Volatility spillovers with spatial effects on the oil and gas market

Karatetskaya Efrosiniya, Lakshina Valeriya

Abstract

The article is devoted to the estimation of volatility spillovers occurred on the oil and gas market taking into account cross-sectional dependence. The latter is implemented via spatial specifications of the BEKK multivariate volatility model. We also use DCC, GO-GARCH and ADCC models as a benchmark.

1 Introduction

In the finance literature the volatility spillovers are an important part of understanding market behavior, price changes and risk measurement. They exist widely through all of financial markets and are characterized by shocks that occur at one market and cause changes in asset prices in other markets. It leads to the situation when current value of a variable depends on past and/or current values of other variables, not only on its past values (Schmidt, 2005). In this case investigation of the cross-market linkages, price co-movements has become a crucial issue for modeling volatility in applications to the tasks set by investors, financial organizations and governments in trading, hedging and financial regulation.

Volatility spillover effects are associated with the spread of market disturbances between countries and sectors as a result of prices, shares, exchange rates or capital flow co-movements and other financial linkages.
among market economies (Dornbusch et al., 2000). For example, volatilities of exchange rates of different currencies can significantly influence the prices of commodities and markets behavior in general (Akman and Bozkurt, 2016).

Systematic modeling of financial volatility has started with the autoregressive conditional heteroscedasticity (ARCH) model in the seminal paper of Engle (1982). To explain correlation transmission and spillover effects ARCH model was generalized to multivariate case in Engle and Kroner (1995). Multivariate GARCH models focus on time-varying conditional variances as well as co-variances, what allows to make better representation of the volatility process.

With a development of econometrics a number of parametric models have been derived to describe the asymmetry of financial volatilities. These models include copulas (Jaworski and Pitera, 2014), DECO-FIEGARCH (Mensi et al., 2017), DCC (Kocaarslan et al., 2017), wavelets (Liu, An, Huang, et al., 2017), complex networks (Liu, An, Li, et al., 2017). Using such specifications some papers document significant volatility spillover between oil and stock markets. Ewing et al. (2002) found the evidence of volatility persistence in both oil and gas markets. They show that volatility in the natural gas sector is directly affected by events in this sector and indirectly by events originating in the oil sector.

El Hedi Arouri et al. (2011), Olson et al. (2014), Serletis and Xu (2016) use BEKK-GARCH model to estimate cross-market volatility spillovers. Their results point to the existence of widespread direct spillover of volatility between oil and stock sector returns whatever the region considered. Moreover, the volatility spillovers across markets are increasing when “the zero lower bound” (situation when the short-run nominal interest rate is at or near zero; liquidity trap) occurs than when it is not, suggesting that unconventional monetary policy at “the zero lower bound” has strengthened the linkages between the crude oil and financial markets.

Aiming to investigating the evolution of mean and volatility spillovers between oil and stock markets Liu, An, Huang, et al. (2017) employ WTI
crude oil prices, the S&P 500 (USA) index and the MICEX index (Russia) with a wavelet-based GARCH–BEKK model to examine the spillover features in frequency dimension. They find the evidence of information transmission between the crude oil market and US stock market that is gradually weakened and mainly maintained in short-term scale. The contacts between two markets gradually disappear at the long-term scales.

However, despite the fact that such models are quite clear and convenient in interpreting the spillover effects, the problem of non-linear growth of the number of estimated parameters (so called “curse of dimensionality”) arises. To overcome this problem Caporin and Paruolo (2015) propose an intermediate form for multivariate GARCH models with restrictions based on spatial dependencies among the assets to build conditional covariances. Spatial BEKK-GARCH is used by Chen and Tian (2017) to explore unidirectional and bidirectional spatial volatility spillover effects among the stock markets based on symbolic transfer entropy. Anatolyev and Khrapov (2016) investigate forecast performance of different types of further restrictions on coefficient matrices in spatial BEKK-GARCH and compare spatial and standard specification of BEKK-GARCH.

The spatial specification of the multivariate model of generalized autoregressive conditional heteroscedasticity (spatial BEKK) allows to take into account both temporal and spatial effects in the dynamics of volatility (Caporin and Paruolo, 2015). Such effects are modeled using weight matrix that is given exogenously and can be defined either as a binary matrix or as a function of the economic distances (Borovkova (n.d.)). There are some evidence that spatial models are good in optimal hedging ratio computation, forecasting performance, modeling the effects of contamination and volatility spillovers in comparison with other multivariate models (Gu et al., 2017, Anatolyev and Khrapov, 2016, Jaworski and Pitera, 2014, Chen and Tian, 2017).

