Monopolistic Competition under Additive Separable Utilities and Demand Uncertainty

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1 Introduction

Utilities with constant elasticity of substitution between products are widely used in modern research to describe monopolistic competition. Within this description, the size of the economy does not affect markups, prices, and outputs, contradicting empirical evidence. Therefore Zhelobodko et al. (2012) introduced an analytical framework to deal with additively separable preferences given in an unspecified functional form. They consider a two-sector economy: firms compete monopolistically in the hi-tech sector and face perfect competition in the traditional sector. In the model, the price can either decrease or increase in response to an expansion of the economy, depending on the elasticity of substitution between goods. The analytical toolkit developed by Zhelobodko et al. (2012) is applicable to problems of taxes (Di Comite et al., 2013), income dispersion (Osharin et al., 2014), and export pricing (Kichko et al., 2014).

In the present paper we enlarge the toolkit introduced by Zhelobodko et al. (2012) to describe equilibrium prices, outputs, and consumers' welfare under uncertainty. Modeling the monopolistic competition after Dixit and Stiglitz (1977) and Krugman (1991), one considers each firm so small with respect to the market that it alone is not able to affect the price index. We posit that the role of specific firms is yet weaker under uncertainty. They not only fail to affect the price index but also know this index just up to some random variable. The corresponding optimization problem is formulated here under general equilibrium modeling.

Addressing effects of uncertainty, we model a two-sector economy. Let a representative consumer choose between hi-tech and traditional goods with a Cobb–Douglas utility function. Introducing uncertainty we assume that firms treat the exponents of this utility as random variables. As a result, firms decide upon their outputs under uncertain demand. We consider a two-stage optimization. Initially, firms choose in advance the amount of goods to produce in order to maximize the expected profit. Later, when the random variables are observed, they adjust the prices. This scheme of decision making is in agreement with the hypothesis stating that a sequence of frequent shocks affects economic activity. A simpler example of this two-stage decision deals with seasonal goods.

We formulate a general equilibrium model and find when it is well posed. In the equilibrium, firms expectedly clear the goods market when adjusting prices. The influence of uncertainty on the equilibrium is linked to the response of the *weighted* elasticity $\mathfrak{S}$ of substitution between hi-tech goods to a change in preferences for them. This weighted $\mathfrak{S}$ is defined as a weighted average of individuals' elasticities of substitution and therefore assigned to the whole society. If $\mathfrak{S}$ and the share z of the income spent for hi-tech goods change co-directionally then uncertainty shifts the equilibrium outputs downwards and moves the number of firms and real prices upwards. If $\mathfrak{S}$ and z go in the opposite directions then uncertainty shifts the equilibrium outputs upwards and moves the number of firms and real prices downwards. The influence of uncertainty on consumer welfare is ambiguous.

2 Model

2.1 Economy

We consider a two-sector economy. Employees' skills required with each sector are supposed to be not inter-changeable. In other words, workers are mobile inside sectors but immobile between them. The number of workers endowed by required skills are pre-defined for both sectors. Both types of labor are homogeneous. These assumptions lead to sector specific wages.

Single-product firms compete monopolistically in the hi-tech sector with L skilled workers. The second sector is traditional. It employs L_a unskilled workers and is characterized by the perfect competition. The index a indicating the sector is "borrowed" from earlier papers that associate traditional sectors with agriculture. Normalizing workers' wages and productivities to 1 in the traditional sector we outline that the price for the traditional good is also 1.

Two types of workers induce two classes $\mathcal{C}$ or $\mathcal{C}_a$ of consumers that represent hi-tech and traditional sectors respectively. All consumers are endowed by identical preferences. The income of low-skilled workers consists only of their wages. However, high-skilled workers are assumed to possess shares of hi-tech firms in equal parts. These shares and wages constitute their income.

The free entry condition in the hi-tech sector together with money and labor balances ends this brief description of the economy.

