Basel IRB Asset and Default Correlation Parameterization

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Abstract

Basel III was finalized in 2017. As was firstly introduced in Basel II (2006), Basel III allows for the use of statistical models or for the internal-ratings-based, IRB, approach. It suggests incorporating different asset correlation (R) functions to assess credit risk for the loan portfolio, or risk-weighted assets, RWA. A shortcoming of the function is that its calibration still dates back to 2004 neglecting risk parameters’ evolution in 15 years. As we show, there is another (tacit) deficiency. The asset correlation function solely depends on the individual (marginal) default probability (PD) disregarding the default rate variance within the loan book (credit pool).

Numerous researches challenged estimating asset correlation in practice; several discussed the default rate variance in the presence of default correlation. However, Basel III did not consider this.

The novelty of the paper is an explicit demonstration of a single parameterization for the IRB asset correlation formula. The asset correlation function does not differ by credit products given the same PD and default rate (DR) variance. Using bivariate normal distribution function provides material bias to default rate variance and default correlation estimates. The existing capital requirements significantly underestimate credit risk (specifically in retail loans’ domain).

JEL Codes: E5, C4, C15, C16, G2.

Keywords: Basel II, IRB, correlated defaults, asset correlation, binomial distribution, Bernoulli trials.

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1. INTRODUCTION

Capital adequacy ratio (CAR) became an internationally accepted indicator of the bank's financial standing in 1988 when the Basel I Accord was adopted (BCBS, 1988). CAR equals to the amount of bank own funds (capital) divided by the amount of risk-weighted assets (RWA). Originally there were several categories of fixed (predefined by the Basel I) risk-weights that were multiplied on assets to derive total RWA amount.

In 2006 Basel II allowed using loan default statistics and mathematical models to estimate risk-weights by banks themselves (BCBS, 2006). The approach was called an internal-ratings-based (IRB) one. The IRB approach is based on (Vasicek, 1987) loan portfolio model. It suggests how to derive the individual credit risk estimate per borrower (in non-retail domain) or facility (in retail domain) as a part of gross portfolio credit risk. Conceptually the IRB approach has two inputs. The first input is the individual (marginal) credit risk estimate (probability of default, PD). A bank is responsible for PD modeling and annual validation. The second input is the asset correlation (AC) function submitted by a regulatory authority. (BCBS, 2005) postulates that AC function parameterization was initially calibrated for all credit product types using 2003 dataset. Then the Basel Committee on banking supervision (BCBS) revised AC function twice: in 2004 for retail exposures (BCBS, 2004) and in 2009 for large financial institutions’ (or systemically important financial institutions, SIFIs) borrowings (BCBS, 2009, p. 30). Thus for the dominant part of credit asset classes – i.e. for the non-retail (specifically corporate) and retail exposures – AC function calibration did not change for 15 years since 2004.

Asset correlation (AC) function allows making the allocation of gross portfolio credit risk to the individual exposures. The AC function varies by credit product types. This means that different AC values exist for different credit products given the same probability of default (PD), or default rate (DR) at the portfolio level, and DR variance. Further on we show that this is wrong, as the asset correlation, probability of default (or default rate) and DR variance are uniquely linked independent of the credit product type.
Basel III is a response to the global financial crisis of 2007-09. In a consultative paper format, the Basel Committee published it in December 2009 and finalized it in December 2017 (BCBS, 2017). Basel III has a range of novelties. Inter alia, it introduces liquidity and leverage risk metrics; it offers corporate revises remuneration rules. Nevertheless, Basel III does not conceptually touch IRB regulation. It limits its scope (e.g. excludes IRB application to equity exposures; removes advanced IRB modeling for banks and very large exposures) and adjusts several quantitative parameters (e.g. output floor, minimum PD estimate (floor), loss given default (LGD) value for non-subordinated exposures). Same time the AC function is left unchanged (except for SIFIs), i.e. it still varies by credit products and comports quantitative parameter estimates based on 2003-04 data sample.

Basel III inherited from Basel II the requirement of regular IRB model validation. For this purpose banks have to evaluate accuracy of the PD models. General idea of such a testing procedure is to compare the portfolio PD forecast with the realized default rate (DR). Here comes a challenge. The conventional binomial test (BCBS, 2005) and its alternative Jeffrey’s interval (Brown, et al., 2001), (EBA, 2019, p. 20) assume default independence, whereas the realized default rate is a consequence of individual credit risk dependencies (i.e. the presence of asset correlation). To overcome this challenge, (BCBS, 2005) and (Blochwitz, et al., 2006) proposes testing procedures incorporating asset correlations. However, the question comes as to how to derive asset correlation. There are numerous papers (for instance, see (Wunderer, 2019) for a comprehensive overview) suggesting approaches to empirically define asset correlation. However, there is no a single theoretically justified universal solution for asset correlation parameterization. Current paper aims at providing such a one. The suggested parameterization enables to objectively define asset correlation based on bank’s own observed data of historical default rate.

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2 On December 16, 2019 the Basel Committee launched the Consolidated Basel Framework. It compiles all the acting regulatory guidance (standards). Though the committee notices certain inconsistencies when joining the standards, it makes no revisions after Basel III with respect to the IRB AC function (BCBS, 2019, p. 6). URL: [https://www.bis.org/bcbs/publ/d491.htm](https://www.bis.org/bcbs/publ/d491.htm) [accessed December 25, 2019].

3 Advanced IRB approach requires banks to develop a wider range of statistical models, not limited to that of probability of default (PD). IRB approach limited to PD models development and validation is called a foundation one.