A number of authors have considered the effects of volatility spillovers between oil and stock markets of different countries: Caporale and Spagnolo (2011) examine linkages between the stock markets of the Czech Re-
public, Hungary, Poland, the UK and Russia; Arouri et al. (2012) — of stock markets in Europe; Lin et al. (2014) — of Ghanaian stock market; Liu, An, Li, et al. (2017) — G20 countries. Thus there is a large number of works related to the effects of volatility spillovers between oil and stock markets of different countries, but not inside one country or industry.

In contrast to previous studies, in this paper we estimate volatility spillovers on the oil and gas sector of stock market taking into account the cross-sectional relationships between companies within the sector. As benchmark we use such multivariate volatility models as the generalized orthogonal GARCH (GO-GARCH), dynamic (DCC) and asymmetric dynamic (ADCC) conditional correlations (for more details on volatility models, see Bauwens et al., 2006).

The article is organized as follows. Section 2 gives a short description of multivariate GARCH models used in the article. Section 3 provides information about the dataset. Section 4 reports estimation results of different volatility model specifications and Section 5 concludes.

2 Multivariate GARCH models

Let $x_t, x_t = (x_{1t}, x_{2t}, \ldots, x_{nt})'$ be a portfolio consisted of $n$ assets at time moment $t, t = 1, \ldots, T$. $x_t$ is represented as a sum of its mathematical expectation $\mathbb{E}(x_t | \mathcal{F}_t)$, conditional on all available at $t - 1$ information, and innovations $y_t, (1)$.

$$x_t = \mathbb{E}(x_t | \mathcal{F}_{t-1}) + u_t, \ t = 1, \ldots, T, \ x_t - (n \times 1) \text{-vector}, \quad (1)$$

Innovations $u_t$ are represented by the product of volatility matrix $H_t$ and idiosyncratic noise term $\varepsilon_t$, which is distributed according to some distribution $f$ with zero mean and additional parameters $\theta$ (2).

$$u_t = H_t^{1/2} \varepsilon_t, \varepsilon_t \sim f(0, \theta), \quad (2)$$
Volatility models differ in the specification of dynamic variance-covariance matrix $H_t$.

### 2.1 DCC-GARCH

The DCC model is defined as follows:

$$ H_t = D_t R_t D_t, D_t = diag(\sqrt{h_t}) $$  

$$ h_{it} = \alpha_{i0} + \sum_{q=1}^{Q_i} \alpha_{iq} y_{i,t-q}^2 + \sum_{q=1}^{P_i} \beta_{ip} h_{i,t-p} $$  

$$ R_t = (diagQ_t)^{-\frac{1}{2}}Q_t(diagQ_t)^{-\frac{1}{2}} $$

$$ Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha u_{t-1}'u_{t-1} + \beta Q_{t-1} $$

this model meets the requirement of stationarity as long as $\alpha + \beta < 1$, $\alpha, \beta > 0$

where $u_t : n \times 1$ vector of innovations  
$H_t : n \times n$ matrix of conditional variances  
$H_t^{-1} : n \times n$ matrix of a Cholesky factorization of $H_t$  
$D_t : n \times n$ diagonal matrix of conditional standard deviations of $u_t$  
$\bar{Q}_t : n \times n$ unconditional variance matrix of $u_t$  
$R_t : n \times n$ conditional correlation matrix with unities on the main diagonal

### 2.2 ADCC-GARCH

DCC model was extended by Cappiello et al. (2006) to the case where asymmetries in the conditional correlations between series can be estimated.

To achieve that matrix $Q_t$ is defined as follows:
\[ Q_t = (1 - \alpha - \beta)\bar{Q} - \gamma\bar{N} + \alpha u_{t-1}u'_{t-1} + \gamma \eta_{t-1}\eta'_{t-1} + \beta Q_{t-1} \quad (7) \]

where \( \eta_{t-1} = I[u_t < 0] \odot u_t; \)

\( I(\cdot) \) is an indicator function which takes on value 1 if the argument is true and 0 otherwise, \( \odot \) is the Hadamard product;

\[ \bar{N} = [\eta_{t-1}\eta'_{t-1}] . \]

Positive definiteness of \( Q_t \) is ensured by condition: \( \alpha + \beta + \lambda\gamma < 1 \), where \( \lambda \) is maximum eigenvalue of \( [\bar{Q}_{t-1}^{-\frac{1}{2}}\bar{N}\bar{Q}_{t-1}^{-\frac{1}{2}}] \).