2.2 Demand

Let N be the number (specifically, "mass") of varieties of the differentiated product produced in the hi-tech sector. A representative consumer from class $\mathcal{C}$ (or $\mathcal{C}_a$) forms a demand A (or, respectively, A_a) for the products of the traditional sector and a demand $q(x)$ (or, respectively, $q_a(x)$) for specific varieties $x \in [0, N]$ in two stages.

First, relying on a Cobb–Douglas utility function she decides what a part of her income to spend for hi-tech goods. More precisely, let M be the aggregate utility from consumption of the variety of the hi-tech goods. Then a consumer from $\mathcal{C}$ constrained by her income y maximizes

$$U = M^z A^{1-z} \to \max, \quad z \in (0, 1). \tag{1}$$

In consumers' problem (1), z is fixed. In our setting, when figuring out the output, firms do not know the value of z. In order to model the uncertainty in z they introduce a random variable ζ with the support $Z \subset (0, 1)$ and the probability density function $f(z)$.

Returning to (1), we recall that consumers with income y set her spending for hi-tech and traditional goods to zy and $(1-z)y$ respectively. In particular, facing prices $p_a = 1$ for the traditional good A, she chooses

$$A = (1 - z)y.$$

Secondly, she maximizes

$$M = \int_0^N u\big(q(x)\big)\, dx \to \max \tag{2}$$

with respect to continuous function $q(x)$, where four times continuously differentiable function u with $u' > 0$ and $u'' < 0$ represents her lower-tier utility. Given prices $p(x)$ for hi-tech varieties x, her budget constrain is

$$\int_0^N p(x)q(x)\,dx \le zy. \tag{3}$$

We denote

$$\mathcal{E}_f(\varkappa) = \frac{f'(\varkappa)\varkappa}{f(\varkappa)}$$

the elasticity of a function $f(\varkappa)$ depending on the variable $\varkappa$. Then the first order condition (see (Zhelobodko et al., 2012) for details) of problem (2), (3) can be written in the form

$$\mathcal{E}_q(p) = -\sigma(q), \tag{4}$$

where

$$\sigma(q) = -\frac{u'(q)}{qu''(q)} \tag{5}$$

is interpreted as the elasticity of substitution between varieties of the differentiated product. In general, $q = q(x)$ depends on varieties x. The optimal demand turns inequality (3) into equality

$$\int_0^N p(x)q(x)\,dx = zy. \tag{6}$$

2.3 Output decision making

Setting and timing. We recall that firms solving optimization problem (2), (3) find individual demands $q(x)$ and $q_a(x)$ also depending on z, income and prices. Then the profit $\pi(x,z)$ of each firm also depends on z, and firm producing the variety x maximizes its expected profit $\bar{\pi}(x)$: here and further on for any function g of the random variable ζ we use the notation

$$\bar{g} = \mathbf{E}(g) = \int_Z g(z)f(z)\,dz$$

From the assumption that firms choosing their output and deciding upon the wages under uncertain demand will price their varieties when the demand is revealed we can conclude that the outputs and wages are fixed in advance as the best response to all feasible values of the uncertain demand. That is why the output and wages are deterministic, whereas the prices are random as a function of ζ.

A two-stage modeling fits this sequence of events. Firms choose their output and price at the first and second stages respectively. The choice is performed by the backward induction. Firms anticipate the price adjustment occurred at the second stage considering the output as a parameter. This allows to predict an optimal price strategy. With this strategy in mind, firms choose the output at the first stage.

As soon as the supply is found, it can be used to describe the prices. We note that at the first stage, decisions are made under uncertainty, whereas at the second stage, all quantities become deterministic.

This scheme is considered step-by-step. The label x of the varieties is dropped in this section to simplify the notation.