4 Output floor is the percent of the standardized (fixed) risk-weight that acts as a minimum benchmark for IRB (variable, based on statistical models) risk-weight. Thus the maximum of the two risk-weights (output floor and IRB one) goes for CAR computation.
and DR variance. This is exactly what IRB approach targets in its very essence, i.e. to reflect banks’ own
default statistics.\(^5\)

There are two primarily findings coming from the suggested parameterization that significantly
improve the accuracy of regulatory credit risk assessment at the portfolio level and eliminate the two
above mentioned shortcomings of the Basel IRB risk-weighting formula (let us refer to it as the ‘Basel
IRB framework’) with respect to AC function. First, we show that the AC function is uniquely linked to
the portfolio DR and DR variance. It is a mere property of DR probability distribution, and not a feature
of a particular credit product. This means that actual values of asset correlation may vary by credit
products only if there are differences in default rates or in DR variances, being independent of the credit
product type as a third determinant for AC value. Second, the offered solution enables to recalibrate the
AC function with the recent data available to credit institution, not preserving outdated parameter values
from 2003-04 dataset.

To guide the reader on how we got the AC function parameterization, the paper has the following
structure. Section 2 presents literature overview. It consists of two parts. Part 1 briefly shows the Basel
IRB framework that takes asset correlation as an input. Asset and default correlation definitions are
discussed there. Part 2 discusses how various researchers handled the issue of the asset and default
correlation evaluation. The material shortcomings of the approaches are summed up as an output. Section
3 proves the suggested universal parameterization (analytical dependence) of default rate, DR variance
and asset correlation. Section 4 runs the robustness check for the suggested parameterization using
simulated data with negative, neutral and positive correlation. Section 5 presents evidence from the
publicly available data to demonstrate the impact of the suggested parameterization. Particularly, we
show that there are situations when the existing IRB approach and 2003-04 AC function produce five
times more excessive credit risk capital requirements than it is needed. Same time we show cases when
the existing capital requirements more than twice underestimate credit risk (specifically in retail domain).
Section 6 summarizes the paper novelties and its contribution to the field.

\(^5\) “... banks that have received supervisory approval to use the IRB approach may rely on their own internal
estimates of risk components in determining the capital requirement for a given exposure” [bold and italics from
2. LITERATURE REVIEW

PART 1. BASEL IRB FRAMEWORK

(Vasicek, 1987) model underlies the IRB approach in Basel II and III accords. (Merton, 1974) model forms the basis for the Vasicek one. Thus, Merton defined default for the company as a situation when the market value of its publicly traded assets goes below the amount of its debt. Vasicek used Merton definition and expanded it by saying that the market value of the i-th company asset \( (Z_i) \) depends on two factors: systemic \( (X) \) and idiosyncratic \( (\epsilon_i) \) ones as follows in (1).

\[
(1) \quad Z_i = X\sqrt{R} + \epsilon_i\sqrt{1-R}
\]

where \( Z_i \sim N(0; 1) \) - asset value of the i-th borrower,

\( X \sim N(0; 1) \) - systemic factor,

\( \epsilon_i \sim N(0; 1) \) - idiosyncratic factor,

\( R \) - dependence parameter.

(Vasicek, 1987, p. 2) assumes that systemic factor contributes to asset market value with a share of square root of \( R \). As a note for future discussion, he does not call \( R \) neither an asset, nor a default correlation, just an 'exposure to a common factor'. He suggests no parameterization for \( R \).

Terms ‘asset and default correlations’ appear in the working paper No. 14 (BCBS, 2005). Specifically, \( R \) is called asset correlation. We will cover the ‘default correlation’ definition from (BCBS, 2005) later in Part 2 of the literature review. However, the working paper has a different notation that is conventional to the probability theory (we write it below in terms corresponding to the ones already introduced above).

\[
(2) \quad Z_i = rX + \epsilon_i\sqrt{1-r^2}
\]

Formula (2) is conventional to the generation of a normally distributed random variable \( Z \) that co-depends with another normally distributed random variable \( X \) with the correlation coefficient \( (r) \). To link formula (1) to formula (2) we have to define \( R = r^2 \). Thus \( r \) stands for the mathematical correlation (let us refer to it as default correlation, DC, as (Foulcher, et al., 2005) also do), whereas \( R \) stands for
regulatory asset correlation (AC). By such a definition, asset correlation $R$ is always positive. However, the default correlation might still be negative. From the credit risk management perspective formula (1) does not allow to account for risk diversification when there is a negative correlation for credit risk events. Formula (2) does allow for risk diversification, as we show below, specifically in case of a finite and not large number (ca. one thousand) of observations (or total number of borrowers, credit facilities). (Gordy, 2000, pp. 147, eq. C.1) and (Gordy & Heitfield, 2010, pp. 46, eq. 3) also use formula (2) allowing for negative correlation coefficient ($r$).

However, the Basel Committee decides to proceed with more conservative formula (1). Formula (1) serves the baseline for the following ultimate IRB risk-weighting formula\(^6\) of Basel II and then Basel III assuming there is an infinite number of infinitely small exposures, i.e. the portfolio is perfectly granular.\(^7\)

$$R_W_i = N \left( \frac{N^{-1}(PD_i) + N^{-1}(0.999)\sqrt{R}}{\sqrt{1-R}} \right)$$

$R_W_i$ – capital requirement for i-th borrower or facility; acts as a multiplier for assets within the CAR denominator;

0.999 - confidence level for systemic risk-factor (X) realization;

$N(\ )$ – standard normal probability distribution function; \(N^{-1}(\ )\) – its inverse;

PD - probability of default lying in the range of $[0; 1]$, stands for the idiosyncratic risk-factor value.