In literature DCC model is usually used for small-scale applications (4 or 5), since it becomes cumbersome to analyze hundreds of assets, though that can be important for financial applications (Jaworski and Pitera, 2014; Guidolin, 2016).

Apart from the DCC model that assume dynamic correlations, multivariate BEKK-GARCH model, that have been proposed by Engle and Kroner (1995), gained high popularity, but also is subject to the dimension curse.

The solution to the dimensionality problem of multivariate GARCH models was proposed in Caporin and Paruolo (2015). The authors apply spatial matrix for the parameter matrices in BEKK volatility equation (8), decreasing the number of parameters to \( O(n) \). A detailed description of the properties of spatial matrices can be find in Caporin and Paruolo (2015).

### 2.3 Spatial BEKK-GARCH

The spatial BEKK model has following structure:

\[ H_t = C'C + A'y_t y_t'A + B'H_{t-1}B \quad (8) \]

where \( n \times n \) coefficient matrices \( A, B, C, D \) are defined as (9) and are equal to AR(1) component of spatial autoregressive model LeSage and
\[ A = \text{diag}(a_0) + \text{diag}(a_1)W, \quad (9) \]

\[ B = \text{diag}(b_0) + \text{diag}(b_1)W, \quad (10) \]

where \( a_0, b_0, a_1, b_1 \) are \( n \times 1 \) vectors of parameters, \( W \) — weight matrix.

The constant \( C'C \) in (8) is borrowed from the spatial error model (see LeSage and Pace, 2009).

\[ C'C = D^{-1} \text{diag}(d_0)(D')^{-1}, \quad (11) \]

where \( D \) is also a spatial matrix with \( d_0 \) equal to identity matrix. The main idea of such parametrization is to define volatility spillovers across assets that belong to the same group (see below).

In general \( A \) shows news impact or innovations effects in a volatility matrix \( H_t \). At the same time matrix \( B \) contains covariance feedback effects (Anatolyev and Khrapov, 2016). We use scalar homogeneous specification, where matrices of parameters are modeled as follows:

\[ a_0 = \alpha_0 1_n, \quad a_1 = \alpha_1 1_n \]

\[ b_0 = \beta_0 1_n, \quad b_1 = \beta_1 1_n \]

\[ d_0 = \text{free}, \quad d_1 = \delta_1 1_n \]

Such modification brings the spatial BEKK closer to the diagonal BEKK model defining equal impact of spillovers for each grouping across all assets, but reduce the number of parameters.

The elements of the weight matrix reflect the force of potential interactions between the assets. When determining the elements of a matrix it is natural to use principle that nearby neighbors exert the greatest influence. Fernandez (2007) aims at deepening researchers’ understanding
of economic distance using spatial methodology with high level of efficiency. Arnold et al. (2013) introduce the approach to model three different types of spatial dependence in stock returns: a general dependence, dependence within industrial branches and based on geographic locations. The most commonly used methods for weight calculation are administrative-territorial specification, the method of moving windows, fixed and adaptive kernels (Chasco Yrigoyen et al., 2007; Münstix et al., 2014).

In this paper spatial weight matrices are modelled in two different ways. First type of matrices is computing as follows:

$$w_{ij} = \begin{cases} 1, & \text{if } j \text{ is neighbour for } i; \\ 0, & \text{else.} \end{cases}$$

(12)

Second type of matrices is based on computation of weights with bi-square kernel (see details in Balash et al., 2011).

$$w_{ij} = \begin{cases} \left(1 - \left(\frac{d_{ij}}{b}\right)^2\right)^2, & \text{if } j \text{ is one of } m \text{ neighbours for } i; \\ 0, & \text{else.} \end{cases}$$

(13)

In this matrix element $d_{ij}$ — is the distance between assets that can be calculated as:

$$d_{ij} = \left(\frac{1}{n} \sum_{1}^{n} \left(p_i^{(n)} - p_j^{(n)}\right)^2\right)^{\frac{1}{2}}$$

(14)

where $n$ — number of indicators that used for distance defining.