Market clearance at the second stage. At the second stage, given the output s considered as a parameter, firms maximize their (deterministic) profit π equaled to revenue minus costs. Since the optimal prices are to be chosen for a pre-defined supply it follows that the aggregate demand, in general, could deviate from this supply. Then under linear costs the profit is

$$\pi = ps - mws - \varphi w \quad \text{if } s < Q, \tag{7}$$

$$\pi = pQ - mws - \varphi w \quad \text{if } s \geq Q, \tag{8}$$

where w, m, φ and Q are the wage, inverse productivity, fixed cost, and aggregate demand in the hi-tech respectively. The aggregate demand generated by all consumers has the form

$$Q = qL + q_a L_a \tag{9}$$

Let us introduce a *weighted elasticity of substitution* $\mathfrak{S}$

$$\mathfrak{S}(q, q_a) = \frac{qL}{Q}\sigma(q) + \frac{q_a L_a}{Q}\sigma(q_a). \tag{10}$$

Then differentiating (9) with respect to p and using (4) and (**??**) we find the marginal aggregate demand:

$$\mathcal{E}_Q(p) = -\mathfrak{S}(q, q_a). \tag{11}$$

The following proposition establishes that adjusting prices at the second stage, firms predictably clear the market.

Proposition 1. *Let the elasticity of substitution* σ *be greater than* 1. *Then each firm chooses prices that equalize the supply and the aggregate demand:* $s = Q$. *The latter as a function of the prices is given by* (9) *and satisfy* (11).

We recall that at the first stage (when demand is unknown), individuals' demands are random from firms' point of view. Nevertheless, since $s = Q$ and the supply is deterministic, it follows that the aggregate demand given by (9) is also deterministic. In other words, a specific outcome z of the random variable ζ does not affect the aggregate demand Q.

Profit maximization at the first stage. We assume that each firm maximizes its expected profit $\bar{\pi}$. According to Proposition 1, the profit is

$$\pi_z = p_z s - mws - w\varphi \tag{12}$$

for any value $z \in Z$. The dependence of x is dropped here.

We denote $S_1(p, z, N, y)$ a solution of consumers' optimization problem (1)–(3). The market clearance condition $s = Q$ and equation (9) imply that

$$s = S_1(p, z, N, y)L + S_1(p, z, N, y_a)L_a.$$

Let

$$p = S_2(z, s, N, y, y_a) \tag{13}$$

be the solution of the last equation with respect to the prices p. Since the output s and the wages w do not depend on z but prices p do, firms' optimization problem is

$$\bar{\pi} = \left(\int_Z S_2(z, s, N, y, y_a)\, dz\right) s - mws - w\varphi. \tag{14}$$

Labor and money balances. We limit ourself by description of symmetric equilibrium with respect to x. Then the balance of labor describes that the number of workers employed by any firm times the number of the firms is equal to the number of the skilled workers:

$$(ms + \varphi)N = L. \tag{15}$$

Choosing the output, firms equalize the expected profit to zero. However, the observed profit, in general, deviates from zero. So, we include this deviation into the balance of money assuming that the skilled workers possess the shares of the firms in equal parts. Namely, the income of the skilled workers consists of the wages and firm profits (or losses)

$$y = w + \pi N/L, \tag{16}$$

whereas the income of the workers employed in the traditional sector consists only of their wages:

$$y_a = w_a = 1. \tag{17}$$

We assume the free entry condition: firms enter the market until the expected profit falls to zero:

$$\bar{\pi} = 0. \tag{18}$$

Equilibrium. Functions $\{\hat{q}_z, \hat{q}_{a,z}, \hat{p}_z, \hat{\pi}_z\}$ in variable z with the support Z and values $\{\hat{s}, \hat{N}, \hat{w}\}$ constitute a symmetric equilibrium if the following conditions are satisfied:
(i) For any fixed $z \in Z$, $\hat{q}_z = S_1(\hat{p}_z, z, \hat{N}, \hat{y}_z)$, $\hat{q}_{a,z} = S_1(\hat{p}_z, z, \hat{N}, 1)$, where the income y_z is given by equation (16) with $N = \hat{N}$, $w = \hat{w}$, and $\pi = \hat{\pi}_z$;
(ii) For any fixed $z \in Z$, the variables $\hat{s}$ solves optimization problem (14) with $N = \hat{N}$, $y_a = 1$, and y_z given by Equation (16), where $\pi = \hat{\pi}_z$;
(iii) $\hat{p}_z = S_2(z, \hat{s}, \hat{N}, \hat{w} + \hat{\pi}_z\hat{N}/L, 1)$;
(iv) balances (15), (18) are satisfied;
(v) $\hat{\pi}_z$ is given by (12) with $p_z = \hat{p}_z$, $s = \hat{s}$, and $w = \hat{w}$.