AC function reflects the degree of borrowers’ co- or counter-dependence. We may deem it to be a regulatory novelty as Vasicek did not suggest any functional dependence of $R$ and PD. The IRB AC function is as follows (see par. 53, 54, 64, 118, 120, pp. 63-75 of (BCBS, 2017)).

\(^6\) Without loss of generality, we demonstrate formula (3) without subtraction of expected loss part (-PD) and without multiplication by loss given default (LGD) and exposure at default (EAD). $R_W_i$ in the presented form is the total capital requirement for the credit risk. Regulator requires decomposing it in two parts: expected loss (EL) and unexpected loss (UL). EL goes into the numerator of the CAR formula; UL – to CAR denominator. Later we will show policy implications with respect to total capital requirement. That is why it is sufficient to proceed with formula (3).

\(^7\) We do not wish to discuss here the earlier noticed deficiencies of the IRB approach, including the choice of the confidence level (Zimper, 2014), the individual risk-factors distribution normality assumption (Witzany, 2013), single-systemic factor use (Pykhtin, 2004), non-granularity (Gordy & Howells, 2004), correlation of PD-LGD parameters (Meng, et al., 2010), non-Gaussian joint credit risk distribution (Li, 2000).
\( R_i(PD_i) = R_0 + R_{MIN} \cdot Q(PD_i, \text{coef}) + R_{MAX}(1 - Q(PD_i, \text{coef})) \)\(^{(4)}\)

\( Q(PD_i, \text{coef}) \) - weight as a function of PD and calibration coefficient (coef) in the following form

\( Q(PD_i, \text{coef}) = \frac{1 - e^{-\text{coef} \cdot PD_i}}{1 - e^{-\text{coef}}} \) \(^{(5)}\)

- \( R \) - asset correlation; \( R_0 \) - fixed value if \( R \) is independent of PD;
- \( R_{MIN} \) - minimum value of asset correlation attained at the largest PD;
- \( R_{MAX} \) - maximum value of asset correlation attained at the smallest PD;

The asset correlation function varies by product types. Basel working paper 14 (BCBS, 2005, p. 51) postulates that \( R \) values were calibrated ca. 15 years ago using quantitative impact study (QIS) 3 (see Annex 1 for parameters calibrated by the Basel Committee).\(^8\) This raises an issue whether the preserved \( R \) functions' calibration is representative and, if it is not, then how we should recalibrate it.

As we can see from (4), \( R \) depends only on PD. The Basel Committee wanted to reflect the following idea. The higher PD is, the lower the \( R \) value is. This means, the less creditworthy the borrower is, the less its credit risk is augmented when crisis comes in. The reason for this is that one’s risk assessment is already relatively high. Figure 1 has the visual representation of various \( R \) functions.

The key implication of AC functions variation by product types is the fact that capital requirements differ given the same default statistics. Annex 3 offers two examples (see row 7): for mean DR equal to 5% and 10%. For instance, for mean DR of 5% and DR variance of 1% the capital requirement (\( RW_i \)) – i.e. the proxy for the mean DR and a bonus for asset (default) correlation – varies from the lowest of 14.73% of the credit exposure amount if it is qualifying revolving retail (QRR) loan to 33.11% if it is a loan to a SIFI.

The Vasicek model provides the link between independent individual credit risk assessment (PD) and credit risk assessment as a part of an aggregate credit portfolio, or a risk-weight (RW). Consider three simplified illustrations to the Vasicek model concept. First, if the borrowers are fully independent, individual credit risk assessment equals to the probability of default (PD) corresponding to

\(^{8}\)BCBS distributed QIS 3 data collection template in November 2002 and collected it in 2003. URL: [https://www.bis.org/bcbs/qis/qis3.htm](https://www.bis.org/bcbs/qis/qis3.htm) [accessed: December 13, 2019].
that borrower or facility. Second, if the borrowers are co-dependent in their credit repayment behavior, then the individual credit risk as a part of portfolio credit risk equals to the probability of default (PD) and some positive add-on (bonus). Third, if the borrowers are counter-dependent, then the individual credit risk assessment should equal to the probability of default and a negative add-on (malus) of portfolio credit risk component.

Figure 1. Asset correlation per credit product types according to Basel II (III).

To sum up, the IRB framework has two material shortcomings. First, it does not reflect the benefits from negative default dependencies. Second, the asset correlation function’s parameterization may be outdated and may require validation and recalibration. Latter seems natural as banks annually validate their inputs to the Basel IRB framework (see par. 425, 427, 443, 476, 501, 531 at (BCBS, 2006)).

PART 2. APPROACHES TO ASSET AND DEFAULT CORRELATION MODELING

There are several milestones in literature when we trace asset and default correlation modeling. To list those chronologically, they are (Vasicek, 1987), (Duffie & Singleton, 1999), (Li, 2000), (Nagpal & Bahar, 2001), (Lopez, 2002), (Duffie & Singleton, 2003), (BCBS, 2005), (Blochwitz, et al., 2006), (Patel & Pereira, 2008), (Ozdemir & Miu, 2009), (Duellmann, et al., 2010), (Li, et al., 2015), (Wunderer, 2019).