The optimal number of $m$-nearest neighbors is found by comparing the quality of models for different values of $m$. As the distance increases, the relationship between assets is declining. The influence of neighbors on the $i$th-element defines with a respect of the $b$ — distance to the farthest neighbor.

Each element in $i$-row of matrix $W := (w_{ij})$ should show the share of
influence of j-neighbor on i. To achieve this effect, the rows of matrix $W$ should be normalized to unit:

$$w_{ij} = \begin{cases} 
w_{ij} / \sum_{j=1}^{n} w_{ij}, & \text{if } \sum_{j=1}^{n} w_{ij} > 0; \\
0, & \text{else.}
\end{cases}$$

(15)

2.4 GO-GARCH

The GO-GARCH model is a special case of the BEKK model (Weide, 2002). In this model the volatility matrix is parametrized as follows:

$$H_t = XV_tX',$$  \hspace{1cm} (16)

where $V_t$ — diagonal $n \times n$ matrix with $v_t = c + A(y \odot y) + Bv_{t-1}$ on the main diagonal, $X$ — $n \times n$ matrix based in singular value decomposition (see Weide, 2002 for detail), $A, B$ — $n \times n$ diagonal matrices of parameters with a restriction $a_{ii,t} + b_{ii,t} < 1$.

3 Data description

Our sample covers the data of the 67 companies from the oil and gas sector in 13 countries. The countries under consideration are classified as upper-middle income level and higher income according to the World bank classification. The dates range is from April 27, 2015 until January 18, 2018 apart from the public holidays, therefore the full sample contains 634 observations. Financial indicators, namely company market capitalization, gross profit and total assets, are taken on 2016 year. Countries of Headquarters include Argentina, China, Colombia, Gabon, Kuwait, Nigeria, Qatar, Russia, Saudi Arabia, South Africa, Thailand, Turkey and Peru.

List of TRBC Activity Codes is presented in table 5.
All the data are obtained from the Thomson Reuters Eikon$^1$.

We construct two weight matrices. The first one is based on the criteria of belonging to the country and activity (by TRBC); the second one — on the economic distance calculated as a difference in market capitalization, total assets and profit.

The calculations were carried out on Amazon Elastic Compute Cloud service with RStudio Amazon Machine Image (AMI) installed.$^2$ The AMI contains R version 3.3.1 running on Ubuntu 16.04 LTS and R packages “rmgarch” for GARCH estimation and “optimx” to log-likelihood maximization (Ghalanos, 2015; Nash and Varadhan, 2011; Nash, 2014).

4 Empirical results

We estimate parameters for multivariate GARCH models and compare them to see what model demonstrates the highest explaining power in modelling volatility spillovers.

The models under consideration are estimated by means of maximum likelihood method. The log-likelihood function $LL$ defined in (17).

$$LL = - \frac{1}{2} \sum_{t=1}^{T} \left( \log(\text{det}H_t) + y'_t H_t^{-1} y_t \right)$$

This method fits well on time series for both linear and non-linear models (Brooks, 2008). Estimations obtained by this method are consistent and asymptotically normal but require a modified calculation of standard errors (Greene, 2003).

For general fitting information following criteria are used ($m$ — number of parameters in model): Akaike information criterion:

$$\text{AIC} = \frac{-LL}{N} + \frac{2m}{N}$$

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$^1$https://eikon.thomsonreuters.com/
$^2$https://aws.amazon.com/
Bayesian information criterion:

\[ \text{BIC} = \frac{-LL}{N} + \frac{2m \log_e N}{N} \]  \hspace{1cm} (19)

Hannan–Quinn information criterion:

\[ \text{HQIC} = \frac{-LL}{N} + \frac{2m \log_e (\log_e N)}{N} \]  \hspace{1cm} (20)

Shibata information criterion:

\[ \text{SIC} = \frac{-LL}{N} + \log_e \left( \frac{N + 2m}{N} \right) \]  \hspace{1cm} (21)

Tables 1, 2, 3, 4, present the results of comparison of three volatility models based on mentioned criteria. Apparently, the spatial structure in the BEKK-GARCH volatility equation improve explaining power of the model according to the chosen criteria. In our sample for all in four criteria scalar spatial BEKK-GARCH has the highest values of information criteria for both types of weight matrices.