Let us introduce some assumptions on individual elasticity of substitution σ and the support Z of distribution of ζ.

Assumption 1. Let the elasticity of substitution σ satisfy the following condition:

$$\sigma'(q) < \frac{m}{2\varphi} \min\{L, L_a\} \tag{19}$$

We distinguish the cases of increasing and decreasing functions σ. If σ decreases we assume that

$$\sigma(q) > 1 \tag{20}$$

If σ increases we assume a stronger condition: there exists $\delta > 0$ such that

$$\sigma'(q)q < \delta \leq \sigma(q) - 1. \tag{21}$$

Assumption 2. Let q and q_a satisfy the first order conditions of firm's optimization problem. Then

$$1/2 < \frac{\sigma(q)}{\sigma(q_a)} < 2. \tag{22}$$

Assumptions 1 and 2 are used to justify that the first order conditions of firm's optimization problem have a unique solution and, respectively, the second order conditions are satisfied. Zhelobodko et al. (2012) introduced an analogue of Assumption 1 to prove the existence and uniqueness of equilibrium in their model without uncertainty. Conditions (19)-(21) are evidently valid for CES preferences when $\sigma' = 0$. Therefore these conditions describe possible deviations from CES preferences. The inequality (19) implies that the economy, which size is measured by L and L_a, is relatively large.

Nevertheless, we could formulate a new Assumption *in model primitives* that leads to Assumption 2.

Assumption 2′. Let the function $\sigma(\kappa)$ be decreasing. Then we assume that the function $\kappa\sigma(\kappa)$ is increasing. Let also the support Z of the distribution of ζ consist of such $z \in [0, 1]$ that

$$\frac{L}{L + L_a} < z < \frac{2L}{2L + L_a}. \tag{23}$$

Assumption $2'$ is technical. It means that a decreasing function σ cannot decrease too quickly. Assumption $2'$ also narrows a feasible range of uncertainty. This range is wider in the economy with a larger traditional sector (L_a/L is bigger). We stress that Assumption $2'$ is sufficient but not necessary: for a broad class of utilities the existence and uniqueness of equilibrium is established without them.

We introduce another technical assumption on the elasticity of substitution σ.

Assumption 3. Put,

$$v(\kappa) = \sigma'(\kappa)\kappa + \sigma(\kappa). \tag{24}$$

We assume that v is a monotonic function.

Proposition 2. *Let Assumptions 1 and 2 or 1 and 2′ be valid. Then a symmetrical equilibrium exists. If, additionally, Assumption 3 is satisfied then* *the equilibrium is unique*[1]. *The following equalities are valid in equilibrium:*

$$Q = s = \frac{\varphi(\mathfrak{S}^* - 1)}{m} \tag{25}$$

$$N = \frac{L}{\varphi \mathfrak{S}^*}, \tag{26}$$

$$p = \frac{z}{1-z} \cdot \frac{m L_a \mathfrak{S}^*}{(\mathfrak{S}^* - 1)L}, \tag{27}$$

$$w = \mathbf{E}\left(\frac{\zeta}{1-\zeta}\right) \cdot \frac{L_a}{L}, \tag{28}$$

$$q = \frac{Qz}{L} \quad \textit{and} \quad q_a = \frac{Q(1-z)}{L_a}. \tag{29}$$

where

$$\mathfrak{S}^* = \mathbf{E}\left(\frac{\zeta}{1-\zeta}\right)\left(\int \frac{z}{1-z} \cdot \frac{f(z)}{\mathfrak{S}(q(z), q_a(z))}\, dz, \right)^{-1}. \tag{30}$$

According to Proposition 2, a general equilibrium still exists under uncertainty. We have introduced sufficient conditions that underlie uniqueness of the equilibrium. They involve monotonicity of the weighted elasticity of substitution between hi-tech goods with respect to z, which represents the share of income spent for them.