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9 It is a rare case that we did not come across before and is not allowed by the Basel IRB framework realization as we have shown above.
(Duffie & Singleton, 1999) are the first authors to mention the correlation of defaults as a formal term. They discuss the importance of such a feature for the credit portfolio management by showing the increased DR variance in its presence. However, they do not attempt calibrating it to historical data. Later (Li, 2000) and (Duffie & Singleton, 2003) discuss how copulas might be applied to simulate joint default dependence. Though both use Gaussian copula, (Li, 2000) merely shows the methodology of how to apply it, whereas (Duffie & Singleton, 2003) claim that Gaussian copula is inadequate in fitting the data.

(BCBS, 2005, p. 51) suggests modification of a PD model accuracy validation procedure (i.e. binomial test) given the presence of asset correlation. It still warns that direct use of recommended asset values from QIS 3 (see Annex 1) would significantly extend the confidence interval. This implies making conclusions that the model is accurate more often than is really needed or expected by someone. As a sort of response to (BCBS, 2005), (Blochwitz, et al., 2006, p. 295) suggest using default, not asset correlation in the Basel IRB risk weighting formula. (Blochwitz, et al., 2006, p. 295) also note that in practice default correlation may be up to 3%. This coincides for the French data estimates by (Foulcher, et al., 2005) and for German companies by (Hamerle & Rosch, 2006, p. 21). Both values are much lower than the values from the Basel AC function resulting in a compromise, i.e. the confidence interval becomes larger than in case of default independence, though smaller than if validators used Basel II AC values from Annex 1. As a result, validators deem models to be accurate less often than if when they used Annex 1 for correlation R values.

(Lopez, 2002) and (Patel & Pereira, 2008) try to investigate the drivers of asset and default correlation. (Lopez, 2002) runs a more descriptive analysis, whereas (Patel & Pereira, 2008) undertake a regression one. However, (Lopez, 2002) obtains asset correlation values that are comparable to the regulatory one. The methodology of (Patel & Pereira, 2008) is, on opposite, non-observable and non-verifiable as they use a proxy for default correlation by taking an equivalent of principal components from the correlation matrixes formed from the copula dependence parameters where latter are computed on the comparable five year intervals. The major shortcoming of (Patel & Pereira, 2008) work as they do neither demonstrate the distribution of default rates, nor the derived correlations or their proxies, nor run a goodness-of-forecast testing for the obtained so called default correlation proxy-values.
Let us more closely review approach to default correlation (DC) definition. (Nagpal & Bahar, 2001) offer DC parameterization of the type for two categories of credit risk (ratings) A and B:

\[
DC = \frac{PD(Y_i; Y_j) - PD(Y_i)PD(Y_j)}{\sqrt{PD(Y_i)(1-PD(Y_i))} \sqrt{PD(Y_j)(1-PD(Y_j))}} = \frac{PD(Y_i; Y_j) - PD^2}{PD(1-PD)}
\]

where \(PD(Y_i; Y_j)\) is the joint default probability for the two random variables (variates), r.v., \(Y_i\) and \(Y_j\):

\[
Y_i = Y_j = \begin{cases} 
0, & 1 - PD \\
1, & PD 
\end{cases}
\]

and \(Y_i = Y_j = 1\) is the default (success) event occurrence for a single borrower (facility);

\(PD\) – marginal default probability.

(BCBS, 2005, p. 48)\(^{10}\) offers transformed DC formula:\(^{11}\)

\[
DC = Corr(Y_i; Y_j) = \frac{N(y, y, R) - PD^2}{PD(1-PD)}
\]

Where \(N(y, y, R)\) – bivariate standard normal cumulative density function (cdf) for two r.v. values of \(\gamma = N^{-1}(PD)\) and asset correlation \(R = Corr(Z_i; Z_j)\):

When one compares (6) to (8), one may conclude that (BCBS, 2005, p. 48) assumes the following ((Gordy, 2000, pp. 148, eq. C.4) suggests a proof for it, albeit he merely introduces it like an assumption):

\[
P(Y_i; Y_j) = N(y, y, R)
\]

Then \(N(y, y, R)\) resembles the concept of upper (lower) tail dependence index (see (Nelsen, 1999), for instance). Such a transition in (8) is not justified, though it is often reproduced (e.g. (Foulcher, et al., 2005, pp. 5, formulas (2.5), (2.6)), (Gordy & Heitfield, 2010, p. 47), (Wunderer, 2019)).

For future discussion, let us remember the modification coming from the (BCBS, 2005, p. 48) formula (8):

\[
N(y, y, R) - PD^2 = DC \cdot PD \cdot (1 - PD)
\]

\(^{10}\) Typical values of \(N(y, y, R)\) are available in Annex 5
\(^{11}\) Author was unable to find the publicly available first version, prior to this updated one if not to consider a page within (BCBS, 2000, pp. 38-39).
(Duellmann, et al., 2010) find that asset correlation values differ when one derives it either from asset price (quote) data or from the default time series. (Wunderer, 2019, pp. 4, eq. 2.4) tries to explain this observation. He supposes that the portfolio inhomogeneity is the cause. To prove it, he departs from the following theoretical binary r.v. variance definition:

\[(11) \quad Var(Y_i) = PD \cdot (1 - PD)\]

and DR definition:

\[(12) \quad DR = \frac{\sum_{i=1}^{n} Y_i}{n}\]

to come to the following DR variance as first shown in (Gordy & Heitfield, 2010, pp. 58, eq. 20):

\[(13) \quad Var(DR) = N(\gamma, \gamma, R) - PD^2 + \frac{PD - N(\gamma, \gamma, R)}{n}\]

He does not write this, but if we let n go to infinity, we obtain exactly the same component that (BCBS, 2005) presented, i.e.

\[(14) \quad Var(DR) = N(\gamma, \gamma, R) - PD^2.\]

If one joins formula (10) and the statement (14), then one can obtain that

\[(15) \quad DC = \frac{Var(DR)}{PD \cdot (1 - PD)}\]

As we show later, formula (15) is the correct linkage for the default correlation and default probability for the case when the number of borrowers goes to infinity.

