative Value	Value
Cumulative loss value	-465.87
Cumulative gain value	392.07
Net value	-73.81

Portfolio approach

- We group by level of correlation MICEX shares.
- For each group we identify a representative shares (which maximises correlation in the group).

We identify the five shares:

- Ros Agro
- MFON MegaFon
- Sberbank Rossii
- Rossiyskiye Seti
- GMK Norilskiy Nikel

We assume the investor wants to beat the MICEX.

Results

For each MICEX bin we show the optimal portfolio combination.

Return	Optimal frequency	w1	w2	w3	w4	w5
> 0	0.60	0.10	0.08	0.50	0.18	0.13
> 0.01	0.51	-1.19	-0.40	1.99	0.60	-0.00
> 0.02	0.35	0.32	0.35	0.83	-2.31	1.81
> 0.03	0.36	0.02	0.11	-0.85	4.09	-2.37