Formula (25) represents an equation with respect to s. The right hand side depends on s implicitly. The value z^* of the random variable ζ, which is unknown but fixed, specifies the demands q and q_a that are related to s by equation (9) with $Q = s$.

Formulae (29) can be written with the output s instead of the aggregate demand Q, which does not depend on z. Therefore an ability to increase spending for hi-tech goods at expense of the traditional good (i.e., an increase of z) shifts the demands of the skilled and unskilled workers for hi-tech goods into two opposite directions: q goes up and q_a goes down.

The logic underlying the opposite shifts of the individual demands q and q_a is simple. A price increase for hi-tech goods follows a growth of preferences for them. These changes in preferences and prices differently affect skilled and unskilled workers. Firms' higher profit following larger prices is distributed only among skilled workers. Then under a fixed output, skilled workers demand more hi-tech goods whereas unskilled workers demand less.

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[1] The left inequality in (23) is still required for the uniqueness if the other part of Assumption $2'$ is not satisfied.

3 Comparison with deterministic case

Formulation of the problem. The equilibrium variables are obtained as the best response to all possible values of the random variable ζ. For each its particular value z the determinstic general equilibrium problem can be formulated and solved. We compare the equilibrium under uncertainty with the equilibrium in the deterministic case. The latter case is considered with $z_0 = \mathbf{E}\zeta$. We add the index zero to the equilibrium variables obtained in the deterministic case. Then the outputs and the number of firms are

$$s_0 = \frac{\varphi\big(\mathfrak{S}(\mathbf{E}\zeta) - 1\big)}{m}, \tag{31}$$

$$N_0 = \frac{L}{\varphi\mathfrak{S}(\mathbf{E}\zeta)} \tag{32}$$

These formulae correspond to (25) and (26).

Main result.

Proposition 3. *Let us assume that Assumptions 1 – 2′ are satisfied. If the function $v(q)$ given by (24), decreases then*

$$s < s_0, \quad N > N_0, \quad \frac{\bar{p}}{w} > \frac{p_0}{w_0}. \tag{33}$$

On the contrary, let function $v(q)$ increase and additionally

$$\frac{L}{L_a} > 2\mathcal{E}_\sigma(q) \quad \forall q. \tag{34}$$

Then

$$s > s_0, \quad N < N_0, \quad \frac{\bar{p}}{w} < \frac{p_0}{w_0}. \tag{35}$$

According to proposition 3, in response to an appearance of uncertainty, the outputs and the number of firms drift in opposite directions. Both directions of shift are feasible: either the output decreases and the number of firms increases or the output increases and the number of firms decreases. The type of the response depends on properties of the function $v(q)$.

The model is interpreted if skilled workers get larger income than unskilled workers. This means that $q > q_a$. Then the response of the economy depends on the sign of $d\mathfrak{S}/dz$. This derivative describes a direction in which the weighted elasticity of substitution $\mathfrak{S}$ moves when preferences for hi-tech goods specified by z vary. If $\mathfrak{S}$ and z move co-directionally then a monotonic v increases and (33) is valid. Inequalities (35) are valid in the opposite case. We combine this reasoning into the following Proposition.

Proposition 4. *If* $d\mathfrak{S}/dz$ *is positive then* (33) *is valid. From inequalities* $d\mathfrak{S}/dz < 0$ *and* (34) *it follows that* (35) *is valid.*

Role of the elasticity of substitution. Zhelobodko et al. (2012) reveal an important role of the elasticity of substitution between hi-tech goods in problems concerning responses of the economy to changes in market size. They find that under decreasing elasticity of substitution prices goes down when the economy enlarges.