To sum up, we have shown that existing literature even from the first glance provides the asset correlation parameterization in (15). However, (BCBS, 2019) does not consider it. To prove that formula (15) is correct, let us proceed independently by researching the correlated binary random variates’ probability distribution and by studying its properties. As a robustness (sanity) check, we model several distributions with various marginal default (success) probabilities and various correlation parameters. Then we compare the suggested parameterization with the parameter estimates when the data generation process is known. As an alternative, we benchmark our results to that of (Wunderer, 2019).
We show the material deficiencies of (Wunderer, 2019), specifically overestimation of asset correlation \((R)\) and of capital requirements for more correlated and more risky borrowers.

3. THEORETICAL PART

Let us depart from the formula for the variance of r.v. As a start, consider the variance of two binary r.v., (Kelbert & Sukhov, 2010, p. 78) or (Van Der Geest, 2005, p. 145):

\[
\text{Var}(Y_1 + Y_2) = \text{Var}(Y_1) + \text{Var}(Y_2) + 2 \cdot \text{Cov}(Y_1; Y_2).
\]

The variance of a sum of \(n\) binary r.v. equals:

\[
\text{Var}(n \cdot Y) = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{Cov}(Y_i; Y_j) = n \cdot \text{Var}(Y_i) + n(n - 1) \text{Cov}(Y_i; Y_j)
\]

Where for a single binary r.v. \(Y_i\), the variance is mentioned in formula (11).

If \(r\) is a correlation coefficient between \(Y_i\) and \(Y_j\) and \(\text{Var}(Y_i) = \text{Var}(Y_j)\), then their covariance equals to:

\[
\text{Cov}(Y_i; Y_j) = r \cdot \sqrt{\text{Var}(Y_i)} \cdot \sqrt{\text{Var}(Y_j)} = r \cdot PD \cdot (1 - PD)
\]

Putting (11) and (18) into (17) gives us

\[
\text{Var}(n \cdot Y_i) = n \cdot PD \cdot (1 - PD) + n(n - 1) \cdot r \cdot PD \cdot (1 - PD)
\]

In case of independence we simply have \(\text{Var}(n \cdot Y_i) = n \cdot PD \cdot (1 - PD)\) (Kelbert & Sukhov, 2010, p. 83). (Witt, 2014, p. 4268) departs from the same standing point to derive the formula the distribution of correlated Bernoulli trials, or the absolute number of defaults. However, he does not do the next steps that we undertake. First, he does not look at the distribution of default rate. Second, he does not provide explicit parameterization for correlation function. As a consequence, he does not evaluate impact on capital requirements, as available in section 5 of the current paper.

Consider the default rate (DR) as in formula (12). For estimating credit risk at the portfolio level, we substitute PD with its sample estimate in (19), i.e. with the mean value of DR \((PD = \overline{DR})\). Then the DR variance is:

\[
\text{Var}(DR) = \frac{\text{Var}(n \cdot Y_i)}{n^2} = \frac{1}{n} \cdot \overline{DR} \cdot (1 - \overline{DR}) \cdot (1 + (n - 1) \cdot r)
\]

From here we may derive the correlation value floor, that was noted by (Preisser & Qaqish, 2014):
This means that when \( n \) goes to infinity, one cannot expect negative (default) correlation as a single parameter over a credit portfolio. However, even for a thousand borrowers a minor negative correlation is feasible. So the Basel IRB framework should better allow for a negative (default) correlation when \( n \) is finite (granular), ca. less than one thousand of observations.

Let us derive the (default) correlation value \( (r) \) from (20).\(^{12}\) Then we have

\[
(22) \quad r = \left( \frac{n}{n-1} \right) \left( \frac{\text{Var}(DR)}{\frac{\partial DR}{\partial (1-DR)}} \right) - \left( \frac{1}{n-1} \right)
\]

When one wishes to obtain asset correlation \( R \), it is sufficient to take the power of two for \( r \), i.e.

\[
(23) \quad (23) \ R = r^2.
\]

Jointly we suggest formulas (22), (23) to substitute formulas (4), (5). If we simplify it by taking the limit for \( n \) going to infinity, then the proxy for the default correlation is as follows in (24), or in (15) as earlier mentioned.

\[
(24) \quad r = \frac{\text{Var}(DR)}{\frac{\partial DR}{\partial (1-DR)}}
\]

As the reader may remember, here in formula (24) we obtain the result of formula (15) that is a combination of (BCBS, 2005) and (Wunderer, 2019). It interesting to mention the survey by (Wehrspohn, 2004) where he argues that PD contributes more to the losses amount than the correlation. As one may see from (24), there is no ground for such a statement as both PD and \( r \) multiplicatively contribute to default rate variance.

For the regulatory purposes, we suggest the following modification to the Basel IRB risk-weighting formula instead of formula (3) to additionally allow for the negative (default) correlation, i.e. to take \( r \) directly in the numerator instead of using the square root of its squared value.

\[
(25) \quad RW_i = N \left( \frac{N^{-1}(PD_i) + r \cdot N^{-1}(0.999)}{\sqrt{1-r^2}} \right)
\]

\(^{12}\) One may claim that the following derivation follows from (Gordy, 2000, pp. 146, eq. B.4). This is equally true as it may follow from the existing textbook son the probability theory. Therefore, the major shortcoming of (Gordy, 2000) is that he does not provide the ultimate explicit formula for the default correlation \( r \).
where \( r \) comes from formula (22) as formula (24) does not allow negative correlation with infinite number of observations.