<table>
<thead>
<tr>
<th>Model</th>
<th>m</th>
<th>Log-likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEKK-GARCH (activity and country)</td>
<td>72</td>
<td>-42114</td>
</tr>
<tr>
<td>DCC-GARCH</td>
<td>2549</td>
<td>-59707</td>
</tr>
<tr>
<td>GO-GARCH</td>
<td>4690</td>
<td>-58406</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>BIC</th>
<th>HQIC</th>
<th>SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEKK-GARCH (activity and country)</td>
<td>199</td>
<td>201</td>
<td>200</td>
<td>199</td>
</tr>
<tr>
<td>DCC-GARCH</td>
<td>294</td>
<td>319</td>
<td>304</td>
<td>285</td>
</tr>
<tr>
<td>GO-GARCH</td>
<td>298</td>
<td>410</td>
<td>316</td>
<td>279</td>
</tr>
</tbody>
</table>
Table 3: Number of parameters and Log-likelihood (full sample)

<table>
<thead>
<tr>
<th>Model</th>
<th>m</th>
<th>Log-likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEKK-GARCH (economics distance)</td>
<td>72</td>
<td>-61079</td>
</tr>
<tr>
<td>DCC-GARCH</td>
<td>2549</td>
<td>-88020</td>
</tr>
<tr>
<td>GO-GARCH</td>
<td>4690</td>
<td>-87094</td>
</tr>
</tbody>
</table>

Table 4: Information criteria (full-sample)

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>BIC</th>
<th>HQIC</th>
<th>SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEKK-GARCH (economics distance)</td>
<td>193</td>
<td>194</td>
<td>193</td>
<td>193</td>
</tr>
<tr>
<td>DCC-GARCH</td>
<td>286</td>
<td>304</td>
<td>293</td>
<td>280</td>
</tr>
<tr>
<td>GO-GARCH</td>
<td>290</td>
<td>370</td>
<td>302</td>
<td>278</td>
</tr>
</tbody>
</table>

Results point to the existence of volatility spillover among assets grouping by TRBC activity (Figure 1). Large outliers are concentrating near the period of high oil Brent price volatility (for example, in the Q3 of 2015 and beginning of 2016) in all groups. Thus, the interdependence between volatility of companies from different activities increases during the period of oil price turbulence. The dynamics of volatility covariances between one asset and the rest of the various groups is similar, the outliers also occur at the same period of time through all of groups.

5 Summary and concluding remarks

Finally, estimation of spatial BEKK gave some results providing evidence that companies in oil and gas sector are integrated. We outline the presence of volatility spillover in oil and gas market and estimate it for groups of assets made by different criteria.

Future research in this field can be focused on the forecasting of the volatility and using results in building financial strategies, for example, hedging. Also it can be useful to decompose daily price changes (returns and volatility) into daytime (open to close) and overnight (close to open)
returns. In the case of investigation different countries, the daytime segment in one market is a subset of the overnight segment of the other market. Correction of such discrepancy is essential for clean information transmission. Furthermore, the effects can be separated in the long run and the short run period.
References


## Appendix

### Table 5: TRBC Activity Codes

<table>
<thead>
<tr>
<th>Code</th>
<th>Activity</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>5010202010</td>
<td>Oil &amp; Gas Exploration and Production (NEC)</td>
<td>Activity1</td>
</tr>
<tr>
<td>5010201010</td>
<td>Integrated Oil &amp; Gas</td>
<td>Activity2</td>
</tr>
<tr>
<td>5010202013</td>
<td>Natural Gas Exploration &amp; Production - Onshore</td>
<td>Activity3</td>
</tr>
<tr>
<td>5010203010</td>
<td>Oil &amp; Gas Refining and Marketing (NEC)</td>
<td>Activity4</td>
</tr>
<tr>
<td>5010203011</td>
<td>Petroleum Refining</td>
<td>Activity5</td>
</tr>
<tr>
<td>5010203012</td>
<td>Gasoline Stations</td>
<td>Activity6</td>
</tr>
<tr>
<td>5010202011</td>
<td>Oil Exploration &amp; Production - Onshore</td>
<td>Activity7</td>
</tr>
<tr>
<td>5010202015</td>
<td>Unconventional Oil &amp; Gas Production</td>
<td>Activity8</td>
</tr>
</tbody>
</table>

Source: Thomson Reuters.
Figure 1: Volatility covariances