Our analysis gives additional evidence that the elasticity of substitution affects main economic variables. Even if only the case of decreasing elasticities is relevant, the emergence of uncertainty can lead to both an increase and a decrease of the outputs. Indeed, the function $v(q)$ decreases if, and only if, one of the following two conditions is valid:

$$\begin{cases} \sigma' > 0 \\ \mathcal{E}_{\sigma'} < -2 \end{cases} \quad \text{(D1)} \qquad \text{or} \quad \begin{cases} \sigma' < 0 \\ \mathcal{E}_{\sigma'} > -2 \end{cases} \quad \text{(D2)}.$$

On the contrary, the function $v(q)$ increases when one of other two conditions are valid:

$$\begin{cases} \sigma' > 0 \\ \mathcal{E}_{\sigma'} > -2 \end{cases} \quad \text{(I1)} \qquad \text{or} \quad \begin{cases} \sigma' < 0 \\ \mathcal{E}_{\sigma'} < -2 \end{cases} \quad \text{(I2)}.$$

Utilities

$$u_1(q) = 2\sqrt{2q+1} + \log \frac{\sqrt{2q+1}-1}{\sqrt{2q+1}+1} \tag{36}$$

and

$$u_2(q) = \log\left(q + 1 + \sqrt{q^2+2q}\right). \tag{37}$$

give examples of utilities with the increasing and, respectively, decreasing function $v(q)$.

4 Welfare and CES utilities

Consumers' welfare is understood as an aggregate indirect utility:

$$W = LU + L_a U_a,$$

where U and U_a are the utilities of the skilled and unskilled workers respectively. Assumption about CES-form of consumers' lower-tier preferences:

$$M = \left(\int_0^N q^\gamma(x) dx \right)^{1/\gamma}. \tag{38}$$

simplifies the computation of the welfare. It is important to note that M defined by (38) is not a partial case of (2). Nevertheless form (38), as standard in the monopolistic competition theory, is used here. In particular, this form does not alter the equilibrium prices, outputs, and the number of firms. Under CES-preferences involving (38) the weighted elasticity of substitution $\mathfrak{S}^*$ is equal to $1/(1-\gamma)$, which is independent of the parameter ζ. Then formulae (25), (27), and (26) are still valid (Shapoval and Goncharenko, 2014a) but the influence of uncertainty on the prices, outputs, and the number of firms, which is observed only through $\mathfrak{S}^*$, disappears now. Substituting equilibrium variables into U and U_a we find that for any revealed value z of the parameter ζ the welfare is

$$W(z) = \left(\frac{N^{1/\gamma} Q}{L_a} \right)^z L_a, \tag{39}$$

where N, Q, L_a, and γ does not depend on z. Let $W_0(z_0)$ be the welfare found for the economy without uncertainty, where, now deterministic, $\zeta = z_0$. A standard algebra (see, f.e., **?**) confirms that $W_0(z_0) = W(z_0)$. This is a consequence of the CES-preferences.

We compare the expected welfare $\mathbf{E}(W(\zeta))$ with the welfare found for the economy with some known value of ζ. As in the previous section, the mean value of ζ, $z_0 = \mathbf{E}(\zeta)$, is chosen as a reference point. Since $W(z)$ given by equation (39) is convex, Jensen's inequality leads to the following result.

Proposition 5. *Under CES-preferences the expected welfare is greater than or equal to the welfare found in the deterministic problem with $z_0 = \mathbf{E}(\zeta)$.*

$$\mathbf{E}(W(\zeta)) \geq W_0(\mathbf{E}(\zeta)). \tag{40}$$

Proposition 5 gives evidence that consumers' welfare under uncertain demand can be *greater* than the corresponding welfare in the deterministic case. There are two effects underlying this phenomenon. First, uncertainty, in general, could negatively affect the model outputs chosen under a limited knowledge of demand. However, when consumers are endowed by CES-preferences, the outputs do not depend on the (random) parameter ζ. Therefore at each fixed value z of the parameter ζ, firms choose the same prices and outputs as they would choose in the deterministic case with $\zeta = z$. As a result, values $W(z)$ averaged in the left hand side of inequality (40) coincide with the welfare found in the deterministic economy with the parameter ζ equalled to z_0. In other words, the left hand side of (40) represents averaging of the welfares attained in the deterministic economy over all possible values of the parameter ζ. Then the CES-form of preferences eliminates the effect of uncertainty. The second effect is described by Jensen's inequality in (40): when uncertainty is absent, the averaging is in favor of consumers. Jensen's inequality is based on the Cobb–Douglas form of the upper-tier utility, which leads to the convexity of W as a function of z in equation (39).