There may be counterarguments against allowing for the negative default correlation in (25). For instance, one may recall the lower boundary for the correlation value from (21), i.e. for the large number of borrowers the correlation \( r \) is non-negative. We need to underline here that the presence of the negative default correlation is feasible in the case of low default portfolios (LDP), i.e. when the number of borrowers is limited. As an illustration, one may imagine an IRB segment of lending to one hundred large hedge funds which have different investment strategies. Then a situation with \( r \approx -1\% \) is feasible.

Let us proceed with the robustness check of the proposed parametrization for the IRB default correlation \( r \).

### 4. ROBUSTNESS CHECK: SIMULATED DATA

To obtain correlated binary r.v.’ probability density distributions, we use genetic algorithm described by (Kruppa, et al., 2018). There are alternative approaches for joint correlated binary r.v. distribution generation. Those include (Lunn & Davies, 1998), (Zaigraev & Kaniowski, 2013), (Preisser & Qaqish, 2014), (Witt, 2014), (Bakbergenuly, et al., 2016), (Chiu, et al., 2017). Though (Witt, 2014), (Chiu, et al., 2017), (Kruppa, et al., 2018) allow getting distributions of negatively correlated binary outcomes, (Kruppa, et al., 2018) algorithm is easier to implement out of the three. So we proceed with it.

(Kruppa, et al., 2018) algorithm has two steps. First, we generate a vector with required marginals, i.e. with the needed number of zeros (non-defaults) and ones (defaults) per columns. For simplicity we take ten variates (\( n = 10 \)) and the number of rows (observations) equals to 100. Second, one starts randomly perturbing (exchanging places) within any column so that marginals do not change, but the correlation changes. We introduce error as the square root of the sum of element-wise differences of two correlation matrices: the one under perturbation and the target one. The target correlation matrix has \( r \) values everywhere except the principal diagonal. For simplicity, we consider the matrix differences only for its half, i.e. for below the diagonal elements only. The optimization procedure is iterative and may not always converge, as (Kruppa, et al., 2018) note. That is why we set limits on the acceptable error.
(10%) and the maximum number of iterations (10 thousand). We report both in Annex 2 for the found solution (see columns 7 and 6, respectively).

For simulation purpose, we choose four values of marginal PD (5, 10, 25, 50%) and six values of correlation r (-10, -5, 0, +5, +10, +20). For each of combinations of PD and r we obtain a DR distribution. For each parameter set we generate new initial DR distribution. That is why there may be statistically insignificant differences in the mean DR values given the same PD input in Annex 2 (compare columns 3 and 8).

As a side-thought, as correlation does not affect the marginals and the mean DR, we had the right to substitute marginal PDs with the portfolio-level DR in formula (20).

One may notice from Figure 2, that for significantly positive default correlations the DR probability density distribution becomes wider and bimodal. (Witt, 2014, p. 4273) is the first to show bimodality. Then (Li, et al., 2015) independently notices the same fact when they observe that the frequency of high and low risk categories (ratings) is higher, than that of mid-risk categories.

When the default correlation is negative, the DR distribution becomes narrow and goes to a degenerate state to concentrate around marginal PD value. Illustration from Figure 2 supports the statements by (Preisser & Qaqish, 2014). They say that the variance of correlated r.v. distributions decreases for low (negative) correlation and increases – for the high (positive) one.

![Figure 2](image_url)

(a) PD = 20%

---

13 Witt, 2014) does neither demonstrate this, nor consider in his examples, though he accepts that negative correlation of binary r.v. (Bernoulli trials) is feasible. In contrast to (Witt, 2014), (Van Der Geest, 2005) deals with negative correlation for illustrative purposes.
Figure 2. Illustration of default rate distributions for various PD and correlations of binary random variates.

Note: # 1s – the number of defaults (ones).

Compare Figure 2 to the statement by (Ozdemir & Miu, 2009, pp. 70-71). They claim that through-the-cycle (TTC) rating philosophy implies wider DR distribution, whereas the point-in-time (PIT) rating philosophy leads to a narrow DR distribution. This data simulation exercise vividly illustrates that (Ozdemir & Miu, 2009) claim is correct if and only if they associate TTC rating philosophy with higher default correlation and PIT one – with lower one.

Figure 3 depicts the dependence of asset correlation values against the marginal PD ones. Several lines correspond to different values of DR variance. We advise the reader to compare Figure 3 to Figure 1. As for the suggested approach illustration in Figure 3, we present values from formula (23), i.e. the squared values ($r^2$) for visual comparability with Figure 1.
One may see that the regulatory R formula (4), (5) in Figure 1 generally reflects the concept of declining asset correlation value for higher PD values when PD is less than 50%. However, correlated binary r.v. do not demonstrate the patterns used in regulatory regulation. First, regulatory values stay flat for PD in excess of 50% at Figure 1 whereas in fact they start upward sloping (see Figure 3). Second, there is a single dependence of R, PD, and the number n of borrowers within the credit pool. It varies only for different values of DR variance at Figure 3. It cannot vary because of some fourth determinant like credit product type as in Figure 1. We may conclude that variation of the existing capital requirements by credit product types (remember reference to Annex 3) may have an incentivizing intent from regulator to banks, but it cannot definitely be an implication from the statistical properties of the DR probability density distribution.

As you can see from Figure 4, the suggested approach in formula (20) mostly ideally predicts the DR variance values and quite closely fits the input correlation r value. The minor discrepancies result from the fact that optimized (iterated) correlation matrix for the generated joint distribution of binary variables does not always converge to the target correlation matrix.\textsuperscript{14}

\textsuperscript{14} One may note high value of error, in excess of its limits when the algorithm reached the maximum number of iterations, see column 7 in Annex 2.
As for (Wunderer, 2019) approach, it suffers from the three major deficiencies observed during the above simulation study (see Annex 2, columns 9-11 for DR variance comparison and columns 14-16 for correlation r comparison):

1. It overestimates variance for the low (including negative) correlation and/or low PD. From risk-management perspective that may be fine as the approach produces the more conservative estimate of risk (proxied by DR variance, for instance) than it really is;

2. It underestimates variance for the high (positive) correlation and/or high PD. This is not acceptable for risk-management as the risk is underestimated compared to its true value.

3. As a consequence of the two above points, correlation r value is overestimated for high PD and/or high correlation. Thus the existing Basel IRB framework - all else being equal - incentivizes to lend more to low-risky borrowers (low PD) and in more diversified sectors (low correlation r) making it less appealing to emerging economies overall and for SME lending in particular.

The mentioned deficiencies imply that we do not recommend using (Wunderer, 2019) approach as a proxy estimate for asset correlation (R) in the Basel IRB framework.
5. APPLICATION TO REAL-WORLD: REGULATORY IMPACT

We have chosen publicly available data from the international rating agencies of (Moody’s, 2018, p. 27) and (S&P Global Ratings, 2019, p. 3) to apply the suggested approach and compare the resulting capital requirements to the existing ones. We specifically estimate mean DR and DR variance on two subsamples:

- till 2003 inclusive (as if it formed basis for (BCBS, 2004) AC functions’ calibration); and
- till 2018 (as if we updated the calibration using recent data).

As the dominant part of the international credit rating agencies’ clients’ pool consists of the non-retail (corporate) exposures, we assign regulatory corporate exposure asset class when evaluating capital requirements for them (see row 1 in Annex 1). Annex 4 provides tabulated results for the time period covered by the data and the default rate mean and variance.

![Graph showing comparison of suggested and existing capital requirements with respect to the coefficient of variation (CV) for the default rate.](image)

**Figure 5.** Comparison of suggested and existing capital requirements with respect to the coefficient of variation (CV) for the default rate.

Note: Horizontal axis has the coefficient of variance (CV). CV is the ratio of variance to the mean value. (Miller, 1991) discusses its properties.

First, we find that the higher the coefficient of variance (CV) is, the closer the capital requirement from the suggested approach (cf. (22) and (25)) is to the existing regulatory approach (cf.
(3), (4), (5)). However, even in the extreme case out of the presented ones, the existing capital requirements are still ca. five times higher than that originating from the suggested approach.\textsuperscript{15}

Second, the existing capital requirements do not adequately captured the default data evolution from 2003 onwards. From one side, for the speculative grades (SG) we observe decrease in mean DR and DR variance disregarding the fact of the world financial crisis of 2007-09 presence in the dataset. Existing capital requirements do not fall as sharply as they should (compare rows 22 to 29 in Annex 4). We may accept this from risk-management point of view as capital does not rapidly decrease either. However, from another side, for the investment graded (IG) borrowers we observe increase in mean DR and DR variance (e.g. for the Moody’s data). The existing capital requirements in such a situation do not rise with the same pace (+20.5% from 2003 to 2018) as they are expected according to the suggested approach (+28.2%). In contrast with the situation of speculative grades, this one is not acceptable from the risk-management perspective as capital requirements’ growth rate lags behind the growth rate of credit risk. Nevertheless, this may seem also acceptable from risk-management perspective as the existing approach overestimate the credit risk amount in absolute terms ca. 50 times for IG grades (see row 16 in Annex 4).

Third, we wanted to verify whether the situation of excessive existing capital requirements takes place for any combination of credit risk parameters (mean DR and DR variance). It turned out that this is not true always. For this purpose we ran illustrative calculations of capital requirements for different credit product types for two values of mean DR (5% and 10%) and two values of DR variance (1% and 2%). Annex 3 presents the capital requirements in rows: No. 7 (for the existing regulatory approach in formulas 3-5) and No. 10 (for the suggested parameterization in formulas 22, 25). Row 11 of Annex 3 reports the ratio of the suggested capital requirements to the existing one. When such a ratio exceeds 100%, we have the situation of credit risk underestimation according to existing capital requirements. So far we may see that when the coefficient of variance (CV), or the ratio of DR variance to mean DR value, is above 40% inclusively, the existing regulatory approach underestimate credit risk (mostly twice). The areas of most drastic underestimation are plastic cards (QRR) and other retail loans (see values of 239% and 209% in row 11 of Annex 3a, respectively).

\textsuperscript{15} Maximum value on the vertical axis at Figure 5 is 20%, i.e. suggested capital requirements (RW) form one fifth of the existing regulatory ones.
6. CONCLUSION

Basel III and Basel Framework have inherited the IRB approach concept from Basel II. However, the Basel committee does not change the asset correlation treatment keeping parameters outdated for 15 years. Let us list the key paper contributions to the literature. Some of them may seem intuitive, simple and straightforward. However, they were not explicitly shown or claimed before.