Under unspecified lower-tier preferences, uncertainty does affect the outputs, Proposition 3. After observing the demand, firms adjust their prices and improve consequences of the output choice but fail to achieve the best result that is feasible in the deterministic case. The economy exhibits some imperfection and the welfare becomes less that it could be. If imperfection had been absent the expected welfare would exceed $W(\mathbf{E}(\zeta))$ again. Market imperfections decrease the welfare to an ambiguous level that depends on the preferences. The computation of this level is possible only with specifications of the preferences.

5 Conclusion

In the paper we have developed a model of monopolistic competition under demand uncertainty with a general utility at the lower-tier and the Cobb–Douglas preferences at the upper-tier. We recall that merely departing from CES-functions at the lower-tier level leads Zhelobodko et al. (2012) to a general theory of monopolistic competition that explains pro- and anti-competitive price effects. With the Cobb–Douglas upper-tier preferences we avoid a routine algebra but still keep the effects found by Zhelobodko et al. (2012).

There are skilled and unskilled workers in the economy. Their income is type-specific. Acting as consumers the workers are homogeneous. According to (Shapoval and Goncharenko, 2014b), a unique equilibrium in such economy exists for a broad class of utilities. The equilibrium variables are described with the weighted elasticity of substitution $\mathfrak{S}$ between hi-tech goods, which is a function of the equilibrium individual demands q and q_a.

Firms are additionally assumed to be partly informed about the exponents in the Cobb–Douglas preferences, which specify shares ζ and $1-\zeta$ of the income spent for hi-tech and traditional goods. Firms choose the volume of their production before those shares are revealed. When the value z of the share ζ becomes known, the firms adjust prices to clear the market. The individual demands $q(z)$ and $q_a(z)$ depend on the revealed value of ζ. If the weighted elasticity of substitution as a function of q and q_a is different from a constant then changes in z affects $\mathfrak{S}(z) = \mathfrak{S}(q(z), q_a(z))$ through q and q_a, formula (10). The behavior of $\mathfrak{S}(z)$ as a function of z determines the influence of uncertainty on the equilibrium. We establish that emergence of uncertainty can both increase and decrease the equilibrium output. Namely, if $\mathfrak{S}(z)$ monotonically increases then the output goes down in the presence of uncertainty, and vice versa, Proposition 4.

Ireland (1985) surprisingly finds that under some preferences of consumers, the appearance of uncertainty can increase consumers' welfare. Reproducing this effect in our model, we are able to explain it. Indeed, strategies of market agents are chosen before information (which is the value z of the parameter ζ in the model) becomes fully available. This choice performed in advance can be worse than the choice made under revealed value z. Then for each specific z uncertainty either decreases the welfare, $W(z) < W_0(z)$, or leaves it as it is, $W(z) = W_0(z)$. Averaging over all possible values of ζ keeps the inequality between the welfares:

$$\mathbf{E}(W(\zeta) \leq \mathbf{E}(W_0(\zeta)), \tag{41}$$

in which the equality is feasible for CES-preferences, Proposition 3.

Nevertheless, comparison of the *expected* welfare $\mathbf{E}(W(\zeta))$ with the welfare found for revealed *specific* value z of ζ is not well posed. What is this z? The mean $z = \mathbf{E}\zeta$ seems to be an appropriate candidate. Jensen's inequality admits that $\mathbf{E}(W_0(\zeta)) > W_0(\mathbf{E}\zeta)$. Therefore inequality

$$\mathbf{E}(W(\zeta) \leq W_0(\mathbf{E}\zeta) \tag{42}$$

is not guaranteed.

Thus, linking uncertainty to some costs (Ireland, 1985), one reasonably expects a negative influence of uncertainty on the welfare. But this influence is given by inequality (41) not (42).

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