First, though previous researchers were close to finding a solution, we are the first ones to explicitly show a single correct parameterization for the default correlation, DC, \((r)\) function that is readily available for use by the regulators.

\[
    r = \left( \frac{n}{n-1} \right) \left( \frac{\text{Var}(DR)}{DR \cdot (1 - DR)} \right) - \left( \frac{1}{n-1} \right)
\]

We repeat the above formula from (22) for convenience. It naturally originates from the statistical properties of the correlated Bernoulli trials distributions, though no one has not previously shown it explicitly in the form needed for the regulatory purposes. The asset correlation \((R)\) would then equal to the default rate correlation squared value \((R = r^2)\).

Second, consequently, we demonstrate that regulatory formula for asset correlation does not rise for default probability in excess of 50% (compare Figure 1 and Figure 3).

Third, we show that the asset correlation estimates do not differ depending on the credit product type given the same mean default rate (DR) and DR variance. As an implication, correlation estimate cannot differ depending on the data source (asset values or default time series are used). In brief, rationale for the proof is that mathematical correlation is a single feature (characteristic) of the default rate probability distribution. We assume that inability to obtain such a parameterization came from one of the two origins. First, there may be a methodologically wrong supposition that joint probability for binary random variates (defaults) equals to the value of the bivariate standard normal distribution for the conditioned floating variates (asset values). Second, there may be a regulatory intent to provide differentiated incentive.

Fourth, we argue that Basel IRB framework has to consider the correlation parameter directly, not the square root from a squared value, i.e. \(r\) instead of \(\sqrt{R} = \sqrt{r^2}\), in the numerator of the expression.
within the standard normal distribution function for $RW_i$. This allows considering negative default correlation stemming from the above expression for $r$ (we repeat formula (25) from the above):

$$RW_i = N\left(\frac{N^{-1}(PD_i) + r \cdot N^{-1}(0.999)}{\sqrt{1 - r^2}}\right)$$

Fifth, we demonstrate that the approach to calibrate default rate variance using bivariate normal distribution cdf of (Gordy & Heitfield, 2010, pp. 58, eq. 20) and (Wunderer, 2019, pp. 4, eq. 2.4) has material shortcomings and should not be used in practice.

Sixth, in addition to already known fact of the default rate bimodality in the presence of positive correlation (for instance, see (Witt, 2014, p. 4273)), we show the implications of the negative correlation, i.e. the narrowing of default rate distribution. As a result, we show that (Ozdemir & Miu, 2009, pp. 70-71) are wrong when suggesting the width of default rate distribution to vary due to the choice of the rating philosophy where in fact the default correlation is the driver for the distribution width.

Seventh, we find default rate coefficient of variance, CV (ratio of variance to mean value) is the determinant to judge whether current capital requirements sufficiently cover credit risk. When CV is around 30% the credit risk is nearly closely captured. From one side, when CV is lower than 30%, the existing capital requirements are in excess of actual credit risk amount. From another side, when CV is higher than 30%, the existing capital requirements significantly underestimate it. Such policy implications were not shown before.
### Annex 1. Basel II (III) IRB AC function parameters

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\( \Phi_2 = N(\gamma, \gamma, R) \) - joint bivariate standard normal distribution function (see Annex 5);
iter., # - number of iterations used to obtain the output (limit of 10 thousand iterations was set);
error – attained square root of sum of differences for two correlation matrixes: target and actual;
AVar – analytically derived (predicted) variance;
indep – as if the correlation of binary variates equaled to nil;
column 14 equals to column 4; it is replicated for visible convenience to compare to columns 15 and 16.
Suggested – approach from the current paper;
Wunderer – approach proposed by (Wunderer, 2019);

### Notations used in Annex 2

- CV – coefficient of variation, ratio of DR variance to its mean value.
- K – capital requirement to cover credit risk, equals to risk-weight (RW)

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Annex 3a. Existing regulatory vs. suggested approach (illustrative case, mean DR=5%).

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#### Suggested approach (K1)

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#### Comparison: Suggested vs. Existing Capital Requirements

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Annex 3b. Existing regulatory vs suggested approach (illustrative case, mean DR=10%).

### Data Descriptives

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<th>Mortgage</th>
<th>Other retail</th>
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### Suggested approach (K1)

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### Comparison: Suggested vs. Existing Capital Requirements

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Annex 4. Existing regulatory vs suggested approach (empirical data).

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<th>S&amp;P</th>
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<td>48.70%</td>
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<td>14</td>
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<td>16</td>
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<td>R1 = r1 ^ 2</td>
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<td>0.049%</td>
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### Comparison: Suggested vs. Existing Capital Requirements

| #  |  | Moody’s | S&P | Moody’s | S&P | Moody’s | S&P |
|----|  |---------|-----|---------|-----|---------|-----|
| 16 | K1/K0 | 2.3%   | 2.5%| 3.0%   | 2.9%| 7.5%   | 7.4%|
| 17 | CV    | 0.1%   | 0.2%| 0.1%   | 0.2%| 0.6%   | 0.7%| 0.7%   | 0.7%| 1.3%   | 1.5%| 1.6%   | 1.8%|
Annex 5. Values of bivariate joint standard normal cumulative density function ($\Phi_2$ or $N(\gamma, \gamma, R)$ where $\gamma = N^{-1}(PD)$, $R = Corr(Z_i; Z_j)$), in percentage points (pp) out of 100.

<table>
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<th>-100</th>
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<th>-50</th>
<th>-25</th>
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<th>+10</th>
<th>+15</th>
<th>+20</th>
<th>+25</th>
<th>+50</th>
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